International Tables for Crystallography (2016). Vol. A, Table 1.4.2.6, p. 54.

Table 1.4.2.6

Right coset decomposition of space group $P2_1/c$, No. 14 (unique axis b, cell choice 1) with respect to the normal subgroup of translations $\mathcal{T}$

The numbers u_1, u_2 and u_3 are positive or negative integers.

x	y	z	$\bar{x}$	$y+\frac{1}{2}$	$\bar{z}+\frac{1}{2}$	$\bar{x}$	$\bar{y}$	$\bar{z}$	x	$\bar{y}+\frac{1}{2}$	$z+\frac{1}{2}$
$x+1$	y	z	$\bar{x}+1$	$y+\frac{1}{2}$	$\bar{z}+\frac{1}{2}$	$\bar{x}+1$	$\bar{y}$	$\bar{z}$	$x+1$	$\bar{y}+\frac{1}{2}$	$z+\frac{1}{2}$
$x+2$	y	z	$\bar{x}+2$	$y+\frac{1}{2}$	$\bar{z}+\frac{1}{2}$	$\bar{x}+2$	$\bar{y}$	$\bar{z}$	$x+2$	$\bar{y}+\frac{1}{2}$	$z+\frac{1}{2}$
	$\vdots$			$\vdots$			$\vdots$			$\vdots$	
x	$y+1$	z	$\bar{x}$	$y+\frac{3}{2}$	$\bar{z}+\frac{1}{2}$	$\bar{x}$	$\bar{y}+1$	$\bar{z}$	x	$\bar{y}+\frac{3}{2}$	$z+\frac{1}{2}$
$x+1$	$y+1$	z	$\bar{x}+1$	$y+\frac{3}{2}$	$\bar{z}+\frac{1}{2}$	$\bar{x}+1$	$\bar{y}+1$	$\bar{z}$	$x+1$	$\bar{y}+\frac{3}{2}$	$z+\frac{1}{2}$
$x+2$	$y+1$	z	$\bar{x}+2$	$y+\frac{3}{2}$	$\bar{z}+\frac{1}{2}$	$\bar{x}+2$	$\bar{y}+1$	$\bar{z}$	$x+2$	$\bar{y}+\frac{3}{2}$	$z+\frac{1}{2}$
	$\vdots$			$\vdots$			$\vdots$			$\vdots$	
x	$y+2$	z	$\bar{x}$	$y+\frac{5}{2}$	$\bar{z}+\frac{1}{2}$	$\bar{x}$	$\bar{y}+2$	$\bar{z}$	x	$\bar{y}+\frac{5}{2}$	$z+\frac{1}{2}$
$x+1$	$y+2$	z	$\bar{x}+1$	$y+\frac{5}{2}$	$\bar{z}+\frac{1}{2}$	$\bar{x}+1$	$\bar{y}+2$	$\bar{z}$	$x+1$	$\bar{y}+\frac{5}{2}$	$z+\frac{1}{2}$
$x+2$	$y+2$	z	$\bar{x}+2$	$y+\frac{5}{2}$	$\bar{z}+\frac{1}{2}$	$\bar{x}+2$	$\bar{y}+2$	$\bar{z}$	$x+2$	$\bar{y}+\frac{5}{2}$	$z+\frac{1}{2}$
	$\vdots$			$\vdots$			$\vdots$			$\vdots$	
x	y	$z+1$	$\bar{x}$	$y+\frac{1}{2}$	$\bar{z}+\frac{3}{2}$	$\bar{x}$	$\bar{y}$	$\bar{z}+1$	x	$\bar{y}+\frac{1}{2}$	$z+\frac{3}{2}$
$x+1$	y	$z+1$	$\bar{x}+1$	$y+\frac{1}{2}$	$\bar{z}+\frac{3}{2}$	$\bar{x}+1$	$\bar{y}$	$\bar{z}+1$	$x+1$	$\bar{y}+\frac{1}{2}$	$z+\frac{3}{2}$
$x+2$	y	$z+1$	$\bar{x}+2$	$y+\frac{1}{2}$	$\bar{z}+\frac{3}{2}$	$\bar{x}+2$	$\bar{y}$	$\bar{z}+1$	$x+2$	$\bar{y}+\frac{1}{2}$	$z+\frac{3}{2}$
	$\vdots$			$\vdots$			$\vdots$			$\vdots$	
x	$y+1$	$z+1$	$\bar{x}$	$y+\frac{3}{2}$	$\bar{z}+\frac{3}{2}$	$\bar{x}$	$\bar{y}+1$	$\bar{z}+1$	x	$\bar{y}+\frac{3}{2}$	$z+\frac{3}{2}$
$x+1$	$y+1$	$z+1$	$\bar{x}+1$	$y+\frac{3}{2}$	$\bar{z}+\frac{3}{2}$	$\bar{x}+1$	$\bar{y}+1$	$\bar{z}+1$	$x+1$	$\bar{y}+\frac{3}{2}$	$z+\frac{3}{2}$
$x+2$	$y+1$	$z+1$	$\bar{x}+2$	$y+\frac{3}{2}$	$\bar{z}+\frac{3}{2}$	$\bar{x}+2$	$\bar{y}+1$	$\bar{z}+1$	$x+2$	$\bar{y}+\frac{3}{2}$	$z+\frac{3}{2}$
	$\vdots$			$\vdots$			$\vdots$			$\vdots$	
$x+u_1$	$y+u_2$	$z+u_3$	$\bar{x}+u_1$	$y+u_2+\frac{1}{2}$	$\bar{z}+u_3+\frac{1}{2}$	$\bar{x}+u_1$	$\bar{y}+u_2$	$\bar{z}+u_3$	$x+u_1$	$\bar{y}+u_2+\frac{1}{2}$	$z+u_3+\frac{1}{2}$
	$\vdots$			$\vdots$			$\vdots$			$\vdots$	

