$R\bar{3}$ No. 148 $R\bar{3}$ C_{3i}^2

HEXAGONAL AXES

Generators selected (1); $t(1,0,0)$; $t(0,1,0)$; $t(0,0,1)$; $t(\frac{2}{3},\frac{1}{3},\frac{1}{3})$; (2); (4)

General position

Multiplicity, Wyckoff letter, Site symmetry	Coordinates
	$(0,0,0)+$ $(\frac{2}{3},\frac{1}{3},\frac{1}{3})+$ $(\frac{1}{3},\frac{2}{3},\frac{2}{3})+$
18 f 1	(1) x,y,z (2) $\bar{y},x-y,z$ (3) $\bar{x}+y,\bar{x},z$ (4) $\bar{x},\bar{y},\bar{z}$ (5) $y,\bar{x}+y,\bar{z}$ (6) $x-y,x,\bar{z}$

I Maximal *translationengleiche* subgroups

[2] $R3$ (146)	(1; 2; 3)+		
[3] $R\bar{1}$ (2, $P\bar{1}$)	(1; 4)+	$\mathbf{a},\mathbf{b},1/3(-\mathbf{a}-2\mathbf{b}+\mathbf{c})$	

II Maximal *klassengleiche* subgroups

- **Loss of centring translations**

[3] $P\bar{3}$ (147)	1; 2; 3; 4; 5; 6		
[3] $P\bar{3}$ (147)	1; 2; 3; (4; 5; 6) + $(\frac{1}{3},\frac{2}{3},\frac{2}{3})$		2/3, 1/3, 1/3
[3] $P\bar{3}$ (147)	1; 2; 3; (4; 5; 6) + $(\frac{2}{3},\frac{1}{3},\frac{1}{3})$		1/3, 2/3, 2/3

- **Enlarged unit cell**

[2] $\mathbf{a}'=-\mathbf{b}$, $\mathbf{b}'=\mathbf{a}+\mathbf{b}$, $\mathbf{c}'=2\mathbf{c}$			
$R\bar{3}$ (148)	$\langle 2;\ 4\rangle$	$-\mathbf{b},\mathbf{a}+\mathbf{b},2\mathbf{c}$	
$R\bar{3}$ (148)	$\langle 2;\ 4+(0,0,1)\rangle$	$-\mathbf{b},\mathbf{a}+\mathbf{b},2\mathbf{c}$	$0,0,1/2$
[4] $\mathbf{a}'=-2\mathbf{b}$, $\mathbf{b}'=2\mathbf{a}+2\mathbf{b}$			
$R\bar{3}$ (148)	$\langle 2;\ 4\rangle$	$-2\mathbf{b},2\mathbf{a}+2\mathbf{b},\mathbf{c}$	
$R\bar{3}$ (148)	$\langle 2+(1,-1,0);\ 4+(2,0,0)\rangle$	$-2\mathbf{b},2\mathbf{a}+2\mathbf{b},\mathbf{c}$	$1,0,0$
$R\bar{3}$ (148)	$\langle 2+(1,2,0);\ 4+(0,2,0)\rangle$	$-2\mathbf{b},2\mathbf{a}+2\mathbf{b},\mathbf{c}$	$0,1,0$
$R\bar{3}$ (148)	$\langle 2+(2,1,0);\ 4+(2,2,0)\rangle$	$-2\mathbf{b},2\mathbf{a}+2\mathbf{b},\mathbf{c}$	$1,1,0$

- **Series of maximal isomorphic subgroups**

$[p]$ $\mathbf{c}'=p\mathbf{c}$			
$R\bar{3}$ (148)	$\langle 2;\ 4+(0,0,2u)\rangle$ prime $p>4$; $0\le u<p$ p conjugate subgroups for $p=6n-1$	$-\mathbf{b},\mathbf{a}+\mathbf{b},p\mathbf{c}$	$0,0,u$
$R\bar{3}$ (148)	$\langle 2;\ 4+(0,0,2u)\rangle$ prime $p>6$; $0\le u<p$ p conjugate subgroups for $p=6n+1$	$\mathbf{a},\mathbf{b},p\mathbf{c}$	$0,0,u$
$[p^2]$ $\mathbf{a}'=-p\mathbf{b}$, $\mathbf{b}'=p\mathbf{a}+p\mathbf{b}$			
$R\bar{3}$ (148)	$\langle 2+(u+v,-u+2v,0);\ 4+(2u,2v,0)\rangle$ p prime; $p=2$ or $p=6n-1$; $0\le u<p$; $0\le v<p$ p^2 conjugate subgroups	$-p\mathbf{b},p\mathbf{a}+p\mathbf{b},\mathbf{c}$	$u,v,0$
$[p=q^2+r^2-qr]$ $\mathbf{a}'=(q-r)\mathbf{a}-r\mathbf{b}$, $\mathbf{b}'=r\mathbf{a}+q\mathbf{b}$			
$R\bar{3}$ (148)	$\langle 2+(u,-u,0);\ 4+(2u,0,0)\rangle$ prime $p=6n+1$; $q>0$; $r>0$; $q\neq r$; $q+r=3n'+1$; $0\le u<p$ p conjugate subgroups for each pair of q and r	$(q-r)\mathbf{a}-r\mathbf{b},r\mathbf{a}+q\mathbf{b},\mathbf{c}$	$u,0,0$

