

$R\bar{3}$

No. 148

 $R\bar{3}$
 C_{3i}^2

HEXAGONAL AXES

Generators selected (1); $t(1,0,0)$; $t(0,1,0)$; $t(0,0,1)$; $t(\frac{2}{3},\frac{1}{3},\frac{1}{3})$; (2); (4)

General position

 Multiplicity,
Wyckoff letter,
Site symmetry

Coordinates

 18 f 1

 $(0,0,0)+ (\frac{2}{3},\frac{1}{3},\frac{1}{3})+ (\frac{1}{3},\frac{2}{3},\frac{2}{3})+$

 (1) x,y,z (2) $\bar{y},x-y,z$ (3) $\bar{x}+y,\bar{x},z$
 (4) \bar{x},\bar{y},\bar{z} (5) $y,\bar{x}+y,\bar{z}$ (6) $x-y,x,\bar{z}$
I Maximal translationengleiche subgroups

 [2] $R\bar{3}$ (146) (1; 2; 3)+

 [3] $R\bar{1}$ (2, $P\bar{1}$) (1; 4)+

 $\mathbf{a}, \mathbf{b}, 1/3(-\mathbf{a} - 2\mathbf{b} + \mathbf{c})$
II Maximal klassengleiche subgroups

• Loss of centring translations

| | | |
|----------------------|--|---------------|
| [3] $P\bar{3}$ (147) | 1; 2; 3; 4; 5; 6 | |
| [3] $P\bar{3}$ (147) | 1; 2; 3; (4; 5; 6) + $(\frac{1}{3}, \frac{2}{3}, \frac{2}{3})$ | 2/3, 1/3, 1/3 |
| [3] $P\bar{3}$ (147) | 1; 2; 3; (4; 5; 6) + $(\frac{2}{3}, \frac{1}{3}, \frac{1}{3})$ | 1/3, 2/3, 2/3 |

• Enlarged unit cell

 [2] $\mathbf{a}' = -\mathbf{b}$, $\mathbf{b}' = \mathbf{a} + \mathbf{b}$, $\mathbf{c}' = 2\mathbf{c}$
 $R\bar{3}$ (148) $\langle 2; 4 \rangle$ $-\mathbf{b}, \mathbf{a} + \mathbf{b}, 2\mathbf{c}$
 $R\bar{3}$ (148) $\langle 2; 4 + (0,0,1) \rangle$ $-\mathbf{b}, \mathbf{a} + \mathbf{b}, 2\mathbf{c}$

0, 0, 1/2

 [4] $\mathbf{a}' = -2\mathbf{b}$, $\mathbf{b}' = 2\mathbf{a} + 2\mathbf{b}$
 $R\bar{3}$ (148) $\langle 2; 4 \rangle$ $-2\mathbf{b}, 2\mathbf{a} + 2\mathbf{b}, \mathbf{c}$
 $R\bar{3}$ (148) $\langle 2 + (1, -1, 0); 4 + (2, 0, 0) \rangle$ $-2\mathbf{b}, 2\mathbf{a} + 2\mathbf{b}, \mathbf{c}$

1, 0, 0

 $R\bar{3}$ (148) $\langle 2 + (1, 2, 0); 4 + (0, 2, 0) \rangle$ $-2\mathbf{b}, 2\mathbf{a} + 2\mathbf{b}, \mathbf{c}$

0, 1, 0

 $R\bar{3}$ (148) $\langle 2 + (2, 1, 0); 4 + (2, 2, 0) \rangle$ $-2\mathbf{b}, 2\mathbf{a} + 2\mathbf{b}, \mathbf{c}$

1, 1, 0

• Series of maximal isomorphic subgroups

 [p] $\mathbf{c}' = p\mathbf{c}$
 $R\bar{3}$ (148) $\langle 2; 4 + (0,0,2u) \rangle$ $-\mathbf{b}, \mathbf{a} + \mathbf{b}, p\mathbf{c}$

