International Tables for Crystallography (2006). Vol. A, Table 15.2.1.1, p. 881.

Table 15.2.1.1. *Euclidean normalizers of the plane groups*

For the restrictions of the cell metric of the two oblique plane groups see text and Fig. 15.2.1.3.

Plane group $\mathcal{G}$			Euclidean normalizer $\mathcal{N}_{\mathcal{E}}(\mathcal{G})$		Additional generators of $\mathcal{N}_{\mathcal{E}}(\mathcal{G})$			
No.	Hermann–Mauguin symbol	Cell metric	Symbol	Basis vectors	Translations	Twofold rotation	Further generators	Index of $\mathcal{G}$ in $\mathcal{N}_{\mathcal{E}}(\mathcal{G})$
1	$p1$	General	p^22	$\varepsilon_1\mathbf{a},\ \varepsilon_2\mathbf{b}$	$r, 0; 0, s$	$-x, -y$		$\infty^2 \cdot 2 \cdot 1$
		$a < b,\ \gamma = 90°$	p^22mm	$\varepsilon_1\mathbf{a},\ \varepsilon_2\mathbf{b}$	$r, 0; 0, s$	$-x, -y$	$-x,\ y$	$\infty^2 \cdot 2 \cdot 2$
		$2\cos\gamma = -a/b$, $90 < \gamma < 120°$	c^22mm	$\varepsilon_1\mathbf{a},\ \varepsilon_2(\frac{1}{2}\mathbf{a} + \mathbf{b})$	$r, 0; 0, s$	$-x, -y$	$x - y, -y$	$\infty^2 \cdot 2 \cdot 2$
		$a = b$, $90 < \gamma < 120°$	c^22mm	$\varepsilon_1(\mathbf{a} - \mathbf{b}),\ \varepsilon_2(\mathbf{a} + \mathbf{b})$	$r, 0; 0, s$	$-x, -y$	y, x	$\infty^2 \cdot 2 \cdot 2$
		$a = b, \gamma = 90°$	p^24mm	$\varepsilon\mathbf{a},\ \varepsilon\mathbf{b}$	$r, 0; 0, s$	$-x, -y$	$-x,\ y;\ y,\ x$	$\infty^2 \cdot 2 \cdot 4$
		$a = b, \gamma = 120°$	p^26mm	$\varepsilon\mathbf{a},\ \varepsilon\mathbf{b}$	$r, 0; 0, s$	$-x, -y$	$y,\ x;\ x,\ x - y$	$\infty^2 \cdot 2 \cdot 6$
2	$p2$	General	$p2$	$\frac{1}{2}\mathbf{a}, \frac{1}{2}\mathbf{b}$	$\frac{1}{2}, 0; 0, \frac{1}{2}$			$4 \cdot 1 \cdot 1$
		$a < b, \gamma = 90°$	$p2mm$	$\frac{1}{2}\mathbf{a}, \frac{1}{2}\mathbf{b}$	$\frac{1}{2}, 0; 0, \frac{1}{2}$		$-x,\ y$	$4 \cdot 1 \cdot 2$
		$2\cos\gamma = -a/b$, $90 < \gamma < 120°$	$c2mm$	$\frac{1}{2}\mathbf{a}, \frac{1}{2}\mathbf{a} + \mathbf{b}$	$\frac{1}{2}, 0; 0, \frac{1}{2}$		$x - y,\ -y$	$4 \cdot 1 \cdot 2$
		$a = b$, $90 < \gamma < 120°$	$c2mm$	$\frac{1}{2}(\mathbf{a} - \mathbf{b}), \frac{1}{2}(\mathbf{a} + \mathbf{b})$	$\frac{1}{2}, 0; 0, \frac{1}{2}$		$y,\ x$	$4 \cdot 1 \cdot 2$
		$a = b, \gamma = 90°$	$p4mm$	$\frac{1}{2}\mathbf{a}, \frac{1}{2}\mathbf{b}$	$\frac{1}{2}, 0; 0, \frac{1}{2}$		$-x,\ y;\ y,\ x$	$4 \cdot 1 \cdot 4$
