

D_3^7

R32

No. 155

R32

HEXAGONAL AXES

Generators selected (1); $t(1,0,0)$; $t(0,1,0)$; $t(0,0,1)$; $t(\frac{2}{3}, \frac{1}{3}, \frac{1}{3})$; (2); (4)

General position

Multiplicity,

Wyckoff letter,

Site symmetry

Coordinates

18 f 1(0,0,0)+ $(\frac{2}{3}, \frac{1}{3}, \frac{1}{3})$ + $(\frac{1}{3}, \frac{2}{3}, \frac{2}{3})$ +(1) x, y, z (2) $\bar{y}, x-y, z$ (3) $\bar{x}+y, \bar{x}, z$ (4) y, x, \bar{z} (5) $x-y, \bar{y}, \bar{z}$ (6) $\bar{x}, \bar{x}+y, \bar{z}$ I Maximal *translationengleiche* subgroups

[2] R31 (146, R3)	(1; 2; 3)+	
[3] R12 (5, C121)	(1; 4)+	$1/3(-\mathbf{a}+\mathbf{b}-2\mathbf{c}), -\mathbf{a}-\mathbf{b}, \mathbf{c}$
[3] R12 (5, C121)	(1; 5)+	$1/3(-\mathbf{a}-2\mathbf{b}-2\mathbf{c}), \mathbf{a}, \mathbf{c}$
[3] R12 (5, C121)	(1; 6)+	$1/3(2\mathbf{a}+\mathbf{b}-2\mathbf{c}), \mathbf{b}, \mathbf{c}$

II Maximal *klassengleiche* subgroups

• Loss of centring translations

[3] $P3_221$ (154)	1; 4; (2; 6)+ $(\frac{1}{3}, \frac{2}{3}, \frac{2}{3})$; (3; 5)+ $(\frac{2}{3}, \frac{1}{3}, \frac{1}{3})$	$2/3, 2/3, 0$
[3] $P3_221$ (154)	1; 5; (2; 4)+ $(\frac{1}{3}, \frac{2}{3}, \frac{2}{3})$; (3; 6)+ $(\frac{2}{3}, \frac{1}{3}, \frac{1}{3})$	$1/3, 0, 1/3$
[3] $P3_221$ (154)	1; 6; (2; 5)+ $(\frac{1}{3}, \frac{2}{3}, \frac{2}{3})$; (3; 4)+ $(\frac{2}{3}, \frac{1}{3}, \frac{1}{3})$	$0, 1/3, 2/3$
[3] $P3_121$ (152)	1; 4; (2; 6)+ $(\frac{2}{3}, \frac{1}{3}, \frac{1}{3})$; (3; 5)+ $(\frac{1}{3}, \frac{2}{3}, \frac{2}{3})$	$1/3, 1/3, 0$
[3] $P3_121$ (152)	1; 5; (2; 4)+ $(\frac{2}{3}, \frac{1}{3}, \frac{1}{3})$; (3; 6)+ $(\frac{1}{3}, \frac{2}{3}, \frac{2}{3})$	$2/3, 0, 2/3$
[3] $P3_121$ (152)	1; 6; (2; 5)+ $(\frac{2}{3}, \frac{1}{3}, \frac{1}{3})$; (3; 4)+ $(\frac{1}{3}, \frac{2}{3}, \frac{2}{3})$	$0, 2/3, 1/3$
[3] $P321$ (150)	1; 2; 3; 4; 5; 6	
[3] $P321$ (150)	1; 2; 3; (4; 5; 6)+ $(\frac{1}{3}, \frac{2}{3}, \frac{2}{3})$	$2/3, 1/3, 1/3$
[3] $P321$ (150)	1; 2; 3; (4; 5; 6)+ $(\frac{2}{3}, \frac{1}{3}, \frac{1}{3})$	$1/3, 2/3, 2/3$

