

D_3^7
 $R32$

No. 155

 $R32$

HEXAGONAL AXES

Generators selected (1); $t(1,0,0)$; $t(0,1,0)$; $t(0,0,1)$; $t(\frac{2}{3},\frac{1}{3},\frac{1}{3})$; (2); (4)

General position

 Multiplicity,
Wyckoff letter,
Site symmetry

Coordinates

 18 f 1

 $(0,0,0)+ (\frac{2}{3},\frac{1}{3},\frac{1}{3})+ (\frac{1}{3},\frac{2}{3},\frac{2}{3})+$
 (1) x,y,z (2) $\bar{y},x-y,z$ (3) $\bar{x}+y,\bar{x},z$
 (4) y,x,\bar{z} (5) $x-y,\bar{y},\bar{z}$ (6) $\bar{x},\bar{x}+y,\bar{z}$
I Maximal translationengleiche subgroups

[2] $R31$ (146, $R3$)	(1; 2; 3)+	
{ [3] $R12$ (5, $C121$)	(1; 4)+	$1/3(-\mathbf{a}+\mathbf{b}-2\mathbf{c}), -\mathbf{a}-\mathbf{b}, \mathbf{c}$
	(1; 5)+	$1/3(-\mathbf{a}-2\mathbf{b}-2\mathbf{c}), \mathbf{a}, \mathbf{c}$
	(1; 6)+	$1/3(2\mathbf{a}+\mathbf{b}-2\mathbf{c}), \mathbf{b}, \mathbf{c}$

II Maximal klassengleiche subgroups

• Loss of centring translations

{ [3] $P3_221$ (154)	1; 4; (2; 6) + $(\frac{1}{3}, \frac{2}{3}, \frac{2}{3})$; (3; 5) + $(\frac{2}{3}, \frac{1}{3}, \frac{1}{3})$	$2/3, 2/3, 0$
	1; 5; (2; 4) + $(\frac{1}{3}, \frac{2}{3}, \frac{2}{3})$; (3; 6) + $(\frac{2}{3}, \frac{1}{3}, \frac{1}{3})$	$1/3, 0, 1/3$
	1; 6; (2; 5) + $(\frac{1}{3}, \frac{2}{3}, \frac{2}{3})$; (3; 4) + $(\frac{2}{3}, \frac{1}{3}, \frac{1}{3})$	$0, 1/3, 2/3$
{ [3] $P3_121$ (152)	1; 4; (2; 6) + $(\frac{2}{3}, \frac{1}{3}, \frac{1}{3})$; (3; 5) + $(\frac{1}{3}, \frac{2}{3}, \frac{2}{3})$	$1/3, 1/3, 0$
	1; 5; (2; 4) + $(\frac{2}{3}, \frac{1}{3}, \frac{1}{3})$; (3; 6) + $(\frac{1}{3}, \frac{2}{3}, \frac{2}{3})$	$2/3, 0, 2/3$
	1; 6; (2; 5) + $(\frac{2}{3}, \frac{1}{3}, \frac{1}{3})$; (3; 4) + $(\frac{1}{3}, \frac{2}{3}, \frac{2}{3})$	$0, 2/3, 1/3$
{ [3] $P321$ (150)	1; 2; 3; 4; 5; 6	
	1; 2; 3; (4; 5; 6) + $(\frac{1}{3}, \frac{2}{3}, \frac{2}{3})$	$2/3, 1/3, 1/3$
	1; 2; 3; (4; 5; 6) + $(\frac{2}{3}, \frac{1}{3}, \frac{1}{3})$	$1/3, 2/3, 2/3$

• Enlarged unit cell

[2] $\mathbf{a}' = -\mathbf{b}, \mathbf{b}' = \mathbf{a} + \mathbf{b}, \mathbf{c}' = 2\mathbf{c}$		
$R32$ (155)	$\langle 2; 4 \rangle$	$-\mathbf{b}, \mathbf{a} + \mathbf{b}, 2\mathbf{c}$
$R32$ (155)	$\langle 2; 4 + (0, 0, 1) \rangle$	$-\mathbf{b}, \mathbf{a} + \mathbf{b}, 2\mathbf{c}$
[4] $\mathbf{a}' = -2\mathbf{b}, \mathbf{b}' = 2\mathbf{a} + 2\mathbf{b}$		$0, 0, 1/2$
$R32$ (155)	$\langle 2; 4 \rangle$	$-2\mathbf{b}, 2\mathbf{a} + 2\mathbf{b}, \mathbf{c}$
$R32$ (155)	$\langle (2; 4) + (1, -1, 0) \rangle$	$-2\mathbf{b}, 2\mathbf{a} + 2\mathbf{b}, \mathbf{c}$
$R32$ (155)	$\langle 2 + (1, 2, 0); 4 + (-1, 1, 0) \rangle$	$-2\mathbf{b}, 2\mathbf{a} + 2\mathbf{b}, \mathbf{c}$
$R32$ (155)	$\langle 4; 2 + (2, 1, 0) \rangle$	$-2\mathbf{b}, 2\mathbf{a} + 2\mathbf{b}, \mathbf{c}$