 0, 0, u
 $p > 4; 0 \leq u < p$
 p conjugate subgroups for prime $p \equiv 2 \pmod{3}$
 $R\bar{3}$ (148) $\langle 2; 4 + (0,0,2u) \rangle$ $\mathbf{a}, \mathbf{b}, p\mathbf{c}$

 0, 0, u
 $p > 6; 0 \leq u < p$
 p conjugate subgroups for prime $p \equiv 1 \pmod{3}$

 [p²] $\mathbf{a}' = -p\mathbf{b}$, $\mathbf{b}' = p\mathbf{a} + p\mathbf{b}$
 $R\bar{3}$ (148) $\langle 2 + (u+v, -u+2v, 0); 4 + (2u, 2v, 0) \rangle$ $-p\mathbf{b}, p\mathbf{a} + p\mathbf{b}, \mathbf{c}$
 $u, v, 0$
 $p > 1; p \equiv 2 \pmod{3}; 0 \leq u < p; 0 \leq v < p$
 p^2 conjugate subgroups for the prime p

 [p = q² + r² - qr] $\mathbf{a}' = (q-r)\mathbf{a} - r\mathbf{b}$, $\mathbf{b}' = r\mathbf{a} + q\mathbf{b}$
 $R\bar{3}$ (148) $\langle 2 + (u, -u, 0); 4 + (2u, 0, 0) \rangle$ $(q-r)\mathbf{a} - r\mathbf{b}, r\mathbf{a} + q\mathbf{b}, \mathbf{c}$
 $u, 0, 0$
 $q > 0; r > 0; q \neq r; q+r \equiv 1 \pmod{3}; p > 6;$
 $0 \leq u < p$
 p conjugate subgroups for each pair of q and r
I Minimal translationengleiche supergroups

 [2] $R\bar{3}m$ (166); [2] $R\bar{3}c$ (167); [4] $Pm\bar{3}$ (200); [4] $Pn\bar{3}$ (201); [4] $Fm\bar{3}$ (202); [4] $Fd\bar{3}$ (203); [4] $Im\bar{3}$ (204); [4] $Pa\bar{3}$ (205); [4] $Ia\bar{3}$ (206)

II Minimal non-isomorphic klassengleiche supergroups

• Additional centring translations

none

• Decreased unit cell

 [3] $\mathbf{a}' = \frac{1}{3}(2\mathbf{a} + \mathbf{b})$, $\mathbf{b}' = \frac{1}{3}(-\mathbf{a} + \mathbf{b})$, $\mathbf{c}' = \frac{1}{3}\mathbf{c}$ $P\bar{3}$ (147)

RHOMBOHEDRAL AXES

Generators selected (1); $t(1,0,0)$; $t(0,1,0)$; $t(0,0,1)$; (2); (4)

General position

Multiplicity,
Wyckoff letter,
Site symmetry

Coordinates

6 *f* 1(1) x, y, z (2) z, x, y (3) y, z, x
(4) $\bar{x}, \bar{y}, \bar{z}$ (5) $\bar{z}, \bar{x}, \bar{y}$ (6) $\bar{y}, \bar{z}, \bar{x}$ I Maximal *translationengleiche* subgroups[2] $R\bar{3}$ (146) 1; 2; 3
[3] $R\bar{1}$ (2, $P\bar{1}$) 1; 4II Maximal *klassengleiche* subgroups

• Loss of centring translations

none

• Enlarged unit cell

[2] $\mathbf{a}' = \mathbf{a} + \mathbf{c}$, $\mathbf{b}' = \mathbf{a} + \mathbf{b}$, $\mathbf{c}' = \mathbf{b} + \mathbf{c}$ $R\bar{3}$ (148) $\langle 2; 4 \rangle$ $\mathbf{a} + \mathbf{c}, \mathbf{a} + \mathbf{b}, \mathbf{b} + \mathbf{c}$ $R\bar{3}$ (148) $\langle 2; 4 + (1, 1, 1) \rangle$ $\mathbf{a} + \mathbf{c}, \mathbf{a} + \mathbf{b}, \mathbf{b} + \mathbf{c}$

