

$P4/nmm$

D_{4h}^7

$4/mmm$

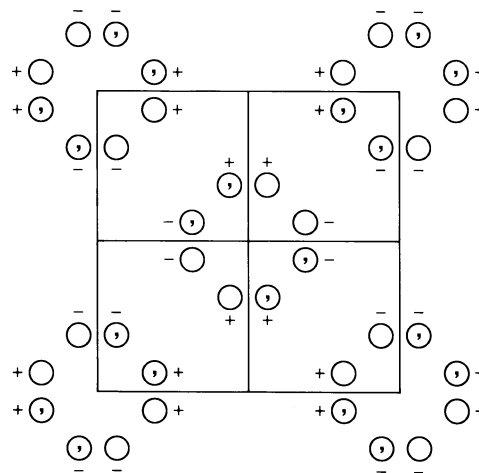
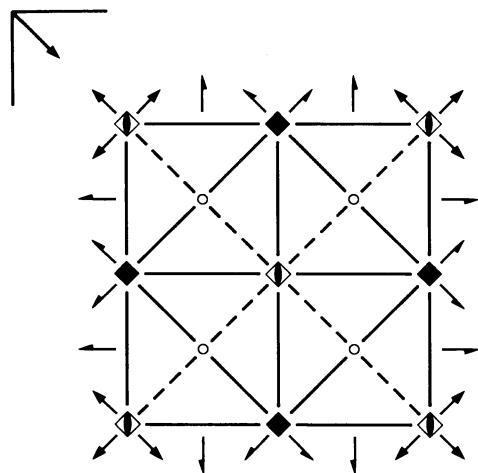
Tetragonal

No. 129

$P 4/n 2_1/m 2/m$

Patterson symmetry $P4/mmm$

ORIGIN CHOICE 1



Origin at $\bar{4}m2$ at $\bar{4}/nm2/g$, at $-\frac{1}{4}, \frac{1}{4}, 0$ from centre ($2/m$)

Asymmetric unit $0 \leq x \leq \frac{1}{2}; 0 \leq y \leq \frac{1}{2}; 0 \leq z \leq \frac{1}{2}; y \leq \frac{1}{2} - x$

Symmetry operations

- | | | | |
|--|---|--|---|
| (1) 1 | (2) 2 $0, 0, z$ | (3) 4^+ $0, \frac{1}{2}, z$ | (4) 4^- $-\frac{1}{2}, 0, z$ |
| (5) $2(0, \frac{1}{2}, 0)$ $\frac{1}{4}, y, 0$ | (6) $2(\frac{1}{2}, 0, 0)$ $x, \frac{1}{4}, 0$ | (7) 2 $x, x, 0$ | (8) 2 $x, \bar{x}, 0$ |
| (9) $\bar{1}$ $\frac{1}{4}, \frac{1}{4}, 0$ | (10) $n(\frac{1}{2}, \frac{1}{2}, 0)$ $x, y, 0$ | (11) $\bar{4}^+$ $0, 0, z; 0, 0, 0$ | (12) $\bar{4}^-$ $0, 0, z; 0, 0, 0$ |
| (13) m $x, 0, z$ | (14) m $0, y, z$ | (15) m $x + \frac{1}{2}, \bar{x}, z$ | (16) $g(\frac{1}{2}, \frac{1}{2}, 0)$ x, x, z |

Generators selected (1); $t(1,0,0)$; $t(0,1,0)$; $t(0,0,1)$; (2); (3); (5); (9)

