

## 1. INTRODUCTION TO SPACE-GROUP SYMMETRY

$P\bar{1}$  is the only one which displays the inversion  $\bar{1}$  explicitly. Sometimes non-conventional centred lattice descriptions may be used, especially when comparing crystal structures.

## 1.4.1.4.4. Monoclinic space groups

Monoclinic space groups have exactly one symmetry direction, often called *the monoclinic axis*. The *b* axis is the symmetry direction of the (most frequently used) conventional setting, called the *b*-axis setting. Another conventional setting has *c* as its symmetry direction (*c*-axis setting). In earlier literature, the unique-axis *c* setting was called the first setting and the unique-axis *b* setting the second setting (*cf.* Section 2.1.3.15). In addition to the primitive lattice *P* there is a centred lattice which is taken as *C* in the *b*-axis setting, *A* in the *c*-axis setting. The (possible) glide reflections are *c* (or *a*). In this volume, more settings are described, *cf.* Sections 1.5.4 and 2.1.3.15 and the space-group tables of Chapter 2.3.

The full HM symbol consists of the lattice symbol and three possible positions for the symmetry directions. The symmetry in the **a** direction is described first, followed by the symmetry in the **b** direction and last in the **c** direction. The two positions of the HM symbol that are not occupied by the monoclinic symmetry direction are marked by 1. The symbol is thus similar to the orthorhombic HM symbol and the monoclinic axis is clearly visible. *P1m1* or *P11m* may designate the same space group but in different settings. *Pm11* is a possible but not conventional setting.

The short HM symbols of the monoclinic space groups are independent of the setting of the space group. They form the *monoclinic standard symbols* and are not variable: *P2*, *P2<sub>1</sub>*, *C2*, *Pm*, *Pc*, *Cm*, *Cc*, *P2/m*, *P2<sub>1</sub>/m*, *C2/m*, *P2/c*, *P2<sub>1</sub>/c* and *C2/c*. Altogether there are 13 monoclinic space-group types.

There are several reasons for the many conventional settings.

- (1) As only one of the three coordinate axes is fixed by symmetry, there are two conventions related to the possible permutations of the other axes.
- (2) The sequence of the three coordinate axes may be chosen because of the lengths of the basis vectors, *i.e.* not because of symmetry.
- (3) If two different crystal structures have related symmetries, one being a subgroup of the other, then it is often convenient to choose a non-conventional setting for one of the structures to make their structural relations transparent. Such similarity happens in particular in substances that are related by a non-destructive phase transition. Monoclinic space groups are particularly flexible in their settings.

## 1.4.1.4.5. Orthorhombic space groups

To the orthorhombic crystal system belong the crystal classes 222, *mm2* and *2/m 2/m 2/m* with the Bravais types of lattices *P*, *C*, *A*, *F* and *I*. Four space groups with a *P* lattice belong to the crystal class 222, ten to *mm2* and 16 to *2/m 2/m 2/m*. Each of the basis vectors marks a symmetry direction; the lattice symbol is followed by characters representing the symmetry operations with respect to the symmetry directions along **a**, **b** and **c**.

We start with the full HM symbols. For a space group of crystal class 222 with a *P* lattice the HM symbol is thus '*PR<sub>1</sub>R<sub>2</sub>R<sub>3</sub>*', where  $R_1, R_2, R_3 = 2$  or  $2_1$ . Conventionally one chooses a setting with the symbols *P222*, *P222<sub>1</sub>*, *P2<sub>1</sub>2<sub>1</sub>2* and *P2<sub>1</sub>2<sub>1</sub>2<sub>1</sub>*.

For the generation of the space groups of this crystal class only two non-translational generators are necessary, say  $R_1$  and  $R_2$ . However, it is not possible to indicate in the HM symbol whether the axes  $R_1$  and  $R_2$  intersect or not. This is decided by the third

(screw) rotation  $R_3$ : if  $R_3 = R_1R_2 = 2$ , the axes  $R_1$  and  $R_2$  intersect, if  $R_3 = 2_1$ , they do not. For this reason,  $R_3$  is sometimes called an *indicator*. However, any two of the three rotations or screw rotations can be taken as the generators and the third one is then the indicator. Mathematically each element of a generating set is a generator independent of its possible redundancy.

