There are also lemmata for the number of subgroups of a certain index. The most important are:

Lemma 1.2.8.2.4. The number of subgroups of index 2 is $2^N - 1$ with $0 \leq N \leq 6$ for space groups and $0 \leq N \leq 4$ for plane groups. The number of *translationengleiche* subgroups of index 2 is $2^M - 1$ with $0 \leq M \leq 3$ for space groups and $0 \leq M \leq 2$ for plane groups. □

Examples are:

$N = 0: \ 2^0 - 1 = 0$ subgroups of index 2 for $p3$, No. 13, and $F23$, No. 196;

$N = 1: \ 2^1 - 1 = 1$ subgroup of index 2 for $p3m1$, No. 14, and $P3$, No. 143; ...;

$N = 4: \ 2^4 - 1 = 15$ subgroups of index 2 for $p2mm$, No. 6, and $P\overline{1}$, No. 2;

$N = 6: \ 2^6 - 1 = 63$ subgroups of index 2 for $Pmmm$, No. 47.

Lemma 1.2.8.2.5. The number of isomorphic subgroups of each space group is infinite and this applies even to the number of maximal isomorphic subgroups. □

Nevertheless, their listing is possible in the form of infinite series. The series are specified by parameters.

Lemma 1.2.8.2.6. For each space group, each maximal isomorphic subgroup $\mathcal{H}$ can be listed as a member of one of at most four series of maximal isomorphic subgroups. Each member is specified by a set of parameters. □

The series of maximal isomorphic subgroups are discussed in Section 2.1.5.

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