I Minimal *translationengleiche* supergroups

[2] $R\bar{3}m$ (166); [2] $R\bar{3}c$ (167); [4] $Pm\bar{3}$ (200); [4] $Pn\bar{3}$ (201); [4] $Fm\bar{3}$ (202); [4] $Fd\bar{3}$ (203); [4] $Im\bar{3}$ (204); [4] $Pa\bar{3}$ (205); [4] $Ia\bar{3}$ (206)

II Minimal non-isomorphic *klassengleiche* supergroups

- **Additional centring translations** none

- **Decreased unit cell**

[3] $\mathbf{a}'=\frac{1}{3}(2\mathbf{a}+\mathbf{b})$, $\mathbf{b}'=\frac{1}{3}(-\mathbf{a}+\mathbf{b})$, $\mathbf{c}'=\frac{1}{3}\mathbf{c}$ $P\bar{3}$ (147)

C_{3i}^2 $R\bar{3}$ No. 148 $R\bar{3}$

RHOMBOHEDRAL AXES

Generators selected (1); $t(1,0,0)$; $t(0,1,0)$; $t(0,0,1)$; (2); (4)

General position

Multiplicity, Wyckoff letter, Site symmetry			Coordinates		
6	f	1	(1) x,y,z	(2) z,x,y	(3) y,z,x
			(4) $\bar{x},\bar{y},\bar{z}$	(5) $\bar{z},\bar{x},\bar{y}$	(6) $\bar{y},\bar{z},\bar{x}$

I Maximal *translationengleiche* subgroups

[2] $R3$ (146)	1; 2; 3
[3] $R\bar{1}$ (2, $P\bar{1}$)	1; 4

II Maximal *klassengleiche* subgroups

- **Loss of centring translations** none

- **Enlarged unit cell**

[2] $\mathbf{a}' = \mathbf{a}+\mathbf{c}$, $\mathbf{b}' = \mathbf{a}+\mathbf{b}$, $\mathbf{c}' = \mathbf{b}+\mathbf{c}$

$R\bar{3}$ (148)	$\langle 2;\ 4\rangle$	$\mathbf{a}+\mathbf{c},\mathbf{a}+\mathbf{b},\mathbf{b}+\mathbf{c}$	
$R\bar{3}$ (148)	$\langle 2;\ 4+(1,1,1)\rangle$	$\mathbf{a}+\mathbf{c},\mathbf{a}+\mathbf{b},\mathbf{b}+\mathbf{c}$	$1/2,1/2,1/2$

[3] $\mathbf{a}' = \mathbf{a}-\mathbf{b}$, $\mathbf{b}' = \mathbf{b}-\mathbf{c}$, $\mathbf{c}' = \mathbf{a}+\mathbf{b}+\mathbf{c}$

$P\bar{3}$ (147)	$\langle 2;\ 4\rangle$	$\mathbf{a}-\mathbf{b},\mathbf{b}-\mathbf{c},\mathbf{a}+\mathbf{b}+\mathbf{c}$	
$P\bar{3}$ (147)	$\langle 2+(1,-1,0);\ 4+(2,0,0)\rangle$	$\mathbf{a}-\mathbf{b},\mathbf{b}-\mathbf{c},\mathbf{a}+\mathbf{b}+\mathbf{c}$	$1,0,0$
$P\bar{3}$ (147)	$\langle 2+(1,0,-1);\ 4+(2,2,0)\rangle$	$\mathbf{a}-\mathbf{b},\mathbf{b}-\mathbf{c},\mathbf{a}+\mathbf{b}+\mathbf{c}$	$1,1,0$

[4] $\mathbf{a}' = \mathbf{a}-\mathbf{b}+\mathbf{c}$, $\mathbf{b}' = \mathbf{a}+\mathbf{b}-\mathbf{c}$, $\mathbf{c}' = -\mathbf{a}+\mathbf{b}+\mathbf{c}$

$R\bar{3}$ (148)	$\langle 2;\ 4\rangle$	$\mathbf{a}-\mathbf{b}+\mathbf{c},\mathbf{a}+\mathbf{b}-\mathbf{c},-\mathbf{a}+\mathbf{b}+\mathbf{c}$	
$R\bar{3}$ (148)	$\langle 2+(1,-2,1);\ 4+(2,-2,0)\rangle$	$\mathbf{a}-\mathbf{b}+\mathbf{c},\mathbf{a}+\mathbf{b}-\mathbf{c},-\mathbf{a}+\mathbf{b}+\mathbf{c}$	$1,-1,0$
$R\bar{3}$ (148)	$\langle 2+(1,1,-2);\ 4+(0,2,-2)\rangle$	$\mathbf{a}-\mathbf{b}+\mathbf{c},\mathbf{a}+\mathbf{b}-\mathbf{c},-\mathbf{a}+\mathbf{b}+\mathbf{c}$	$0,1,-1$
$R\bar{3}$ (148)	$\langle 2+(2,-1,-1);\ 4+(2,0,-2)\rangle$	$\mathbf{a}-\mathbf{b}+\mathbf{c},\mathbf{a}+\mathbf{b}-\mathbf{c},-\mathbf{a}+\mathbf{b}+\mathbf{c}$	$1,0,-1$