• Enlarged unit cell

[2] $\mathbf{a}' = -\mathbf{b}, \mathbf{b}' = \mathbf{a} + \mathbf{b}, \mathbf{c}' = 2\mathbf{c}$		
R32 (155)	$\langle 2; 4 \rangle$	$-\mathbf{b}, \mathbf{a} + \mathbf{b}, 2\mathbf{c}$
R32 (155)	$\langle 2; 4 + (0, 0, 1) \rangle$	$-\mathbf{b}, \mathbf{a} + \mathbf{b}, 2\mathbf{c}$
[4] $\mathbf{a}' = -2\mathbf{b}, \mathbf{b}' = 2\mathbf{a} + 2\mathbf{b}$		
R32 (155)	$\langle 2; 4 \rangle$	$-2\mathbf{b}, 2\mathbf{a} + 2\mathbf{b}, \mathbf{c}$
R32 (155)	$\langle (2; 4) + (1, -1, 0) \rangle$	$-2\mathbf{b}, 2\mathbf{a} + 2\mathbf{b}, \mathbf{c}$
R32 (155)	$\langle 2 + (1, 2, 0); 4 + (-1, 1, 0) \rangle$	$-2\mathbf{b}, 2\mathbf{a} + 2\mathbf{b}, \mathbf{c}$
R32 (155)	$\langle 4; 2 + (2, 1, 0) \rangle$	$-2\mathbf{b}, 2\mathbf{a} + 2\mathbf{b}, \mathbf{c}$

• Series of maximal isomorphic subgroups

[p] $\mathbf{c}' = p\mathbf{c}$		
R32 (155)	$\langle 2; 4 + (0, 0, 2u) \rangle$	$-\mathbf{b}, \mathbf{a} + \mathbf{b}, p\mathbf{c}$
	prime $p > 4; 0 \leq u < p$	$0, 0, u$
	p conjugate subgroups for $p = 6n - 1$	
R32 (155)	$\langle 2; 4 + (0, 0, 2u) \rangle$	$\mathbf{a}, \mathbf{b}, p\mathbf{c}$
	prime $p > 6; 0 \leq u < p$	$0, 0, u$
	p conjugate subgroups for $p = 6n + 1$	
[p^2] $\mathbf{a}' = p\mathbf{a}, \mathbf{b}' = p\mathbf{b}$		
R32 (155)	$\langle 2 + (u+v, -u+2v, 0); 4 + (u-v, -u+v, 0) \rangle$	$-p\mathbf{b}, p\mathbf{a} + p\mathbf{b}, \mathbf{c}$
	p prime; $0 \leq u < p; 0 \leq v < p$	$u, v, 0$
	p^2 conjugate subgroups for $p = 2$ or $p = 6n - 1$	
R32 (155)	$\langle 2 + (u+v, -u+2v, 0); 4 + (u-v, -u+v, 0) \rangle$	$p\mathbf{a}, p\mathbf{b}, \mathbf{c}$
	prime $p > 6; 0 \leq u < p; 0 \leq v < p$	$u, v, 0$
	p^2 conjugate subgroups for $p = 6n + 1$	

I Minimal *translationengleiche* supergroups[2] $R\bar{3}m$ (166); [2] $R\bar{3}c$ (167); [4] $P432$ (207); [4] $P4_232$ (208); [4] $F432$ (209); [4] $F4_132$ (210); [4] $I432$ (211); [4] $P4_332$ (212); [4] $P4_132$ (213); [4] $I4_132$ (214)II Minimal non-isomorphic *klassengleiche* supergroups

• Additional centring translations

none

• Decreased unit cell

[3] $\mathbf{a}' = \frac{1}{3}(2\mathbf{a} + \mathbf{b}), \mathbf{b}' = \frac{1}{3}(-\mathbf{a} + \mathbf{b}), \mathbf{c}' = \frac{1}{3}\mathbf{c}$ $P312$ (149)

RHOMBOHEDRAL AXES

Generators selected (1); $t(1,0,0)$; $t(0,1,0)$; $t(0,0,1)$; (2); (4)