Positions

Multiplicity, Wyckoff letter, Site symmetry	Coordinates	Reflection conditions
16 k 1	(1) x, y, z (2) \bar{x}, \bar{y}, z (3) $\bar{y} + \frac{1}{2}, x + \frac{1}{2}, z$ (4) $y + \frac{1}{2}, \bar{x} + \frac{1}{2}, z$ (5) $\bar{x} + \frac{1}{2}, y + \frac{1}{2}, \bar{z}$ (6) $x + \frac{1}{2}, \bar{y} + \frac{1}{2}, \bar{z}$ (7) y, x, \bar{z} (8) $\bar{y}, \bar{x}, \bar{z}$ (9) $\bar{x} + \frac{1}{2}, \bar{y} + \frac{1}{2}, \bar{z}$ (10) $x + \frac{1}{2}, y + \frac{1}{2}, \bar{z}$ (11) y, \bar{x}, \bar{z} (12) \bar{y}, x, \bar{z} (13) x, \bar{y}, z (14) \bar{x}, y, z (15) $\bar{y} + \frac{1}{2}, \bar{x} + \frac{1}{2}, z$ (16) $y + \frac{1}{2}, x + \frac{1}{2}, z$	General: $hk0 : h + k = 2n$ $h00 : h = 2n$ Special: as above, plus no extra conditions
8 j $\dots m$	$x, x + \frac{1}{2}, z$ $\bar{x} + \frac{1}{2}, x, \bar{z}$ $\bar{x}, \bar{x} + \frac{1}{2}, z$ $x + \frac{1}{2}, \bar{x}, \bar{z}$ $\bar{x}, x + \frac{1}{2}, z$ $x + \frac{1}{2}, x, \bar{z}$ $x, \bar{x} + \frac{1}{2}, z$ $\bar{x} + \frac{1}{2}, \bar{x}, \bar{z}$	no extra conditions
8 i $\dots m$	$0, y, z$ $\frac{1}{2}, y + \frac{1}{2}, \bar{z}$ $0, \bar{y}, z$ $\frac{1}{2}, \bar{y} + \frac{1}{2}, \bar{z}$ $\bar{y} + \frac{1}{2}, \frac{1}{2}, z$ $y, 0, \bar{z}$ $y + \frac{1}{2}, \frac{1}{2}, z$ $\bar{y}, 0, \bar{z}$	no extra conditions
8 h $\dots 2$	$x, x, \frac{1}{2}$ $\bar{x} + \frac{1}{2}, \bar{x} + \frac{1}{2}, \frac{1}{2}$ $\bar{x}, \bar{x}, \frac{1}{2}$ $x + \frac{1}{2}, x + \frac{1}{2}, \frac{1}{2}$ $\bar{x} + \frac{1}{2}, x + \frac{1}{2}, \frac{1}{2}$ $x, \bar{x}, \frac{1}{2}$ $x + \frac{1}{2}, \bar{x} + \frac{1}{2}, \frac{1}{2}$ $\bar{x}, x, \frac{1}{2}$	$hkl : h + k = 2n$
8 g $\dots 2$	$x, x, 0$ $\bar{x} + \frac{1}{2}, \bar{x} + \frac{1}{2}, 0$ $\bar{x}, \bar{x}, 0$ $x + \frac{1}{2}, x + \frac{1}{2}, 0$ $\bar{x} + \frac{1}{2}, x + \frac{1}{2}, 0$ $x, \bar{x}, 0$ $x + \frac{1}{2}, \bar{x} + \frac{1}{2}, 0$ $\bar{x}, x, 0$	$hkl : h + k = 2n$
4 f $2mm$	$0, 0, z$ $\frac{1}{2}, \frac{1}{2}, z$ $\frac{1}{2}, \frac{1}{2}, \bar{z}$ $0, 0, \bar{z}$	$hkl : h + k = 2n$
4 e $\dots 2/m$	$\frac{1}{4}, \frac{1}{4}, \frac{1}{2}$ $\frac{3}{4}, \frac{3}{4}, \frac{1}{2}$ $\frac{1}{4}, \frac{3}{4}, \frac{1}{2}$ $\frac{3}{4}, \frac{1}{4}, \frac{1}{2}$	$hkl : h, k = 2n$
4 d $\dots 2/m$	$\frac{1}{4}, \frac{1}{4}, 0$ $\frac{3}{4}, \frac{3}{4}, 0$ $\frac{1}{4}, \frac{3}{4}, 0$ $\frac{3}{4}, \frac{1}{4}, 0$	$hkl : h, k = 2n$
2 c $4mm$	$0, \frac{1}{2}, z$ $\frac{1}{2}, 0, \bar{z}$	no extra conditions
2 b $\bar{4}m2$	$0, 0, \frac{1}{2}$ $\frac{1}{2}, \frac{1}{2}, \frac{1}{2}$	$hkl : h + k = 2n$
2 a $\bar{4}m2$	$0, 0, 0$ $\frac{1}{2}, \frac{1}{2}, 0$	$hkl : h + k = 2n$

Symmetry of special projectionsAlong [001] $p4mm$

$$\mathbf{a}' = \frac{1}{2}(\mathbf{a} - \mathbf{b}) \quad \mathbf{b}' = \frac{1}{2}(\mathbf{a} + \mathbf{b})$$