In the space groups of crystal class *mm2* the two reflections or glide reflections are the generators, the twofold rotation or screw rotation is generated by composition of the (glide) reflections. The position of the rotation axis relative to the intersection line of the two planes as well as its screw component are determined uniquely by the glide components of the reflections or glide reflections.

The rotation or screw rotation in the HM symbols of space groups of the crystal class *mm2* could be omitted, and were omitted in older HM symbols. Nowadays they are included to make the orthorhombic HM symbols more homogeneous. Conventional symbols are, among others, *Pmm2*, *Pmc2<sub>1</sub>*, *Pba2* and *Pca2<sub>1</sub>*.

The 16 space groups with a *P* lattice in crystal class *2/m 2/m 2/m* are similarly obtained by starting with the letter *P* and continuing with the point-group symbol, modified by the possible replacements  $2_1$  for 2 and *a*, *b*, *c* or *n* for *m*. The conventional symbols are, among others, *P2/m 2/m 2/m*, *P2<sub>1</sub>/m 2/m 2/a*, *P2/m 2/n 2<sub>1</sub>/a*, *P2<sub>1</sub>/b 2<sub>1</sub>/a 2/m* or *P2<sub>1</sub>/n 2<sub>1</sub>/m 2<sub>1</sub>/a*. The symbols *P2/m 2/n 2<sub>1</sub>/a* and *P2<sub>1</sub>/n 2<sub>1</sub>/m 2<sub>1</sub>/a* designate different space-group types, as is easily seen by looking at the screw rotations: *P2/m 2/n 2<sub>1</sub>/a* has screw axes in the direction of **c** only, *P2<sub>1</sub>/n 2<sub>1</sub>/m 2<sub>1</sub>/a* has screw axes in all three symmetry directions.

If the lattice is centred, the constituents in the same symmetry direction are not unique. In this case, according to the 'simplest symmetry operation' rule, in general the simplest operation is chosen, *cf.* Section 1.5.4.

## Examples

In the HM symbol *C2/m 2/c 2<sub>1</sub>/m* there are in addition  $2_1$  screw rotations in the first two symmetry directions; additional glide reflections *b* occur in the first, and *n* in the second and third symmetry directions.

In *I2/b 2/a 2/m*, all rotations 2 are accompanied by screw rotations  $2_1$ ; *b* and *a* are accompanied by *c* and *m* is accompanied by *n*. The symmetry operations that are not listed in the full HM symbol can be derived by composition of the listed operations with a centring translation, *cf.* Section 1.4.2.4.

There are two exceptions to the 'simplest symmetry operation' rule. If the *I* centring is added to the *P* space groups of the crystal class 222, one obtains two different space groups with an *I* lattice, each has 2 and  $2_1$  operations in each of the symmetry directions. One space group is derived by adding the *I* centring to the space group *P222*, the other is obtained by adding the *I* centring to a space group *P2<sub>1</sub>2<sub>1</sub>2<sub>1</sub>*. In the first case the twofold axes intersect, in the second they do not. According to the rules both should get the HM symbol *I222*, but only the space group generated from *P222* is named *I222*, whereas the space group generated from *P2<sub>1</sub>2<sub>1</sub>2<sub>1</sub>* is called *I2<sub>1</sub>2<sub>1</sub>2<sub>1</sub>*. The second exception occurs among the cubic space groups and is due to similar reasons, *cf.* Section 1.4.1.4.8.

The *short HM symbols* for the space groups of the crystal classes 222 and *mm2* are the same as the full HM symbols. In the short HM symbols for the space groups of the crystal class *2/m 2/m 2/m* the symbols for the (screw) rotations are omitted, resulting in the short symbols *Pmmm*, *Pmma*, *Pmna*, *Pbam*, *Pnma*, *Cmcm* and *Ibam* for the space groups mentioned above.