General position

Multiplicity,
Wyckoff letter,
Site symmetry

Coordinates

6 f 1

(1) x, y, z (2) z, x, y (3) y, z, x
(4) $\bar{z}, \bar{y}, \bar{x}$ (5) $\bar{y}, \bar{x}, \bar{z}$ (6) $\bar{x}, \bar{z}, \bar{y}$

I Maximal translationengleiche subgroups

[2] R31 (146, R3)	1; 2; 3	
{ [3] R12 (5, C121)	1; 4	$-\mathbf{a} - \mathbf{c}, -\mathbf{a} + \mathbf{c}, \mathbf{a} + \mathbf{b} + \mathbf{c}$
[3] R12 (5, C121)	1; 5	$-\mathbf{a} - \mathbf{b}, \mathbf{a} - \mathbf{b}, \mathbf{a} + \mathbf{b} + \mathbf{c}$
{ [3] R12 (5, C121)	1; 6	$-\mathbf{b} - \mathbf{c}, \mathbf{b} - \mathbf{c}, \mathbf{a} + \mathbf{b} + \mathbf{c}$

II Maximal klassengleiche subgroups• **Loss of centring translations**

none

• **Enlarged unit cell**[2] $\mathbf{a}' = \mathbf{a} + \mathbf{c}, \mathbf{b}' = \mathbf{a} + \mathbf{b}, \mathbf{c}' = \mathbf{b} + \mathbf{c}$ R32 (155) $\langle 2; 4 \rangle$ $\mathbf{a} + \mathbf{c}, \mathbf{a} + \mathbf{b}, \mathbf{b} + \mathbf{c}$ R32 (155) $\langle 2; 4 + (1, 1, 1) \rangle$ $\mathbf{a} + \mathbf{c}, \mathbf{a} + \mathbf{b}, \mathbf{b} + \mathbf{c}$

1/2, 1/2, 1/2

[3] $\mathbf{a}' = \mathbf{a} - \mathbf{b}, \mathbf{b}' = \mathbf{b} - \mathbf{c}, \mathbf{c}' = \mathbf{a} + \mathbf{b} + \mathbf{c}$ $\left\{ \begin{array}{l} P3_221 (154) \\ P3_221 (154) \\ P3_221 (154) \end{array} \right. \langle 4; 2 + (2, 0, 0) \rangle$ $\mathbf{a} - \mathbf{b}, \mathbf{b} - \mathbf{c}, \mathbf{a} + \mathbf{b} + \mathbf{c}$

2/3, 0, -2/3

 $\left\{ \begin{array}{l} P3_221 (154) \\ P3_221 (154) \end{array} \right. \langle (2; 4) + (1, 0, 1) \rangle$ $\mathbf{a} - \mathbf{b}, \mathbf{b} - \mathbf{c}, \mathbf{a} + \mathbf{b} + \mathbf{c}$

2/3, 0, 1/3

 $\left\{ \begin{array}{l} P3_221 (154) \\ P3_221 (154) \end{array} \right. \langle 2 + (1, 1, 0); 4 + (1, 2, 1) \rangle$ $\mathbf{a} - \mathbf{b}, \mathbf{b} - \mathbf{c}, \mathbf{a} + \mathbf{b} + \mathbf{c}$

2/3, 1, 1/3

 $\left\{ \begin{array}{l} P3_121 (152) \\ P3_121 (152) \end{array} \right. \langle 4; 2 + (1, 0, 0) \rangle$ $\mathbf{a} - \mathbf{b}, \mathbf{b} - \mathbf{c}, \mathbf{a} + \mathbf{b} + \mathbf{c}$

1/3, 0, -1/3

 $\left\{ \begin{array}{l} P3_121 (152) \\ P3_121 (152) \end{array} \right. \langle 2 + (1, -1, 1); 4 + (2, 0, 2) \rangle$ $\mathbf{a} - \mathbf{b}, \mathbf{b} - \mathbf{c}, \mathbf{a} + \mathbf{b} + \mathbf{c}$