Origin at $0, 0, z$ Along [100] $p2mg$

$$\mathbf{a}' = \mathbf{b} \quad \mathbf{b}' = \mathbf{c}$$

Origin at $x, \frac{1}{4}, 0$ Along [110] $p2mm$

$$\mathbf{a}' = \frac{1}{2}(-\mathbf{a} + \mathbf{b}) \quad \mathbf{b}' = \mathbf{c}$$

Origin at $x, x, 0$ **Maximal non-isomorphic subgroups**

I	[2] $P\bar{4}m2$ (115)	1; 2; 7; 8; 11; 12; 13; 14
	[2] $P\bar{4}2_1m$ (113)	1; 2; 5; 6; 11; 12; 15; 16
	[2] $P4mm$ (99)	1; 2; 3; 4; 13; 14; 15; 16
	[2] $P42_12$ (90)	1; 2; 3; 4; 5; 6; 7; 8
	[2] $P4/n11$ ($P4/n$, 85)	1; 2; 3; 4; 9; 10; 11; 12
	[2] $P2/n12/m$ (<i>Cmme</i> , 67)	1; 2; 7; 8; 9; 10; 15; 16
	[2] $P2/n2_1/m1$ (<i>Pmmn</i> , 59)	1; 2; 5; 6; 9; 10; 13; 14

IIa none**IIb** [2] $P4_2/n2m$ ($\mathbf{c}' = 2\mathbf{c}$) (138); [2] $P4_2/nmc$ ($\mathbf{c}' = 2\mathbf{c}$) (137); [2] $P4/ncc$ ($\mathbf{c}' = 2\mathbf{c}$) (130)**Maximal isomorphic subgroups of lowest index****IIc** [2] $P4/nmm$ ($\mathbf{c}' = 2\mathbf{c}$) (129); [9] $P4/nmm$ ($\mathbf{a}' = 3\mathbf{a}, \mathbf{b}' = 3\mathbf{b}$) (129)**Minimal non-isomorphic supergroups****I** none**II** [2] $C4/mmm$ ($P4/mmm$, 123); [2] $I4/mmm$ (139)

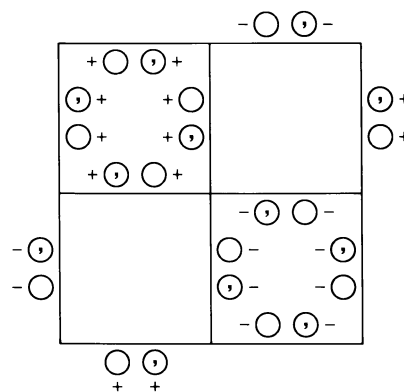
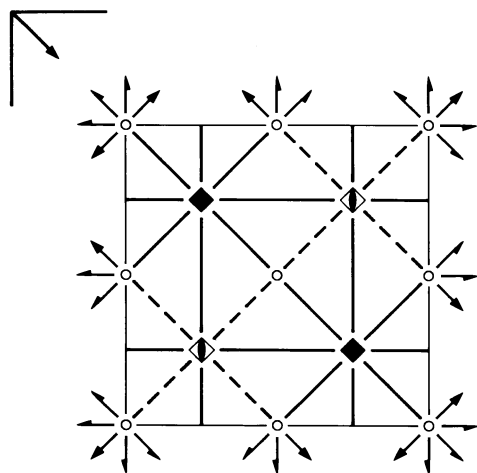
$P4/nmm$ D_{4h}^7 $4/mmm$

Tetragonal

No. 129

 $P 4/n 2_1/m 2/m$ Patterson symmetry $P4/mmm$

ORIGIN CHOICE 2



Origin at centre $(2/m)$ at $n2_1(2/m, 2_1/g)$, at $\frac{1}{4}, -\frac{1}{4}, 0$ from $\bar{4}m2$

Asymmetric unit $-\frac{1}{4} \leq x \leq \frac{1}{4}; -\frac{1}{4} \leq y \leq \frac{1}{4}; 0 \leq z \leq \frac{1}{2}; x \leq y$