• Series of maximal isomorphic subgroups

[p] $\mathbf{c}' = p\mathbf{c}$		
$R32$ (155)	$\langle 2; 4 + (0, 0, 2u) \rangle$ $p > 4; 0 \leq u < p$	$-\mathbf{b}, \mathbf{a} + \mathbf{b}, p\mathbf{c}$
	p conjugate subgroups for prime $p \equiv 2 \pmod{3}$	
$R32$ (155)	$\langle 2; 4 + (0, 0, 2u) \rangle$ $p > 6; 0 \leq u < p$	$\mathbf{a}, \mathbf{b}, p\mathbf{c}$
	p conjugate subgroups for prime $p \equiv 1 \pmod{3}$	
[p^2] $\mathbf{a}' = p\mathbf{a}, \mathbf{b}' = p\mathbf{b}$		
$R32$ (155)	$\langle 2 + (u + v, -u + 2v, 0); 4 + (u - v, -u + v, 0) \rangle$ $p > 1; 0 \leq u < p; 0 \leq v < p$	$-p\mathbf{b}, p\mathbf{a} + p\mathbf{b}, \mathbf{c}$
	p^2 conjugate subgroups for prime $p \equiv 2 \pmod{3}$	
$R32$ (155)	$\langle 2 + (u + v, -u + 2v, 0); 4 + (u - v, -u + v, 0) \rangle$ $p > 6; 0 \leq u < p; 0 \leq v < p$	$p\mathbf{a}, p\mathbf{b}, \mathbf{c}$
	p^2 conjugate subgroups for prime $p \equiv 1 \pmod{3}$	

I Minimal translationengleiche supergroups

 [2] $R\bar{3}m$ (166); [2] $R\bar{3}c$ (167); [4] $P432$ (207); [4] $P4_232$ (208); [4] $F432$ (209); [4] $F4_132$ (210); [4] $I432$ (211); [4] $P4_332$ (212); [4] $P4_132$ (213); [4] $I4_132$ (214)

II Minimal non-isomorphic klassengleiche supergroups

• Additional centring translations

none

• Decreased unit cell

 [3] $\mathbf{a}' = \frac{1}{3}(2\mathbf{a} + \mathbf{b}), \mathbf{b}' = \frac{1}{3}(-\mathbf{a} + \mathbf{b}), \mathbf{c}' = \frac{1}{3}\mathbf{c}$ $P312$ (149)

RHOMBOHEDRAL AXES

Generators selected (1); $t(1,0,0)$; $t(0,1,0)$; $t(0,0,1)$; (2); (4)

General position

Multiplicity,
Wyckoff letter,
Site symmetry

Coordinates

6 f 1

(1) x, y, z (2) z, x, y (3) y, z, x
(4) $\bar{z}, \bar{y}, \bar{x}$ (5) $\bar{y}, \bar{x}, \bar{z}$ (6) $\bar{x}, \bar{z}, \bar{y}$

I Maximal *translationengleiche* subgroups

[2] R31 (146, R3)	1; 2; 3	
{ [3] R12 (5, C121)	1; 4	$-a - c, -a + c, a + b + c$
	1; 5	$-a - b, a - b, a + b + c$
	1; 6	$-b - c, b - c, a + b + c$