1/2, 1/2, 1/2

[3] $\mathbf{a}' = \mathbf{a} - \mathbf{b}$, $\mathbf{b}' = \mathbf{b} - \mathbf{c}$, $\mathbf{c}' = \mathbf{a} + \mathbf{b} + \mathbf{c}$ $P\bar{3}$ (147) $\langle 2; 4 \rangle$ $\mathbf{a} - \mathbf{b}, \mathbf{b} - \mathbf{c}, \mathbf{a} + \mathbf{b} + \mathbf{c}$ $P\bar{3}$ (147) $\langle 2 + (1, -1, 0); 4 + (2, 0, 0) \rangle$ $\mathbf{a} - \mathbf{b}, \mathbf{b} - \mathbf{c}, \mathbf{a} + \mathbf{b} + \mathbf{c}$

1, 0, 0

 $P\bar{3}$ (147) $\langle 2 + (1, 0, -1); 4 + (2, 2, 0) \rangle$ $\mathbf{a} - \mathbf{b}, \mathbf{b} - \mathbf{c}, \mathbf{a} + \mathbf{b} + \mathbf{c}$

1, 1, 0

[4] $\mathbf{a}' = \mathbf{a} - \mathbf{b} + \mathbf{c}$, $\mathbf{b}' = \mathbf{a} + \mathbf{b} - \mathbf{c}$, $\mathbf{c}' = -\mathbf{a} + \mathbf{b} + \mathbf{c}$ $R\bar{3}$ (148) $\langle 2; 4 \rangle$ $\mathbf{a} - \mathbf{b} + \mathbf{c}, \mathbf{a} + \mathbf{b} - \mathbf{c}, -\mathbf{a} + \mathbf{b} + \mathbf{c}$ $R\bar{3}$ (148) $\langle 2 + (1, -2, 1); 4 + (2, -2, 0) \rangle$ $\mathbf{a} - \mathbf{b} + \mathbf{c}, \mathbf{a} + \mathbf{b} - \mathbf{c}, -\mathbf{a} + \mathbf{b} + \mathbf{c}$

1, -1, 0

 $R\bar{3}$ (148) $\langle 2 + (1, 1, -2); 4 + (0, 2, -2) \rangle$ $\mathbf{a} - \mathbf{b} + \mathbf{c}, \mathbf{a} + \mathbf{b} - \mathbf{c}, -\mathbf{a} + \mathbf{b} + \mathbf{c}$

0, 1, -1

 $R\bar{3}$ (148) $\langle 2 + (2, -1, -1); 4 + (2, 0, -2) \rangle$ $\mathbf{a} - \mathbf{b} + \mathbf{c}, \mathbf{a} + \mathbf{b} - \mathbf{c}, -\mathbf{a} + \mathbf{b} + \mathbf{c}$

