

15.4. Normalizers of point groups

BY E. KOCH AND W. FISCHER

Normalizers with respect to the Euclidean or affine group may be defined for any group of isometries (cf. Gubler, 1982a,b). For a point group, however, it seems inadequate to use a supergroup that contains transformations that do not map a fixed point of that point group onto itself. Appropriate supergroups for the definition of normalizers of point groups are the full isometry groups of the sphere, $m\bar{\infty}$, and of the circle, ∞m , in three-dimensional and two-dimensional space (cf. Galiulin, 1978).

These normalizers are listed in Tables 15.4.1.1 and 15.4.1.2. It has to be noticed that the normalizer of a crystallographic point group may contain continuous rotations, *i.e.* rotations with

infinitesimal rotation angle, or noncrystallographic rotations (∞m ; $m\bar{\infty}$, ∞/mm , $8mm$, $12mm$; $8/mmm$, $12/mmm$). In analogy to space groups, these normalizers define equivalence relationships on the ‘Wyckoff positions’ of the point groups (cf. Section 10.1.2). They give also the relation between the different but equivalent morphological descriptions of a crystal.

Table 15.4.1.1. Normalizers of the two-dimensional point groups with respect to the full isometry group of the circle

The upper part refers to the crystallographic, the lower part to the noncrystallographic point groups as listed in Table 10.1.4.1.

Normalizer	Point groups
∞m	1, 2, 4, 3, 6
$12mm$	$6mm$
$8mm$	$4mm$
$6mm$	$3m$
$4mm$	$2mm$
$2mm$	m
∞m	$n, \infty, \infty m$
$(2n)mm$	nmm, nm

Table 15.4.1.2. Normalizers of the three-dimensional point groups with respect to the full isometry group of the sphere

The upper part refers to the crystallographic, the lower part to the noncrystallographic point groups as listed in Table 10.1.4.2.

Normalizer	Point groups
$m\bar{\infty}$	1, $\bar{1}$
$m\bar{3}m$	$222, mmm, 23, m\bar{3}, 432, \bar{4}3m, m\bar{3}m$
∞/mm	$2, m, 2/m, 4, \bar{4}, 4/m, 3, \bar{3}, 6, \bar{6}, 6/m$
$12/mmm$	$622, 6mm, 6/mmm$
$8/mmm$	$422, 4mm, 4/mmm$
$6/mmm$	$32, 3m, \bar{3}m, \bar{6}2m$
$4/mmm$	$mm2, \bar{4}2m$
$m\bar{\infty}$	$2\infty, m\bar{\infty}$
$m\bar{3}5$	$235, m\bar{3}5$
∞/mm	$n, \bar{n}, n/m, \infty, \infty/m, \infty 2, \infty m, \infty/mm$
$(2n)/mmm$	$n22, nmm, n/mmm, n2, nm, \bar{n}m$
n/mmm	$\bar{n}2m$