

$I4_1/a$

$C_{4h}^6$

$4/m$

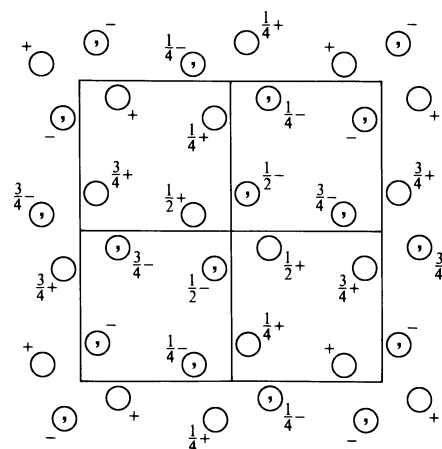
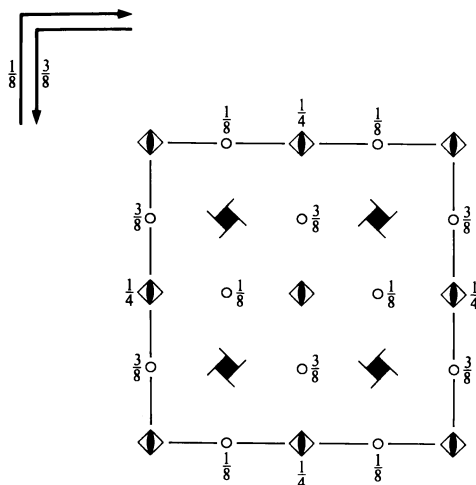
Tetragonal

No. 88

$I4_1/a$

Patterson symmetry  $I4/m$

ORIGIN CHOICE 1



Origin at  $\bar{4}$ , at  $0, -\frac{1}{4}, -\frac{1}{8}$  from  $\bar{1}$

Asymmetric unit  $0 \leq x \leq \frac{1}{4}; 0 \leq y \leq \frac{1}{4}; 0 \leq z \leq 1$

**Symmetry operations**

For  $(0,0,0)^+$  set

- |   |  |   |  |
|---|--|---|--|
| (1) $\bar{1}$                                   | (2) $2(0,0,\frac{1}{2}) \quad \frac{1}{4}, \frac{1}{4}, z$ | (3) $4^+(0,0,\frac{1}{4}) \quad -\frac{1}{4}, \frac{1}{4}, z$ | (4) $4^-(0,0,\frac{3}{4}) \quad \frac{1}{4}, -\frac{1}{4}, z$              |
| (5) $\bar{1} \quad 0, \frac{1}{4}, \frac{1}{8}$ | (6) $a \quad x, y, \frac{3}{8}$                            | (7) $\bar{4}^+ \quad 0, 0, z; \quad 0, 0, 0$                  | (8) $\bar{4}^- \quad 0, \frac{1}{2}, z; \quad 0, \frac{1}{2}, \frac{1}{4}$ |

For  $(\frac{1}{2}, \frac{1}{2}, \frac{1}{2})^+$  set

- |   |                                 |  |  |
|---|---------------------------------|--|--|
| (1) $t(\frac{1}{2}, \frac{1}{2}, \frac{1}{2})$  | (2) $2 \quad 0, 0, z$           | (3) $4^+(0,0,\frac{3}{4}) \quad \frac{1}{4}, \frac{1}{4}, z$               | (4) $4^-(0,0,\frac{1}{4}) \quad \frac{1}{4}, \frac{1}{4}, z$ |
| (5) $\bar{1} \quad \frac{1}{4}, 0, \frac{3}{8}$ | (6) $b \quad x, y, \frac{1}{8}$ | (7) $\bar{4}^+ \quad \frac{1}{2}, 0, z; \quad \frac{1}{2}, 0, \frac{1}{4}$ | (8) $\bar{4}^- \quad 0, 0, z; \quad 0, 0, 0$                 |

**Generators selected** (1);  $t(1,0,0)$ ;  $t(0,1,0)$ ;  $t(0,0,1)$ ;  $t(\frac{1}{2}, \frac{1}{2}, \frac{1}{2})$ ; (2); (3); (5)

