1.3. Printed symbols for symmetry elements

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1.3.1. Printed symbols for symmetry elements and for the corresponding symmetry operations in one, two and three dimensions

For ‘reflection conditions’, see Tables 2.2.13.2 and 2.2.13.3.

Printed symbol	Symmetry element and its orientation	Defining symmetry operation with glide or screw vector
m	Reflection plane, mirror plane	Reflection through the plane
	Reflection line, mirror line (two dimensions)	Reflection through the line
	Reflection point, mirror point (one dimension)	Reflection through the point
a, b or c	‘Axial’ glide plane	Glide reflection through the plane, with glide vector
a	$\perp$ [010] or $\perp$ [001]	$\frac{1}{2}\mathbf{a}$
b	$\perp$ [001] or $\perp$ [100]	$\frac{1}{2}\mathbf{b}$
c †	$\perp$ [100] or $\perp$ [010]	$\frac{1}{2}\mathbf{c}$
	$\perp$ [$1\bar{1}0$] or $\perp$ [110]	$\frac{1}{2}\mathbf{c}$
	$\perp$ [100] or $\perp$ [010] or $\perp$ [$\bar{1}\bar{1}0$]	$\frac{1}{2}\mathbf{c}$ } hexagonal coordinate system
	$\perp$ [$1\bar{1}0$] or $\perp$ [120] or $\perp$ [$\bar{2}\bar{1}0$]	$\frac{1}{2}\mathbf{c}$ } hexagonal coordinate system
e ‡	‘Double’ glide plane (in centred cells only)	*Two* glide reflections through *one* plane, with perpendicular glide vectors
	$\perp$ [001]	$\frac{1}{2}\mathbf{a}$ *and* $\frac{1}{2}\mathbf{b}$
	$\perp$ [100]	$\frac{1}{2}\mathbf{b}$ *and* $\frac{1}{2}\mathbf{c}$
	$\perp$ [010]	$\frac{1}{2}\mathbf{a}$ *and* $\frac{1}{2}\mathbf{c}$
	$\perp$ [$1\bar{1}0$]; $\perp$ [110]	$\frac{1}{2}(\mathbf{a}+\mathbf{b})$ *and* $\frac{1}{2}\mathbf{c}$; $\frac{1}{2}(\mathbf{a}-\mathbf{b})$ *and* $\frac{1}{2}\mathbf{c}$
	$\perp$ [$01\bar{1}$]; $\perp$ [011]	$\frac{1}{2}(\mathbf{b}+\mathbf{c})$ *and* $\frac{1}{2}\mathbf{a}$; $\frac{1}{2}(\mathbf{b}-\mathbf{c})$ *and* $\frac{1}{2}\mathbf{a}$
	$\perp$ [$\bar{1}01$]; $\perp$ [101]	$\frac{1}{2}(\mathbf{a}+\mathbf{c})$ *and* $\frac{1}{2}\mathbf{b}$; $\frac{1}{2}(\mathbf{a}-\mathbf{c})$ *and* $\frac{1}{2}\mathbf{b}$
n	‘Diagonal’ glide plane	Glide reflection through the plane, with glide vector
	$\perp$ [001]; $\perp$ [100]; $\perp$ [010]	$\frac{1}{2}(\mathbf{a}+\mathbf{b})$; $\frac{1}{2}(\mathbf{b}+\mathbf{c})$; $\frac{1}{2}(\mathbf{a}+\mathbf{c})$
	$\perp$ [$1\bar{1}0$] or $\perp$ [$01\bar{1}$] or $\perp$ [$\bar{1}01$]	$\frac{1}{2}(\mathbf{a}+\mathbf{b}+\mathbf{c})$
	$\perp$ [110]; $\perp$ [011]; $\perp$ [101]	$\frac{1}{2}(-\mathbf{a}+\mathbf{b}+\mathbf{c})$; $\frac{1}{2}(\mathbf{a}-\mathbf{b}+\mathbf{c})$; $\frac{1}{2}(\mathbf{a}+\mathbf{b}-\mathbf{c})$
d §	‘Diamond’ glide plane	Glide reflection through the plane, with glide vector
	$\perp$ [001]; $\perp$ [100]; $\perp$ [010]	$\frac{1}{4}(\mathbf{a}\pm\mathbf{b})$; $\frac{1}{4}(\mathbf{b}\pm\mathbf{c})$; $\frac{1}{4}(\pm\mathbf{a}+\mathbf{c})$
	$\perp$ [$1\bar{1}0$]; $\perp$ [$01\bar{1}$]; $\perp$ [$\bar{1}01$]	$\frac{1}{4}(\mathbf{a}+\mathbf{b}\pm\mathbf{c})$; $\frac{1}{4}(\pm\mathbf{a}+\mathbf{b}+\mathbf{c})$; $\frac{1}{4}(\mathbf{a}\pm\mathbf{b}+\mathbf{c})$
	$\perp$ [110]; $\perp$ [011]; $\perp$ [101]	$\frac{1}{4}(-\mathbf{a}+\mathbf{b}\pm\mathbf{c})$; $\frac{1}{4}(\pm\mathbf{a}-\mathbf{b}+\mathbf{c})$; $\frac{1}{4}(\mathbf{a}\pm\mathbf{b}-\mathbf{c})$
g	Glide line (two dimensions)	Glide reflection through the line, with glide vector
	$\perp$ [01]; $\perp$ [10]	$\frac{1}{2}\mathbf{a}$; $\frac{1}{2}\mathbf{b}$
1	None	Identity
2, 3, 4, 6	n-fold rotation axis, n	Counter-clockwise rotation of $360/n$ degrees around the axis (see Note viii)
	n-fold rotation point, n (two dimensions)	Counter-clockwise rotation of $360/n$ degrees around the point
$\bar{1}$	Centre of symmetry, inversion centre	Inversion through the point
$\bar{2}=m$,¶ $\bar{3},\bar{4},\bar{6}$	Rotoinversion axis, $\bar{n}$, and inversion point on the axis††	Counter-clockwise rotation of $360/n$ degrees around the axis, followed by inversion through the point on the axis†† (see Note viii)
2_1 $3_1, 3_2$ $4_1, 4_2, 4_3$ $6_1, 6_2, 6_3, 6_4, 6_5$	n-fold screw axis, n_p	Right-handed screw rotation of $360/n$ degrees around the axis, with screw vector (pitch) (p/n) $\mathbf{t}$; here $\mathbf{t}$ is the shortest lattice translation vector parallel to the axis in the direction of the screw

† In the rhombohedral space-group symbols $R3c$ (161) and $R\bar{3}c$ (167), the symbol c refers to the description with ‘hexagonal axes’; *i.e.* the glide vector is $\frac{1}{2}\mathbf{c}$, along [001]. In the description with ‘rhombohedral axes’, this glide vector is $\frac{1}{2}(\mathbf{a}+\mathbf{b}+\mathbf{c})$, along [111], *i.e.* the symbol of the glide plane would be n: *cf.* Section 4.3.5.
‡ For further explanations of the ‘double’ glide plane e, see Note (x) below.
§ Glide planes d occur only in orthorhombic F space groups, in tetragonal I space groups, and in cubic I and F space groups. They always occur in pairs with alternating glide vectors, for instance $\frac{1}{4}(\mathbf{a}+\mathbf{b})$ and $\frac{1}{4}(\mathbf{a}-\mathbf{b})$. The second power of a glide reflection d is a centring vector.
¶ Only the symbol m is used in the Hermann–Mauguin symbols, for both point groups and space groups.
†† The inversion point is a centre of symmetry if n is odd.