4/3, 0, 2/3

 $\left\{ \begin{array}{l} P3_121 (152) \\ P3_121 (152) \end{array} \right. \langle 2 + (1, 1, -1); 4 + (0, 2, 0) \rangle$ $\mathbf{a} - \mathbf{b}, \mathbf{b} - \mathbf{c}, \mathbf{a} + \mathbf{b} + \mathbf{c}$

1/3, 1, -1/3

 $\left\{ \begin{array}{l} P321 (150) \\ P321 (150) \end{array} \right. \langle 2; 4 \rangle$ $\mathbf{a} - \mathbf{b}, \mathbf{b} - \mathbf{c}, \mathbf{a} + \mathbf{b} + \mathbf{c}$

1, 0, 0

 $\left\{ \begin{array}{l} P321 (150) \\ P321 (150) \end{array} \right. \langle 2 + (1, -1, 0); 4 + (1, 0, 1) \rangle$ $\mathbf{a} - \mathbf{b}, \mathbf{b} - \mathbf{c}, \mathbf{a} + \mathbf{b} + \mathbf{c}$

1, 0, 0

 $\left\{ \begin{array}{l} P321 (150) \\ P321 (150) \end{array} \right. \langle 2 + (1, 0, -1); 4 + (1, 2, 1) \rangle$ $\mathbf{a} - \mathbf{b}, \mathbf{b} - \mathbf{c}, \mathbf{a} + \mathbf{b} + \mathbf{c}$

1, 1, 0

[4] $\mathbf{a}' = \mathbf{a} - \mathbf{b} + \mathbf{c}, \mathbf{b}' = \mathbf{a} + \mathbf{b} - \mathbf{c}, \mathbf{c}' = -\mathbf{a} + \mathbf{b} + \mathbf{c}$ $\left\{ \begin{array}{l} R32 (155) \\ R32 (155) \end{array} \right. \langle 2; 4 \rangle$ $\mathbf{a} - \mathbf{b} + \mathbf{c}, \mathbf{a} + \mathbf{b} - \mathbf{c}, -\mathbf{a} + \mathbf{b} + \mathbf{c}$ $\left\{ \begin{array}{l} R32 (155) \\ R32 (155) \end{array} \right. \langle (2; 4) + (1, -2, 1) \rangle$ $\mathbf{a} - \mathbf{b} + \mathbf{c}, \mathbf{a} + \mathbf{b} - \mathbf{c}, -\mathbf{a} + \mathbf{b} + \mathbf{c}$

1, -1, 0

 $\left\{ \begin{array}{l} R32 (155) \\ R32 (155) \end{array} \right. \langle 2 + (1, 1, -2); 4 + (-1, 2, -1) \rangle$ $\mathbf{a} - \mathbf{b} + \mathbf{c}, \mathbf{a} + \mathbf{b} - \mathbf{c}, -\mathbf{a} + \mathbf{b} + \mathbf{c}$

0, 1, -1

 $\left\{ \begin{array}{l} R32 (155) \\ R32 (155) \end{array} \right. \langle 4; 2 + (2, -1, -1) \rangle$ $\mathbf{a} - \mathbf{b} + \mathbf{c}, \mathbf{a} + \mathbf{b} - \mathbf{c}, -\mathbf{a} + \mathbf{b} + \mathbf{c}$

