The use of the affine and Euclidean normalizers of a space group $\mathcal{R}$ is described in Part 15 of *IT* A. The *affine normalizer*

$$\mathcal{N} := \mathcal{N}_{\mathcal{A}_n}(\mathcal{R}) = \{g \in \mathcal{A}_n \mid g\mathcal{R}g^{-1} = \mathcal{R}\}$$

of an n-dimensional space group $\mathcal{R} \leq \mathcal{A}_n$ acts on the set of all minimal t-supergroups $\mathcal{S}$ of $\mathcal{R}$ by conjugation.

Theorem 1.4.8.5. $\mathcal{N}$ has finitely many orbits on the set of (minimal) t-supergroups $\mathcal{S}$ of $\mathcal{R}$. □

Proof. Let $\mathcal{S}_1, \ldots, \mathcal{S}_m$ be representatives of the $\mathcal{A}_n$-orbits on the set of n-dimensional space groups, *i.e.* of the types of n-dimensional space groups. For $1 \leq i \leq m$ let

$$\mathcal{M}_i := \{\mathcal{R}_{ij} \mid 1 \leq j \leq a_i\}$$

denote the set of all (maximal) t-subgroups of $\mathcal{S}_i$ that are isomorphic to $\mathcal{R}$.

If $\mathcal{S}$ is a (minimal) t-supergroup of the space group $\mathcal{R}$, then there is some $g \in \mathcal{A}_n$ and $i \in \{1, \ldots, m\}$ such that $g\mathcal{S}g^{-1} = \mathcal{S}_i$ and $g\mathcal{R}g^{-1} = \mathcal{R}_{ij} \in \mathcal{M}_i$, hence the pair of space groups $(\mathcal{R}, \mathcal{S}) = (g^{-1}\mathcal{R}_{ij}g, g^{-1}\mathcal{S}_i g)$ for some $i \in \{1, \ldots, m\}$, $\mathcal{R}_{ij} \in \mathcal{M}_i$ and $g \in \mathcal{A}_n$.

If $\mathcal{S}'$ is a second supergroup of $\mathcal{R}$ and $h \in \mathcal{A}_n$ such that $(\mathcal{R}, \mathcal{S}') = (h^{-1}\mathcal{R}_{ij}h, h^{-1}\mathcal{S}_i h)$ for the same i, j, then $h^{-1}g \in \mathcal{N}$ normalizes $\mathcal{R}$. Hence there are at most $\sum_{i=1}^{m} a_i$ orbits of $\mathcal{N}$ on the set of (minimal) t-supergroups of $\mathcal{R}$. QED

This proof also provides an algorithm to determine representatives of the $\mathcal{N}$-orbits of minimal t-supergroups of a given space group $\mathcal{R}$, provided that one knows representatives of all affine classes of space groups and their maximal t-subgroups. For dimensions 2 and 3 these are given in this volume. Since maximal t-subgroups of three-dimensional space groups have index 2, 3 or 4, this also holds for the minimal t-supergroups of these groups.

Up to dimension $n \leq 4$, the minimal t-supergroups and the minimal k-supergroups of a given space group $\mathcal{R} \leq \mathcal{A}_n$ can be obtained with the commands *TSupergroups* and *KSupergroups* in *CARAT* [see also Heidbüchel (2003)].

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