

5.2. TRANSFORMATIONS OF SYMMETRY OPERATIONS

$$\begin{aligned}
 (1) & \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}; & (2) & \begin{pmatrix} \bar{1} & 0 & 0 & \frac{1}{2} \\ 0 & \bar{1} & 0 & 0 \\ 0 & 0 & 1 & \frac{1}{2} \\ 0 & 0 & 0 & 1 \end{pmatrix}; & (3) & \begin{pmatrix} 0 & \bar{1} & 0 & \frac{1}{4} \\ 1 & 0 & 0 & \frac{1}{4} \\ 0 & 0 & 1 & \frac{1}{4} \\ 0 & 0 & 0 & 1 \end{pmatrix}; \\
 (4) & \begin{pmatrix} 0 & 1 & 0 & \frac{1}{4} \\ \bar{1} & 0 & 0 & \frac{1}{4} \\ 0 & 0 & 1 & \frac{3}{4} \\ 0 & 0 & 0 & 1 \end{pmatrix}; & (5) & \begin{pmatrix} 0 & 1 & 0 & \frac{1}{4} \\ 1 & 0 & 0 & \frac{1}{4} \\ 0 & 0 & \bar{1} & \frac{1}{4} \\ 0 & 0 & 0 & 1 \end{pmatrix}; & (6) & \begin{pmatrix} 0 & \bar{1} & 0 & \frac{1}{4} \\ \bar{1} & 0 & 0 & \frac{1}{4} \\ 0 & 0 & \bar{1} & \frac{3}{4} \\ 0 & 0 & 0 & 1 \end{pmatrix}; \\
 (7) & \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \bar{1} & 0 & 0 \\ 0 & 0 & \bar{1} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}; & (8) & \begin{pmatrix} \bar{1} & 0 & 0 & \frac{1}{2} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \bar{1} & \frac{1}{2} \\ 0 & 0 & 0 & 1 \end{pmatrix}.
 \end{aligned}$$

Another set of eight matrices is obtained by adding the *C*-centring translation $\frac{1}{2}, \frac{1}{2}, 0$ to the *w*'s.

From these matrices, one obtains the coordinates of the general position in the *C* cell, for instance from matrix (2)

$$\begin{pmatrix} \tilde{x} \\ \tilde{y} \\ \tilde{z} \\ 1 \end{pmatrix} = \begin{pmatrix} \bar{1} & 0 & 0 & \frac{1}{2} \\ 0 & \bar{1} & 0 & 0 \\ 0 & 0 & 1 & \frac{1}{2} \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} = \begin{pmatrix} -x - \frac{1}{2} \\ -y \\ z + \frac{1}{2} \\ 1 \end{pmatrix}.$$

The eight points obtained by the eight matrices \mathbb{W}' are

$$\begin{aligned}
 (1) & x, y, z; & (2) & -\frac{1}{2} - x, \bar{y}, \frac{1}{2} + z; \\
 (3) & \frac{1}{4} - y, \frac{1}{4} + x, \frac{1}{4} + z; & (4) & \frac{1}{4} + y, -\frac{1}{4} - x, \frac{3}{4} + z; \\
 (5) & \frac{1}{4} + y, \frac{1}{4} + x, \frac{1}{4} - z; & (6) & \frac{1}{4} - y, -\frac{1}{4} - x, \frac{3}{4} - z; \\
 (7) & x, \bar{y}, \bar{z}; & (8) & -\frac{1}{2} - x, y, \frac{1}{2} - z.
 \end{aligned}$$

The other set of eight points is obtained by adding $\frac{1}{2}, \frac{1}{2}, 0$.

In space group $P4_12_12$, the silicon atoms are in special position 4(*a*)..2 with the coordinates *x, x, 0*. Transformed into the *C* cell, the position becomes

$$\begin{aligned}
 & (0, 0, 0) + \left(\frac{1}{2}, \frac{1}{2}, 0\right) + \\
 & x, 0, 0; \quad \frac{1}{2} - x, 0, \frac{1}{2}; \quad \frac{1}{4}, \frac{1}{4} + x, \frac{1}{4}; \quad \frac{1}{4}, \frac{3}{4} - x, \frac{3}{4}.
 \end{aligned}$$

The parameter $x = 0.300$ of the *P* cell has changed to $x = 0.050$ in the *C* cell. For $x = 0$, the special position of the *C* cell assumes the same coordinate triplets as Wyckoff position 8(*a*) $43m$ in space group $Fd\bar{3}m$ (227), *i.e.* this change of the *x* parameter reflects the displacement of the silicon atoms in the cubic to tetragonal phase transition.

References

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