- **Series of maximal isomorphic subgroups**

[p] $\mathbf{a}' = \frac{1}{3}\big((p+1)\mathbf{a}+(p-2)\mathbf{b}+(p+1)\mathbf{c}\big)$, $\mathbf{b}' = \frac{1}{3}\big((p+1)\mathbf{a}+(p+1)\mathbf{b}+(p-2)\mathbf{c}\big)$, $\mathbf{c}' = \frac{1}{3}\big((p-2)\mathbf{a}+(p+1)\mathbf{b}+(p+1)\mathbf{c}\big)$

$R\bar{3}$ (148) $\langle 2;\ 4+(2u,2u,2u)\rangle$ $\mathbf{a}' = \frac{1}{3}\big((p+1)\mathbf{a}\ldots$, see lattice relations
prime $p>4$; $0 \le u < p$ u,u,u
p conjugate subgroups for $p = 6n-1$

[p] $\mathbf{a}' = \frac{1}{3}\big((p+2)\mathbf{a}+(p-1)\mathbf{b}+(p-1)\mathbf{c}\big)$, $\mathbf{b}' = \frac{1}{3}\big((p-1)\mathbf{a}+(p+2)\mathbf{b}+(p-1)\mathbf{c}\big)$, $\mathbf{c}' = \frac{1}{3}\big((p-1)\mathbf{a}+(p-1)\mathbf{b}+(p+2)\mathbf{c}\big)$

$R\bar{3}$ (148) $\langle 2;\ 4+(2u,2u,2u)\rangle$ $\mathbf{a}' = \frac{1}{3}\big((p+2)\mathbf{a}\ldots$, see lattice relations
prime $p>6$; $0 \le u < p$ u,u,u
p conjugate subgroups for $p = 6n+1$

[p^2] $\mathbf{a}' = \frac{1}{3}\big((p+1)\mathbf{a}+(1-2p)\mathbf{b}+(p+1)\mathbf{c}\big)$, $\mathbf{b}' = \frac{1}{3}\big((p+1)\mathbf{a}+(p+1)\mathbf{b}+(1-2p)\mathbf{c}\big)$, $\mathbf{c}' = \frac{1}{3}\big((1-2p)\mathbf{a}+(p+1)\mathbf{b}+(p+1)\mathbf{c}\big)$

$R\bar{3}$ (148) $\langle 2+(u+v,-2u+v,u-2v)$; $\mathbf{a}' = \frac{1}{3}\big((p+1)\mathbf{a}\ldots$, see lattice relations
$4+(2u,-2u+2v,-2v)\rangle$ $u,-u+v,-v$
p prime; $p=2$ or $p=6n-1$; $0 \le u < p$; $0 \le v < p$
p^2 conjugate subgroups

[$p = q^2+r^2-qr$] $\mathbf{a}' = \frac{1}{3}(\alpha\mathbf{a}+\beta\mathbf{b}+\gamma\mathbf{c})$, $\mathbf{b}' = \frac{1}{3}(\gamma\mathbf{a}+\alpha\mathbf{b}+\beta\mathbf{c})$, $\mathbf{c}' = \frac{1}{3}(\beta\mathbf{a}+\gamma\mathbf{b}+\alpha\mathbf{c})$; $\alpha = 2q-r+1$, $\beta = 1-q-r$, $\gamma = 2r+1-q$

$R\bar{3}$ (148) $\langle 2+(u,-2u,u);\ 4+(2u,-2u,0)\rangle$ $\mathbf{a}' = \frac{1}{3}(\alpha\mathbf{a}+\beta\mathbf{b}+\ldots$, see lattice relations
prime $p = 6n+1$; $q>0$; $r>0$; $q \ne r$; $u,-u,0$
$q+r = 3n'+1$; $0 \le u < p$
p conjugate subgroups for each pair of q and r

I Minimal *translationengleiche* supergroups

[2] $R\bar{3}m$ (166); [2] $R\bar{3}c$ (167); [4] $Pm\bar{3}$ (200); [4] $Pn\bar{3}$ (201); [4] $Fm\bar{3}$ (202); [4] $Fd\bar{3}$ (203); [4] $Im\bar{3}$ (204); [4] $Pa\bar{3}$ (205); [4] $Ia\bar{3}$ (206)

II Minimal non-isomorphic *klassengleiche* supergroups

- **Additional centring translations** none

- **Decreased unit cell**

[3] $\mathbf{a}' = \frac{1}{3}(2\mathbf{a}-\mathbf{b}-\mathbf{c})$, $\mathbf{b}' = \frac{1}{3}(-\mathbf{a}+2\mathbf{b}-\mathbf{c})$, $\mathbf{c}' = \frac{1}{3}(\mathbf{a}+\mathbf{b}+\mathbf{c})$ $P\bar{3}$ (147)