**Positions**

Multiplicity, Wyckoff letter, Site symmetry		Coordinates				Reflection conditions	
		$(0,0,0)+ (\frac{1}{2}, \frac{1}{2}, \frac{1}{2})+$					
16	$f$	1	(1) $x, y, z$ (5) $\bar{x}, \bar{y} + \frac{1}{2}, \bar{z} + \frac{1}{4}$	(2) $\bar{x} + \frac{1}{2}, \bar{y} + \frac{1}{2}, z + \frac{1}{2}$ (6) $x + \frac{1}{2}, y, \bar{z} + \frac{3}{4}$	(3) $\bar{y}, x + \frac{1}{2}, z + \frac{1}{4}$ (7) $y, \bar{x}, \bar{z}$	(4) $y + \frac{1}{2}, \bar{x}, z + \frac{3}{4}$ (8) $\bar{y} + \frac{1}{2}, x + \frac{1}{2}, \bar{z} + \frac{1}{2}$	General: $hkl : h + k + l = 2n$ $hk0 : h, k = 2n$ $0kl : k + l = 2n$ $hhl : l = 2n$ $00l : l = 4n$ $h00 : h = 2n$ $h\bar{h}0 : h = 2n$
8	$e$	2..	0,0,z	$0, \frac{1}{2}, z + \frac{1}{4}$	$0, \frac{1}{2}, \bar{z} + \frac{1}{4}$	0,0, $\bar{z}$	Special: as above, plus $hkl : l = 2n + 1$ or $2h + l = 4n$
8	$d$	$\bar{1}$	$0, \frac{1}{4}, \frac{5}{8}$	$\frac{1}{2}, \frac{1}{4}, \frac{1}{8}$	$\frac{3}{4}, \frac{1}{2}, \frac{7}{8}$	$\frac{3}{4}, 0, \frac{3}{8}$	$hkl : l = 2n + 1$ or $h, k = 2n, h + k + l = 4n$
8	$c$	$\bar{1}$	$0, \frac{1}{4}, \frac{1}{8}$	$\frac{1}{2}, \frac{1}{4}, \frac{5}{8}$	$\frac{3}{4}, \frac{1}{2}, \frac{3}{8}$	$\frac{3}{4}, 0, \frac{7}{8}$	
4	$b$	$\bar{4}..$	0,0, $\frac{1}{2}$	$0, \frac{1}{2}, \frac{3}{4}$	$0, \frac{1}{2}, \frac{1}{4}$	$0, \frac{1}{2}, \frac{1}{4}$	$hkl : l = 2n + 1$ or $2h + l = 4n$
4	$a$	$\bar{4}..$	0,0,0	$0, \frac{1}{2}, \frac{1}{4}$			

**Symmetry of special projections**

Along [001]  $p4$   
 $\mathbf{a}' = \frac{1}{2}\mathbf{a}$     $\mathbf{b}' = \frac{1}{2}\mathbf{b}$   
 Origin at 0,0,z

Along [100]  $c2mg$   
 $\mathbf{a}' = \mathbf{b}$     $\mathbf{b}' = \mathbf{c}$   
 Origin at  $x, 0, \frac{3}{8}$

Along [110]  $p2mg$   
 $\mathbf{a}' = \frac{1}{2}(-\mathbf{a} + \mathbf{b})$     $\mathbf{b}' = \frac{1}{2}\mathbf{c}$   
 Origin at  $x, x + \frac{1}{4}, \frac{1}{8}$

**Maximal non-isomorphic subgroups**

**I**     $[2]I\bar{4}(82)$     (1; 2; 7; 8)+  
        $[2]I4_1(80)$     (1; 2; 3; 4)+  
        $[2]I2/a(C2/c, 15)$  (1; 2; 5; 6)+

**IIa** none

**IIb** none

**Maximal isomorphic subgroups of lowest index**

**IIc**    $[3]I4_1/a(\mathbf{c}' = 3\mathbf{c})(88)$ ;  $[5]I4_1/a(\mathbf{a}' = \mathbf{a} + 2\mathbf{b}, \mathbf{b}' = -2\mathbf{a} + \mathbf{b}$  or  $\mathbf{a}' = \mathbf{a} - 2\mathbf{b}, \mathbf{b}' = 2\mathbf{a} + \mathbf{b})(88)$