		$a = b, \gamma = 120°$	$p6mm$	$\frac{1}{2}\mathbf{a}, \frac{1}{2}\mathbf{b}$	$\frac{1}{2}, 0; 0, \frac{1}{2}$		$y,\ x;\ x,\ x - y$	$4 \cdot 1 \cdot 6$
3	$p1m1$		p^12mm	$\frac{1}{2}\mathbf{a},\ \varepsilon\mathbf{b}$	$\frac{1}{2}, 0; 0, s$	$-x, -y$		$(2 \cdot \infty) \cdot 2 \cdot 1$
4	$p1g1$		p^12mm	$\frac{1}{2}\mathbf{a},\ \varepsilon\mathbf{b}$	$\frac{1}{2}, 0; 0, s$	$-x, -y$		$(2 \cdot \infty) \cdot 2 \cdot 1$
5	$c1m1$		p^12mm	$\frac{1}{2}\mathbf{a},\ \varepsilon\mathbf{b}$	$0, s$	$-x, -y$		$\infty \cdot 2 \cdot 1$
6	$p2mm$	$a \neq b$	$p2mm$	$\frac{1}{2}\mathbf{a}, \frac{1}{2}\mathbf{b}$	$\frac{1}{2}, 0; 0, \frac{1}{2}$			$4 \cdot 1 \cdot 1$
		$a = b$	$p4mm$	$\frac{1}{2}\mathbf{a}, \frac{1}{2}\mathbf{b}$	$\frac{1}{2}, 0; 0, \frac{1}{2}$		y, x	$4 \cdot 1 \cdot 2$
7	$p2mg$		$p2mm$	$\frac{1}{2}\mathbf{a}, \frac{1}{2}\mathbf{b}$	$\frac{1}{2}, 0; 0, \frac{1}{2}$			$4 \cdot 1 \cdot 1$
8	$p2gg$	$a \neq b$	$p2mm$	$\frac{1}{2}\mathbf{a}, \frac{1}{2}\mathbf{b}$	$\frac{1}{2}, 0; 0, \frac{1}{2}$			$4 \cdot 1 \cdot 1$
		$a = b$	$p4mm$	$\frac{1}{2}\mathbf{a}, \frac{1}{2}\mathbf{b}$	$\frac{1}{2}, 0; 0, \frac{1}{2}$		y, x	$4 \cdot 1 \cdot 2$
9	$c2mm$	$a \neq b$	$p2mm$	$\frac{1}{2}\mathbf{a}, \frac{1}{2}\mathbf{b}$	$\frac{1}{2}, 0$			$2 \cdot 1 \cdot 1$
		$a = b$	$p4mm$	$\frac{1}{2}\mathbf{a}, \frac{1}{2}\mathbf{b}$	$\frac{1}{2}, 0$		y, x	$2 \cdot 1 \cdot 2$
10	$p4$		$p4mm$	$\frac{1}{2}(\mathbf{a} - \mathbf{b}), \frac{1}{2}(\mathbf{a} + \mathbf{b})$	$\frac{1}{2}, \frac{1}{2}$		y, x	$2 \cdot 1 \cdot 2$
11	$p4mm$		$p4mm$	$\frac{1}{2}(\mathbf{a} - \mathbf{b}), \frac{1}{2}(\mathbf{a} + \mathbf{b})$	$\frac{1}{2}, \frac{1}{2}$			$2 \cdot 1 \cdot 1$
12	$p4gm$		$p4mm$	$\frac{1}{2}(\mathbf{a} - \mathbf{b}), \frac{1}{2}(\mathbf{a} + \mathbf{b})$	$\frac{1}{2}, \frac{1}{2}$			$2 \cdot 1 \cdot 1$
13	$p3$		$p6mm$	$\frac{1}{3}(2\mathbf{a} + \mathbf{b}), \frac{1}{3}(-\mathbf{a} + \mathbf{b})$	$\frac{2}{3}, \frac{1}{3}$	$-x, -y$	y, x	$3 \cdot 2 \cdot 2$
14	$p3m1$		$p6mm$	$\frac{1}{3}(2\mathbf{a} + \mathbf{b}), \frac{1}{3}(-\mathbf{a} + \mathbf{b})$	$\frac{2}{3}, \frac{1}{3}$	$-x, -y$		$3 \cdot 2 \cdot 1$
15	$p31m$		$p6mm$	$\mathbf{a}, \mathbf{b}$		$-x, -y$		$1 \cdot 2 \cdot 1$
16	$p6$		$p6mm$	$\mathbf{a}, \mathbf{b}$			y, x	$1 \cdot 1 \cdot 2$
17	$p6mm$		$p6mm$	$\mathbf{a}, \mathbf{b}$				$1 \cdot 1 \cdot 1$