Symmetry operations

- | | | | |
|------------------------------------|---|---|---|
| (1) 1 | (2) $2 \frac{1}{4}, \frac{1}{4}, z$ | (3) $4^+ \frac{1}{4}, \frac{1}{4}, z$ | (4) $4^- \frac{1}{4}, \frac{1}{4}, z$ |
| (5) $2(0, \frac{1}{2}, 0) 0, y, 0$ | (6) $2(\frac{1}{2}, 0, 0) x, 0, 0$ | (7) $2(\frac{1}{2}, \frac{1}{2}, 0) x, x, 0$ | (8) $2 x, \bar{x}, 0$ |
| (9) $\bar{1} 0, 0, 0$ | (10) $n(\frac{1}{2}, \frac{1}{2}, 0) x, y, 0$ | (11) $\bar{4}^+ \frac{1}{4}, -\frac{1}{4}, z; \frac{1}{4}, -\frac{1}{4}, 0$ | (12) $\bar{4}^- -\frac{1}{4}, \frac{1}{4}, z; -\frac{1}{4}, \frac{1}{4}, 0$ |
| (13) $m x, \frac{1}{4}, z$ | (14) $m \frac{1}{4}, y, z$ | (15) $m x + \frac{1}{2}, \bar{x}, z$ | (16) $m x, x, z$ |

Generators selected (1); $t(1,0,0)$; $t(0,1,0)$; $t(0,0,1)$; (2); (3); (5); (9)

Positions

Multiplicity, Wyckoff letter, Site symmetry	Coordinates	Reflection conditions
		General:
16 k 1	(1) x, y, z (2) $\bar{x} + \frac{1}{2}, \bar{y} + \frac{1}{2}, z$ (3) $\bar{y} + \frac{1}{2}, x, z$ (4) $y, \bar{x} + \frac{1}{2}, z$ (5) $\bar{x}, y + \frac{1}{2}, \bar{z}$ (6) $x + \frac{1}{2}, \bar{y}, \bar{z}$ (7) $y + \frac{1}{2}, x + \frac{1}{2}, \bar{z}$ (8) $\bar{y}, \bar{x}, \bar{z}$ (9) $\bar{x}, \bar{y}, \bar{z}$ (10) $x + \frac{1}{2}, y + \frac{1}{2}, \bar{z}$ (11) $y + \frac{1}{2}, \bar{x}, \bar{z}$ (12) $\bar{y}, x + \frac{1}{2}, \bar{z}$ (13) $x, \bar{y} + \frac{1}{2}, z$ (14) $\bar{x} + \frac{1}{2}, y, z$ (15) $\bar{y} + \frac{1}{2}, \bar{x} + \frac{1}{2}, z$ (16) y, x, z	$hk0 : h + k = 2n$ $h00 : h = 2n$
		Special: as above, plus
8 j $..m$	x, x, z $\bar{x}, x + \frac{1}{2}, \bar{z}$ $\bar{x} + \frac{1}{2}, \bar{x} + \frac{1}{2}, z$ $x + \frac{1}{2}, \bar{x}, \bar{z}$ $\bar{x} + \frac{1}{2}, x, z$ $x + \frac{1}{2}, x + \frac{1}{2}, \bar{z}$ $x, \bar{x} + \frac{1}{2}, z$ $\bar{x}, \bar{x}, \bar{z}$	no extra conditions
8 i $.m.$	$\frac{1}{4}, y, z$ $\frac{3}{4}, y + \frac{1}{2}, \bar{z}$ $\frac{1}{4}, \bar{y} + \frac{1}{2}, z$ $\frac{3}{4}, \bar{y}, \bar{z}$ $\bar{y} + \frac{1}{2}, \frac{1}{4}, z$ $y + \frac{1}{2}, \frac{3}{4}, \bar{z}$ $y, \frac{1}{4}, z$ $\bar{y}, \frac{3}{4}, \bar{z}$	no extra conditions
8 h $..2$	$x, \bar{x}, \frac{1}{2}$ $\bar{x}, x, \frac{1}{2}$ $\bar{x} + \frac{1}{2}, x + \frac{1}{2}, \frac{1}{2}$ $x + \frac{1}{2}, \bar{x} + \frac{1}{2}, \frac{1}{2}$ $x + \frac{1}{2}, x, \frac{1}{2}$ $\bar{x} + \frac{1}{2}, \bar{x}, \frac{1}{2}$ $\bar{x}, \bar{x} + \frac{1}{2}, \frac{1}{2}$ $x, x + \frac{1}{2}, \frac{1}{2}$	$hkl : h + k = 2n$
8 g $..2$	$x, \bar{x}, 0$ $\bar{x}, x, 0$ $\bar{x} + \frac{1}{2}, x + \frac{1}{2}, 0$ $x + \frac{1}{2}, \bar{x} + \frac{1}{2}, 0$ $x + \frac{1}{2}, x, 0$ $\bar{x} + \frac{1}{2}, \bar{x}, 0$ $\bar{x}, \bar{x} + \frac{1}{2}, 0$ $x, x + \frac{1}{2}, 0$	$hkl : h + k = 2n$
4 f $2mm.$	$\frac{3}{4}, \frac{1}{4}, z$ $\frac{1}{4}, \frac{3}{4}, z$ $\frac{1}{4}, \frac{3}{4}, \bar{z}$ $\frac{3}{4}, \frac{1}{4}, \bar{z}$	$hkl : h + k = 2n$
4 e $..2/m$	$0, 0, \frac{1}{2}$ $\frac{1}{2}, \frac{1}{2}, \frac{1}{2}$ $\frac{1}{2}, 0, \frac{1}{2}$ $0, \frac{1}{2}, \frac{1}{2}$	$hkl : h, k = 2n$
4 d $..2/m$	$0, 0, 0$ $\frac{1}{2}, \frac{1}{2}, 0$ $\frac{1}{2}, 0, 0$ $0, \frac{1}{2}, 0$	$hkl : h, k = 2n$
2 c $4mm$	$\frac{1}{4}, \frac{1}{4}, z$ $\frac{3}{4}, \frac{3}{4}, \bar{z}$	no extra conditions
2 b $\bar{4}m2$	$\frac{3}{4}, \frac{1}{4}, \frac{1}{2}$ $\frac{1}{4}, \frac{3}{4}, \frac{1}{2}$	$hkl : h + k = 2n$
2 a $\bar{4}m2$	$\frac{3}{4}, \frac{1}{4}, 0$ $\frac{1}{4}, \frac{3}{4}, 0$	$hkl : h + k = 2n$