II Maximal *klassengleiche* subgroups

• Loss of centring translations

none

• Enlarged unit cell

[2] $a' = a + c, b' = a + b, c' = b + c$		
R32 (155)	$\langle 2; 4 \rangle$	$a + c, a + b, b + c$
R32 (155)	$\langle 2; 4 + (1, 1, 1) \rangle$	$a + c, a + b, b + c$ 1/2, 1/2, 1/2
[3] $a' = a - b, b' = b - c, c' = a + b + c$		
{ P3 ₂ 21 (154)	$\langle 4; 2 + (2, 0, 0) \rangle$	$a - b, b - c, a + b + c$ 2/3, 0, -2/3
	$\langle (2; 4) + (1, 0, 1) \rangle$	$a - b, b - c, a + b + c$ 2/3, 0, 1/3
	$\langle 2 + (1, 1, 0); 4 + (1, 2, 1) \rangle$	$a - b, b - c, a + b + c$ 2/3, 1, 1/3
{ P3 ₁ 21 (152)	$\langle 4; 2 + (1, 0, 0) \rangle$	$a - b, b - c, a + b + c$ 1/3, 0, -1/3
	$\langle 2 + (1, -1, 1); 4 + (2, 0, 2) \rangle$	$a - b, b - c, a + b + c$ 4/3, 0, 2/3
	$\langle 2 + (1, 1, -1); 4 + (0, 2, 0) \rangle$	$a - b, b - c, a + b + c$ 1/3, 1, -1/3
{ P321 (150)	$\langle 2; 4 \rangle$	$a - b, b - c, a + b + c$
	$\langle 2 + (1, -1, 0); 4 + (1, 0, 1) \rangle$	$a - b, b - c, a + b + c$ 1, 0, 0
	$\langle 2 + (1, 0, -1); 4 + (1, 2, 1) \rangle$	$a - b, b - c, a + b + c$ 1, 1, 0
[4] $a' = a - b + c, b' = a + b - c, c' = -a + b + c$		
{ R32 (155)	$\langle 2; 4 \rangle$	$a - b + c, a + b - c, -a + b + c$
	$\langle (2; 4) + (1, -2, 1) \rangle$	$a - b + c, a + b - c, -a + b + c$ 1, -1, 0
	$\langle 2 + (1, 1, -2); 4 + (-1, 2, -1) \rangle$	$a - b + c, a + b - c, -a + b + c$ 0, 1, -1
	$\langle 4; 2 + (2, -1, -1) \rangle$	$a - b + c, a + b - c, -a + b + c$ 1, 0, -1

• Series of maximal isomorphic subgroups

[p] $a' = \frac{1}{3}((p+1)a + (p-2)b + (p+1)c), b' = \frac{1}{3}((p+1)a + (p+1)b + (p-2)c), c' = \frac{1}{3}((p-2)a + (p+1)b + (p+1)c)$		
R32 (155)	$\langle 2; 4 + (2u, 2u, 2u) \rangle$	$a' = \frac{1}{3}((p+1)a \dots, \text{see lattice relations } u, u, u$
	$p > 4; 0 \leq u < p$	
	p conjugate subgroups for prime $p \equiv 2 \pmod{3}$	
[p] $a' = \frac{1}{3}((p+2)a + (p-1)b + (p-1)c), b' = \frac{1}{3}((p-1)a + (p+2)b + (p-1)c), c' = \frac{1}{3}((p-1)a + (p-1)b + (p+2)c)$		
R32 (155)	$\langle 2; 4 + (2u, 2u, 2u) \rangle$	$a' = \frac{1}{3}((p+2)a \dots, \text{see lattice relations } u, u, u$
	$p > 6; 0 \leq u < p$	
	p conjugate subgroups for prime $p \equiv 1 \pmod{3}$	
[p^2] $a' = \frac{1}{3}((p+1)a + (1-2p)b + (p+1)c), b' = \frac{1}{3}((p+1)a + (p+1)b + (1-2p)c), c' = \frac{1}{3}((1-2p)a + (p+1)b + (p+1)c)$		
R32 (155)	$\langle 2 + (u+v, -2u+v, u-2v); 4 + (u-v, -2u+2v, u-v) \rangle$	$a' = \frac{1}{3}((p+1)a \dots, \text{see lattice relations } u, -u+v, -v$
	$p > 1; 0 \leq u < p; 0 \leq v < p$	
	p^2 conjugate subgroups for prime $p \equiv 2 \pmod{3}$	
[p^2] $a' = \frac{1}{3}((2p+1)a + (1-p)b + (1-p)c), b' = \frac{1}{3}((1-p)a + (2p+1)b + (1-p)c), c' = \frac{1}{3}((1-p)a + (1-p)b + (2p+1)c)$		
R32 (155)	$\langle 2 + (u+v, -2u+v, u-2v); 4 + (u-v, -2u+2v, u-v) \rangle$	$a' = \frac{1}{3}((2p+1)a \dots, \text{see lattice relations } u, -u+v, -v$
	$p > 6; 0 \leq u < p; 0 \leq v < p$	
	p^2 conjugate subgroups for prime $p \equiv 1 \pmod{3}$	

I Minimal translationengleiche supergroups

[2] $R\bar{3}m$ (166); [2] $R\bar{3}c$ (167); [4] $P432$ (207); [4] $P4_232$ (208); [4] $F432$ (209); [4] $F4_132$ (210); [4] $I432$ (211); [4] $P4_332$ (212); [4] $P4_132$ (213); [4] $I4_132$ (214)

II Minimal non-isomorphic klassengleiche supergroups

- Additional centring translations

none

- Decreased unit cell

[3] $\mathbf{a}' = \frac{1}{3}(2\mathbf{a} - \mathbf{b} - \mathbf{c})$, $\mathbf{b}' = \frac{1}{3}(-\mathbf{a} + 2\mathbf{b} - \mathbf{c})$, $\mathbf{c}' = \frac{1}{3}(\mathbf{a} + \mathbf{b} + \mathbf{c})$ $P312$ (149)