1, 0, -1

• Series of maximal isomorphic subgroups

[p] $\mathbf{a}' = \frac{1}{3}((p+1)\mathbf{a} + (p-2)\mathbf{b} + (p+1)\mathbf{c})$, $\mathbf{b}' = \frac{1}{3}((p+1)\mathbf{a} + (p+1)\mathbf{b} + (p-2)\mathbf{c})$, $\mathbf{c}' = \frac{1}{3}((p-2)\mathbf{a} + (p+1)\mathbf{b} + (p+1)\mathbf{c})$ $R\bar{3}$ (148) $\langle 2; 4 + (2u, 2u, 2u) \rangle$ $\mathbf{a}' = \frac{1}{3}((p+1)\mathbf{a} \dots$, see lattice relations u, u, u $p > 4; 0 \leq u < p$ p conjugate subgroups for prime $p \equiv 2 \pmod{3}$ [p] $\mathbf{a}' = \frac{1}{3}((p+2)\mathbf{a} + (p-1)\mathbf{b} + (p-1)\mathbf{c})$, $\mathbf{b}' = \frac{1}{3}((p-1)\mathbf{a} + (p+2)\mathbf{b} + (p-1)\mathbf{c})$, $\mathbf{c}' = \frac{1}{3}((p-1)\mathbf{a} + (p-1)\mathbf{b} + (p+2)\mathbf{c})$ $R\bar{3}$ (148) $\langle 2; 4 + (2u, 2u, 2u) \rangle$ $\mathbf{a}' = \frac{1}{3}((p+2)\mathbf{a} \dots$, see lattice relations u, u, u $p > 6; 0 \leq u < p$ p conjugate subgroups for prime $p \equiv 1 \pmod{3}$ [p^2] $\mathbf{a}' = \frac{1}{3}((p+1)\mathbf{a} + (1-2p)\mathbf{b} + (p+1)\mathbf{c})$, $\mathbf{b}' = \frac{1}{3}((p+1)\mathbf{a} + (p+1)\mathbf{b} + (1-2p)\mathbf{c})$, $\mathbf{c}' = \frac{1}{3}((1-2p)\mathbf{a} + (p+1)\mathbf{b} + (p+1)\mathbf{c})$ $R\bar{3}$ (148) $\langle 2 + (u+v, -2u+v, u-2v);$ $\mathbf{a}' = \frac{1}{3}((p+1)\mathbf{a} \dots$, see lattice relations $u, -u+v, -v$ $4 + (2u, -2u+2v, -2v) \rangle$ $p > 1; 0 \leq u < p; 0 \leq v < p$ p^2 conjugate subgroups for prime $p \equiv 2 \pmod{3}$ [$p = q^2 + r^2 - qr$] $\mathbf{a}' = \frac{1}{3}(\alpha\mathbf{a} + \beta\mathbf{b} + \gamma\mathbf{c})$, $\mathbf{b}' = \frac{1}{3}(\gamma\mathbf{a} + \alpha\mathbf{b} + \beta\mathbf{c})$, $\mathbf{c}' = \frac{1}{3}(\beta\mathbf{a} + \gamma\mathbf{b} + \alpha\mathbf{c})$; $\alpha = 2q - r + 1$, $\beta = 1 - q - r$, $\gamma = 2r + 1 - q$ $R\bar{3}$ (148) $\langle 2 + (u, -2u, u); 4 + (2u, -2u, 0) \rangle$ $\mathbf{a}' = \frac{1}{3}(\alpha\mathbf{a} + \beta\mathbf{b} + \dots$, see lattice relations $u, -u, 0$ $q > 0; r > 0; q \neq r; q+r \equiv 1 \pmod{3}; p > 6;$ $0 \leq u < p$ p conjugate subgroups for each pair of q and r I Minimal *translationengleiche* supergroups[2] $R\bar{3}m$ (166); [2] $R\bar{3}c$ (167); [4] $Pm\bar{3}$ (200); [4] $Pn\bar{3}$ (201); [4] $Fm\bar{3}$ (202); [4] $Fd\bar{3}$ (203); [4] $Im\bar{3}$ (204); [4] $Pa\bar{3}$ (205); [4] $Ia\bar{3}$ (206)II Minimal non-isomorphic *klassengleiche* supergroups

• Additional centring translations

none

• Decreased unit cell

[3] $\mathbf{a}' = \frac{1}{3}(2\mathbf{a} - \mathbf{b} - \mathbf{c})$, $\mathbf{b}' = \frac{1}{3}(-\mathbf{a} + 2\mathbf{b} - \mathbf{c})$, $\mathbf{c}' = \frac{1}{3}(\mathbf{a} + \mathbf{b} + \mathbf{c})$ $P\bar{3}$ (147)