Symmetry of special projections

Along [001] $p4mm$

$$\mathbf{a}' = \frac{1}{2}(\mathbf{a} - \mathbf{b}) \quad \mathbf{b}' = \frac{1}{2}(\mathbf{a} + \mathbf{b})$$

Origin at $\frac{1}{4}, \frac{1}{4}, z$

Along [100] $p2mg$

$$\mathbf{a}' = \mathbf{b} \quad \mathbf{b}' = \mathbf{c}$$

Origin at $x, 0, 0$

Along [110] $p2mm$

$$\mathbf{a}' = \frac{1}{2}(-\mathbf{a} + \mathbf{b}) \quad \mathbf{b}' = \mathbf{c}$$

Origin at $x, x, 0$

Maximal non-isomorphic subgroups

I	[2] $P\bar{4}m2$ (115)	1; 2; 7; 8; 11; 12; 13; 14
	[2] $P\bar{4}_2m$ (113)	1; 2; 5; 6; 11; 12; 15; 16
	[2] $P4mm$ (99)	1; 2; 3; 4; 13; 14; 15; 16
	[2] $P4_22$ (90)	1; 2; 3; 4; 5; 6; 7; 8
	[2] $P4/n11$ ($P4/n$, 85)	1; 2; 3; 4; 9; 10; 11; 12
	[2] $P2/n12/m$ ($Cmme$, 67)	1; 2; 7; 8; 9; 10; 15; 16
	[2] $P2/n2_1/m1$ ($Pmnm$, 59)	1; 2; 5; 6; 9; 10; 13; 14

IIa none

IIb [2] $P4_2/n2m$ ($\mathbf{c}' = 2\mathbf{c}$) (138); [2] $P4_2/nmc$ ($\mathbf{c}' = 2\mathbf{c}$) (137); [2] $P4/ncc$ ($\mathbf{c}' = 2\mathbf{c}$) (130)

Maximal isomorphic subgroups of lowest index

IIc [2] $P4/nmm$ ($\mathbf{c}' = 2\mathbf{c}$) (129); [9] $P4/nmm$ ($\mathbf{a}' = 3\mathbf{a}, \mathbf{b}' = 3\mathbf{b}$) (129)

Minimal non-isomorphic supergroups

I none

II [2] $C4/mmm$ ($P4/mmm$, 123); [2] $I4/mmm$ (139)