**Minimal non-isomorphic supergroups**

**I**     $[2]I4_1/amd(141)$ ;  $[2]I4_1/acd(142)$

**II**    $[2]C4_2/a(\mathbf{c}' = \frac{1}{2}\mathbf{c})(P4_2/n, 86)$

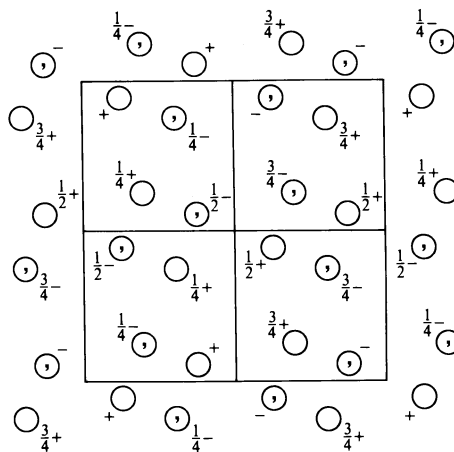
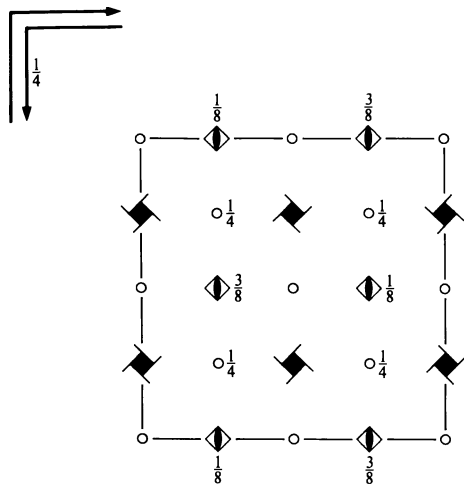
$I4_1/a$  $C_{4h}^6$  $4/m$ 

Tetragonal

No. 88

 $I4_1/a$ Patterson symmetry  $I4/m$ 

ORIGIN CHOICE 2

**Origin** at  $\bar{1}$  on glide plane  $b$ , at  $0, \frac{1}{4}, \frac{1}{8}$  from  $\bar{4}$ **Asymmetric unit**  $0 \leq x \leq \frac{1}{4}; 0 \leq y \leq \frac{1}{4}; 0 \leq z \leq 1$ **Symmetry operations**For  $(0,0,0)^+$  set

- |                           |  |  |  |
|---------------------------|--|--|--|
| (1) $1$                   | (2) $2(0,0,\frac{1}{2}) \quad \frac{1}{4}, 0, z$ | (3) $4^+(0,0,\frac{1}{4}) \quad \frac{1}{4}, \frac{1}{2}, z$                             | (4) $4^-(0,0,\frac{3}{4}) \quad \frac{3}{4}, 0, z$                   |
| (5) $\bar{1} \quad 0,0,0$ | (6) $a \quad x,y,\frac{1}{4}$                    | (7) $\bar{4}^+ \quad \frac{1}{2}, \frac{1}{4}, z; \frac{1}{2}, \frac{1}{4}, \frac{3}{8}$ | (8) $\bar{4}^- \quad 0, \frac{1}{4}, z; 0, \frac{1}{4}, \frac{1}{8}$ |

For  $(\frac{1}{2}, \frac{1}{2}, \frac{1}{2})^+$  set

- |   |                                 |  |  |
|---|---------------------------------|--|--|
| (1) $t(\frac{1}{2}, \frac{1}{2}, \frac{1}{2})$            | (2) $2 \quad 0, \frac{1}{4}, z$ | (3) $4^+(0,0,\frac{3}{4}) \quad -\frac{1}{4}, \frac{1}{2}, z$                              | (4) $4^-(0,0,\frac{1}{4}) \quad \frac{1}{4}, 0, z$                   |
| (5) $\bar{1} \quad \frac{1}{4}, \frac{1}{4}, \frac{1}{4}$ | (6) $b \quad x,y,0$             | (7) $\bar{4}^+ \quad \frac{1}{2}, -\frac{1}{4}, z; \frac{1}{2}, -\frac{1}{4}, \frac{1}{8}$ | (8) $\bar{4}^- \quad 0, \frac{3}{4}, z; 0, \frac{3}{4}, \frac{3}{8}$ |

**Generators selected** (1);  $t(1,0,0)$ ;  $t(0,1,0)$ ;  $t(0,0,1)$ ;  $t(\frac{1}{2}, \frac{1}{2}, \frac{1}{2})$ ; (2); (3); (5)

