International Tables for Crystallography (2006). Vol. A, Section 4.2.3, p. 61.

4.2. Symbols for plane groups (two-dimensional space groups)

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4.2.1. Arrangement of the tables

Comparative tables for the 17 plane groups first appeared in *IT* (1952). The classification of plane groups is discussed in Chapter 2.1. Table 4.2.1.1 lists for each plane group its system, lattice symbol, point group and the plane-group number, followed by the short, full and extended Hermann–Mauguin symbols. Short symbols are included only where different from the full symbols. The next column contains the full symbol for another setting which corresponds to an interchange of the basis vectors **a** and **b**; it is only needed for the rectangular system. Multiple cells *c* and *h* for the square and the hexagonal system are introduced in the last column.

4.2.2. Additional symmetry elements and extended symbols

'Additional symmetry' elements are

(i) rotation points 2, 3 and 4, reproduced in the interior of the cell (*cf.* Table 4.1.2.1 and plane-group diagrams in Part 6);

(ii) glide lines *g* which alternate with mirror lines *m*.

In the extended plane-group symbols, only the additional glide lines *g* are listed: they are due either to *c* centring or to 'inclined' integral translations, as shown in Table 4.1.2.2.

4.2.3. Multiple cells

The *c* cell in the square system is defined as follows:

$$\mathbf{a}' = \mathbf{a} \mp \mathbf{b}; \quad \mathbf{b}' = \pm\mathbf{a} + \mathbf{b},$$

with 'centring points' at 0, 0; $\frac{1}{2}, \frac{1}{2}$. It plays the same role as the three-dimensional *C* cell in the tetragonal system (*cf.* Section 4.3.4).

Likewise, the triple cell *h* in the hexagonal system is defined as follows:

$$\mathbf{a}' = \mathbf{a} - \mathbf{b}; \quad \mathbf{b}' = \mathbf{a} + 2\mathbf{b},$$

with 'centring points' at 0, 0; $\frac{2}{3}, \frac{1}{3}; \frac{1}{3}, \frac{2}{3}$. It is the two-dimensional analogue of the three-dimensional *H* cell (*cf.* Chapter 1.2 and Section 4.3.5).

4.2.4. Group–subgroup relations

The following example illustrates the usefulness of multiple cells.

Example: *p*3*m*1 (14)

The symbol of this plane group, described by the triple cell *h,* is *h*31*m,* where the symmetry elements of the secondary and tertiary positions are interchanged. 'Decentring' the *h* cell gives rise to maximal non-isomorphic *k* subgroups *p*31*m* of index [3], with lattice parameters $a\sqrt{3}, a\sqrt{3}$ (*cf.* Section 4.3.5).

Table 4.2.1.1. *Index of symbols for plane groups*

System and lattice symbol	Point group	No. of plane group	Hermann–Mauguin symbol			Full symbol for other setting	Multiple cell
			Short	Full	Extended		
Oblique	1	1		*p*1			
p	2	2		*p*2			
Rectangular	*m*	3	*pm*	*p*1*m*1		*p*11*m*	
p, *c*		4	*pg*	*p*1*g*1		*p*11*g*	
		5	*cm*	*c*1*m*1	*c*1*m*1 *g*	*c*11*m*	
	2*mm*	6		*p*2*mm*		*p*2*mm*	
		7		*p*2*mg*		*p*2*gm*	
		8		*p*2*gg*		*p*2*gg*	
		9		*c*2*mm*	*c*2*mm* *g g*	*c*2*mm*	
Square	4	10		*p*4			*c*4
p	4*mm*	11		*p*4*mm*	*p*4*mm* *g*		*c*4*mm* *g*
		12		*p*4*gm*	*p*4*gm* *g*		*c*4*mg* *g*
Hexagonal	3	13		*p*3			*h*3
p	3*m*	14		*p*3*m*1	*p*3*m*1 *g*		*h*31*m* *g*
		15		*p*31*m*	*p*31*m* *g*		*h*3*m*1 *g*
	6	16		*p*6			*h*6
	6*mm*	17		*p*6*mm*	*p*6*mm* *g g*		*h*6*mm* *g g*