$P3_121$ No. 152 $P3_121$ D_3^4

Generators selected (1); $t(1,0,0)$; $t(0,1,0)$; $t(0,0,1)$; (2); (4)

General position

Multiplicity, Wyckoff letter, Site symmetry		Coordinates	
6	c 1	(1) x,y,z	(2) $\bar{y},x-y,z+\frac{1}{3}$
		(3) $\bar{x}+y,\bar{x},z+\frac{2}{3}$	
		(4) $y,x,\bar{z}$	(5) $x-y,\bar{y},\bar{z}+\frac{2}{3}$
		(6) $\bar{x},\bar{x}+y,\bar{z}+\frac{1}{3}$	

I Maximal *translationengleiche* subgroups

[2] $P3_111$ (144, $P3_1$)	1; 2; 3		
[3] $P121$ (5, $C121$)	1; 4	$-\mathbf{a}+\mathbf{b},-\mathbf{a}-\mathbf{b},\mathbf{c}$	
[3] $P121$ (5, $C121$)	1; 5	$-\mathbf{a}-2\mathbf{b},\mathbf{a},\mathbf{c}$	$0,0,1/3$
[3] $P121$ (5, $C121$)	1; 6	$2\mathbf{a}+\mathbf{b},\mathbf{b},\mathbf{c}$	$0,0,2/3$

II Maximal *klassengleiche* subgroups

- **Enlarged unit cell**

[2] $\mathbf{c}'=2\mathbf{c}$			
$P3_221$ (154)	$\langle 4;\ 2+(0,0,1)\rangle$	$\mathbf{a},\mathbf{b},2\mathbf{c}$	
$P3_221$ (154)	$\langle (2;\ 4)+(0,0,1)\rangle$	$\mathbf{a},\mathbf{b},2\mathbf{c}$	$0,0,1/2$
[3] $\mathbf{a}'=3\mathbf{a},\ \mathbf{b}'=3\mathbf{b}$			
$H3_121$ (151, $P3_112$)	$\langle 2;\ 4\rangle$	$\mathbf{a}-\mathbf{b},\mathbf{a}+2\mathbf{b},\mathbf{c}$	$0,0,1/3$
[4] $\mathbf{a}'=2\mathbf{a},\ \mathbf{b}'=2\mathbf{b}$			
$P3_121$ (152)	$\langle 2;\ 4\rangle$	$2\mathbf{a},2\mathbf{b},\mathbf{c}$	
$P3_121$ (152)	$\langle (2;\ 4)+(1,-1,0)\rangle$	$2\mathbf{a},2\mathbf{b},\mathbf{c}$	$1,0,0$
$P3_121$ (152)	$\langle 2+(1,2,0);\ 4+(-1,1,0)\rangle$	$2\mathbf{a},2\mathbf{b},\mathbf{c}$	$0,1,0$
$P3_121$ (152)	$\langle 4;\ 2+(2,1,0)\rangle$	$2\mathbf{a},2\mathbf{b},\mathbf{c}$	$1,1,0$

- **Series of maximal isomorphic subgroups**

[p] $\mathbf{c}'=p\mathbf{c}$			
$P3_221$ (154)	$\langle 2+(0,0,\frac{2p}{3}-\frac{1}{3});\ 4+(0,0,2u)\rangle$ prime $p>4$; $0\le u<p$ p conjugate subgroups for $p=6n-1$	$\mathbf{a},\mathbf{b},p\mathbf{c}$	$0,0,u$
$P3_121$ (152)	$\langle 2+(0,0,\frac{p}{3}-\frac{1}{3});\ 4+(0,0,2u)\rangle$ prime $p>6$; $0\le u<p$ p conjugate subgroups for $p=6n+1$	$\mathbf{a},\mathbf{b},p\mathbf{c}$	$0,0,u$
[p^2] $\mathbf{a}'=p\mathbf{a},\ \mathbf{b}'=p\mathbf{b}$			
$P3_121$ (152)	$\langle 2+(u+v,-u+2v,0);\ 4+(u-v,-u+v,0)\rangle$ prime $p\ne 3$; $0\le u<p$; $0\le v<p$ p^2 conjugate subgroups	$p\mathbf{a},p\mathbf{b},\mathbf{c}$	$u,v,0$

I Minimal *translationengleiche* supergroups

[2] $P6_122$ (178); [2] $P6_422$ (181)

II Minimal non-isomorphic *klassengleiche* supergroups

- **Additional centring translations**

[3] $H3_121$ (151, $P3_112$); [3] $R_{\rm obv}32$ (155, $R32$); [3] $R_{\rm rev}32$ (155, $R32$)

- **Decreased unit cell**

[3] $\mathbf{c}'=\frac{1}{3}\mathbf{c}$ $P321$ (150)