**Positions**

Multiplicity, Wyckoff letter, Site symmetry	Coordinates				Reflection conditions
	$(0,0,0)+ (\frac{1}{2}, \frac{1}{2}, \frac{1}{2})+$				General:
16 <i>f</i> 1	(1) $x, y, z$ (5) $\bar{x}, \bar{y}, \bar{z}$	(2) $\bar{x} + \frac{1}{2}, \bar{y}, z + \frac{1}{2}$ (6) $x + \frac{1}{2}, y, \bar{z} + \frac{1}{2}$	(3) $\bar{y} + \frac{3}{4}, x + \frac{1}{4}, z + \frac{1}{4}$ (7) $y + \frac{1}{4}, \bar{x} + \frac{3}{4}, \bar{z} + \frac{3}{4}$	(4) $y + \frac{3}{4}, \bar{x} + \frac{3}{4}, z + \frac{3}{4}$ (8) $\bar{y} + \frac{1}{4}, x + \frac{1}{4}, \bar{z} + \frac{1}{4}$	$hkl : h + k + l = 2n$ $hk0 : h, k = 2n$ $0kl : k + l = 2n$ $hhl : l = 2n$ $00l : l = 4n$ $h00 : h = 2n$ $h\bar{h}0 : h = 2n$
8 <i>e</i> 2..	$0, \frac{1}{4}, z$	$\frac{1}{2}, \frac{1}{4}, z + \frac{1}{4}$	$0, \frac{3}{4}, \bar{z}$	$\frac{1}{2}, \frac{3}{4}, \bar{z} + \frac{3}{4}$	Special: as above, plus $hkl : l = 2n + 1$ or $2h + l = 4n$
8 <i>d</i> $\bar{1}$	$0, 0, \frac{1}{2}$	$\frac{1}{2}, 0, 0$	$\frac{3}{4}, \frac{1}{4}, \frac{3}{4}$	$\frac{3}{4}, \frac{3}{4}, \frac{1}{4}$	$hkl : l = 2n + 1$ or $h, k = 2n, h + k + l = 4n$
8 <i>c</i> $\bar{1}$	$0, 0, 0$	$\frac{1}{2}, 0, \frac{1}{2}$	$\frac{3}{4}, \frac{1}{4}, \frac{1}{4}$	$\frac{3}{4}, \frac{3}{4}, \frac{3}{4}$	
4 <i>b</i> $\bar{4}..$	$0, \frac{1}{4}, \frac{5}{8}$	$\frac{1}{2}, \frac{1}{4}, \frac{7}{8}$	$\left. \begin{array}{l} \frac{3}{4}, \frac{1}{4}, \frac{3}{4} \\ \frac{3}{4}, \frac{3}{4}, \frac{1}{4} \end{array} \right\}$	$\left. \begin{array}{l} \frac{3}{4}, \frac{1}{4}, \frac{1}{4} \\ \frac{3}{4}, \frac{3}{4}, \frac{3}{4} \end{array} \right\}$	$hkl : l = 2n + 1$ or $2h + l = 4n$
4 <i>a</i> $\bar{4}..$	$0, \frac{1}{4}, \frac{1}{8}$	$\frac{1}{2}, \frac{1}{4}, \frac{3}{8}$			

**Symmetry of special projections**

Along  $[001]$   $p4$

$$\mathbf{a}' = \frac{1}{2}\mathbf{a} \quad \mathbf{b}' = \frac{1}{2}\mathbf{b}$$

Origin at  $\frac{1}{4}, 0, z$

Along  $[100]$   $c2mm$

$$\mathbf{a}' = \mathbf{b} \quad \mathbf{b}' = \mathbf{c}$$

Origin at  $x, \frac{1}{4}, \frac{1}{4}$

Along  $[110]$   $p2mg$

$$\mathbf{a}' = \frac{1}{2}(-\mathbf{a} + \mathbf{b}) \quad \mathbf{b}' = \frac{1}{2}\mathbf{c}$$

Origin at  $x, x, 0$

**Maximal non-isomorphic subgroups**

- I**  $[2]I\bar{4}(82)$  (1; 2; 7; 8)+  
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**Minimal non-isomorphic supergroups**

- I**  $[2]I4_1/amd(141)$ ;  $[2]I4_1/acd(142)$

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