Laue class of $\mathcal{G}$. If $\mathcal{N}_{\mathcal{E}}(\mathcal{G})$ is noncentrosymmetric, an intermediate group $\mathcal{L}(\mathcal{G})$ cannot exist.

The groups $\mathcal{K}(\mathcal{G})$ and $\mathcal{L}(\mathcal{G})$ are of special interest in connection with direct methods for structure determination: they cause the parity classes of reflections; $\mathcal{K}(\mathcal{G})$ defines the permissible origin shifts and the parameter ranges for the phase restrictions in the specification of the origin; and $\mathcal{L}(\mathcal{G})$ gives information on possible phase restrictions for the selection of the enantiomorph. For any space (plane) group $\mathcal{G}$, the translation subgroups of $\mathcal{K}(\mathcal{G})$, $\mathcal{L}(\mathcal{G})$, $\mathcal{N}_{\mathcal{E}}(\mathcal{G})$ and even $\mathcal{N}_{\mathcal{A}}(\mathcal{G})$ coincide.

The Euclidean normalizers of the plane groups are listed in Table 15.2.1.1, those of triclinic space groups in Table 15.2.1.2, of monoclinic and orthorhombic space groups in Table 15.2.1.3, and those of all other space groups in Table 15.2.1.4. Herein all settings and choices of cell and origin as tabulated in Parts 6 and 7 are taken into account and, in addition, all metrical specializations giving rise to Euclidean normalizers with enhanced symmetry. Each setting, cell choice, origin or metrical specialization corresponds to one line in the tables. (Exceptions are some orthorhombic space groups with tetragonal metric: if $a = b$ as well as $b = c$ and $c = a$ give rise to a symmetry enhancement of the Euclidean normalizer, only the case $a = b$ is listed in Table 15.2.1.3.)

The first column of Tables 15.2.1.1, 15.2.1.3 and 15.2.1.4 shows the number of the plane group or space group, the second column its Hermann–Mauguin symbol together with information on the setting, cell choice and origin, if necessary. Special metrical conditions affecting the Euclidean normalizer are tabulated in the third column of Tables 15.2.1.1 and 15.2.1.3. The term 'general' means that only the general metrical conditions for the respective crystal system are valid. In Table 15.2.1.4, a corresponding column is superfluous because here a metrical specialization of the space group does not influence the type of the Euclidean normalizer.

The Euclidean normalizer of the space (plane) group is identified in the fourth column of Table 15.2.1.3 (15.2.1.1) or in the third column of Table 15.2.1.4. As Euclidean normalizers are groups of motions, they can normally be designated by Hermann–Mauguin symbols. If, however, the origin of the space (plane) group is not fixed by symmetry (examples: $P4$, $P1m1$, $P1$), the Euclidean normalizer contains continuous translations in one, two or three (one or two) independent directions. In these cases, P^1, B^1, C^1, P^2 or

references