5. TRANSFORMATIONS IN CRYSTALLOGRAPHY

Fig. 5.1.3.5. Tetragonal lattices, projected along [001]. (a) Primitive cell $P$ with $a, b, c$ and the $C$-centred cells $C_1$ with $a_1, b_1, c$ and $C_2$ with $a_2, b_2, c$. Origin for all three cells is the same. (b) Body-centred cell $F$ with $a, b, c$ and the $F$-centred cells $F_1$ with $a_1, b_1, c$ and $F_2$ with $a_2, b_2, c$. Origin for all three cells is the same.

Fig. 5.1.3.6. Unit cells in the rhombohedral lattice: same origin for all cells. The basis of the rhombohedral cell is labelled $a, b, c$. Two settings of the triple hexagonal cell are possible with respect to a primitive rhombohedral cell: the so-called reverse setting of triple hexagonal cell $a_1, b_1, c$; $a_2, b_2, c$; $a_3, b_3, c$. Projection along $c$. (c) Primitive rhombohedral cell (- - - lower edges), $a, b, c$ in relation to the primitive hexagonal cell $a_1, b_1, c$; $a_2, b_2, c$; $a_3, b_3, c$. Projection along $c'$. (d) Primitive rhombohedral cell (- - - lower edges), $a, b, c$ in relation to the three triple hexagonal cells in reverse setting $a_1, b_1, c'$; $a_2, b_2, c'$; $a_3, b_3, c'$. Projection along $c'$.

\[
\begin{pmatrix}
    x' \\
    y' \\
    z'
\end{pmatrix} = \begin{pmatrix}
    x \\
    y \\
    z
\end{pmatrix}
\begin{pmatrix}
    Q_{11} & Q_{12} & Q_{13} & q_1 \\
    Q_{21} & Q_{22} & Q_{23} & q_2 \\
    Q_{31} & Q_{32} & Q_{33} & q_3 \\
    0 & 0 & 0 & 1
\end{pmatrix}
\]

The inverse of the augmented matrix $P$ is the augmented matrix $P^\dagger$ which contains the matrices $P$ and $p$, specifically,

\[
P = P^\dagger = \begin{pmatrix}
P & p \\
p & 1
\end{pmatrix} = \begin{pmatrix}
Q^{-1} & -Q^{-1}q \\
0 & 1
\end{pmatrix}.
\]

The advantage of the use of $(4 \times 4)$ matrices is that a sequence of affine transformations corresponds to the product of the correspond-