1, 0, -1

• **Series of maximal isomorphic subgroups**[p] $\mathbf{a}' = \frac{1}{3}((p+1)\mathbf{a} + (p-2)\mathbf{b} + (p+1)\mathbf{c}), \mathbf{b}' = \frac{1}{3}((p+1)\mathbf{a} + (p+1)\mathbf{b} + (p-2)\mathbf{c}), \mathbf{c}' = \frac{1}{3}((p-2)\mathbf{a} + (p+1)\mathbf{b} + (p+1)\mathbf{c})$ R32 (155) $\langle 2; 4 + (2u, 2u, 2u) \rangle$ $\mathbf{a}' = \frac{1}{3}((p+1)\mathbf{a} \dots, \text{ see lattice relations}$ u, u, u prime $p > 4; 0 \leq u < p$ p conjugate subgroups for $p = 6n - 1$ [p] $\mathbf{a}' = \frac{1}{3}((p+2)\mathbf{a} + (p-1)\mathbf{b} + (p-1)\mathbf{c}), \mathbf{b}' = \frac{1}{3}((p-1)\mathbf{a} + (p+2)\mathbf{b} + (p-1)\mathbf{c}), \mathbf{c}' = \frac{1}{3}((p-1)\mathbf{a} + (p-1)\mathbf{b} + (p+2)\mathbf{c})$ R32 (155) $\langle 2; 4 + (2u, 2u, 2u) \rangle$ $\mathbf{a}' = \frac{1}{3}((p+2)\mathbf{a} \dots, \text{ see lattice relations}$ u, u, u prime $p > 6; 0 \leq u < p$ p conjugate subgroups for $p = 6n + 1$ [p^2] $\mathbf{a}' = \frac{1}{3}((p+1)\mathbf{a} + (1-2p)\mathbf{b} + (p+1)\mathbf{c}), \mathbf{b}' = \frac{1}{3}((p+1)\mathbf{a} + (p+1)\mathbf{b} + (1-2p)\mathbf{c}), \mathbf{c}' = \frac{1}{3}((1-2p)\mathbf{a} + (p+1)\mathbf{b} + (p+1)\mathbf{c})$ R32 (155) $\langle 2 + (u+v, -2u+v, u-2v);$ $\mathbf{a}' = \frac{1}{3}((p+1)\mathbf{a} \dots, \text{ see lattice relations}$ $u, -u+v, -v$ $4 + (u-v, -2u+2v, u-v) \rangle$ p prime; $0 \leq u < p; 0 \leq v < p$ p^2 conjugate subgroups for $p = 2$ or $p = 6n - 1$ [p^2] $\mathbf{a}' = \frac{1}{3}((2p+1)\mathbf{a} + (1-p)\mathbf{b} + (1-p)\mathbf{c}), \mathbf{b}' = \frac{1}{3}((1-p)\mathbf{a} + (2p+1)\mathbf{b} + (1-p)\mathbf{c}), \mathbf{c}' = \frac{1}{3}((1-p)\mathbf{a} + (1-p)\mathbf{b} + (2p+1)\mathbf{c})$ R32 (155) $\langle 2 + (u+v, -2u+v, u-2v);$ $\mathbf{a}' = \frac{1}{3}((2p+1)\mathbf{a} \dots, \text{ see lattice relations}$ $u, -u+v, -v$ $4 + (u-v, -2u+2v, u-v) \rangle$ prime $p > 6; 0 \leq u < p; 0 \leq v < p$ p^2 conjugate subgroups for $p = 6n + 1$

I Minimal *translationengleiche* supergroups

[2] $R\bar{3}m$ (166); [2] $R\bar{3}c$ (167); [4] $P432$ (207); [4] $P4_232$ (208); [4] $F432$ (209); [4] $F4_132$ (210); [4] $I432$ (211); [4] $P4_332$ (212); [4] $P4_132$ (213); [4] $I4_132$ (214)

II Minimal non-isomorphic *klassengleiche* supergroups

- Additional centring translations

none

- Decreased unit cell

[3] $\mathbf{a}' = \frac{1}{3}(2\mathbf{a} - \mathbf{b} - \mathbf{c})$, $\mathbf{b}' = \frac{1}{3}(-\mathbf{a} + 2\mathbf{b} - \mathbf{c})$, $\mathbf{c}' = \frac{1}{3}(\mathbf{a} + \mathbf{b} + \mathbf{c})$ $P312$ (149)