set of matrix–column pairs $\{(\boldsymbol{W},\boldsymbol{w})\}$. The set $\mathcal{T}_{\mathcal{G}} = \{(\boldsymbol{I},\boldsymbol{t})\}$ of all translations forms the *translation subgroup* $\mathcal{T}_{\mathcal{G}} \triangleleft \mathcal{G}$, which is a normal subgroup of $\mathcal{G}$ of finite index $[i]$. If $(\boldsymbol{W},\boldsymbol{w})$ is a fixed symmetry operation, then all the products $\mathcal{T}_{\mathcal{G}}(\boldsymbol{W},\boldsymbol{w}) = \{(\boldsymbol{I},\boldsymbol{t})(\boldsymbol{W},\boldsymbol{w})\} = \{(\boldsymbol{W},\boldsymbol{w}+\boldsymbol{t})\}$ of translations with $(\boldsymbol{W},\boldsymbol{w})$ have the same rotation part $\boldsymbol{W}$. Conversely, every symmetry operation W of $\mathcal{G}$ with the same matrix part $\boldsymbol{W}$ is represented in the set $\mathcal{T}_{\mathcal{G}}(\boldsymbol{W},\boldsymbol{w})$. The infinite set of symmetry operations $\mathcal{T}_{\mathcal{G}}(\boldsymbol{W},\boldsymbol{w})$ is called a coset of the right coset decomposition of $\mathcal{G}$ with respect to $\mathcal{T}_{\mathcal{G}}$, and $(\boldsymbol{W},\boldsymbol{w})$ its coset representative. In this way, the symmetry operations of $\mathcal{G}$ can be distributed into a finite set of infinite cosets, the elements of which are obtained by the combination of a coset representative $(\boldsymbol{W}_j,\boldsymbol{w}_j)$ and the infinite set $\mathcal{T}_{\mathcal{G}} = \{(\boldsymbol{I},\boldsymbol{t})\}$ of translations (*cf.* Section 1.3.3.2):

$$\mathcal{G} = \mathcal{T}_{\mathcal{G}} \cup \mathcal{T}_{\mathcal{G}}(\boldsymbol{W}_2,\boldsymbol{w}_2) \cup \cdots \cup \mathcal{T}_{\mathcal{G}}(\boldsymbol{W}_m,\boldsymbol{w}_m) \cup \cdots \cup \mathcal{T}_{\mathcal{G}}(\boldsymbol{W}_i,\boldsymbol{w}_i), \quad (1.4.2.1)$$

where $(\boldsymbol{W}_1,\boldsymbol{w}_1) = (\boldsymbol{I},\boldsymbol{o})$ is omitted. Obviously, the coset representatives $(\boldsymbol{W}_j,\boldsymbol{w}_j)$ of the decomposition $(\mathcal{G}:\mathcal{T}_{\mathcal{G}})$ represent in a clear and compact way the infinite number of symmetry operations of the space group $\mathcal{G}$. Each coset in the decomposition $(\mathcal{G}:\mathcal{T}_{\mathcal{G}})$ is characterized by its linear part $\boldsymbol{W}_j$ and its entries differ only by lattice translations. The translations $(\boldsymbol{I},\boldsymbol{t}) \in \mathcal{T}_{\mathcal{G}}$ form the first coset with the identity $(\boldsymbol{I},\boldsymbol{o})$ as a coset representative. The symmetry operations with rotation part $\boldsymbol{W}_2$ form the second coset *etc*. The number of cosets equals the number of different matrices $\boldsymbol{W}_j$ of the symmetry operations of the space group. This number $[i]$ is always finite and is equal to the order of the point group $\mathcal{P}_{\mathcal{G}}$ of the space group (*cf.* Section 1.3.3.2).

For each space group, a set of coset representatives $\{(\boldsymbol{W}_j,\boldsymbol{w}_j), 1 \le j \le [i]\}$ of the decomposition $(\mathcal{G}:\mathcal{T}_{\mathcal{G}})$ is listed under the general-position block of the space-group tables. In general, any element of a coset may be chosen as a coset representative. For convenience, the representatives listed in the space-group tables are always chosen such that the components $w_{j,k}$, $k = 1, 2, 3$, of the translation parts $\boldsymbol{w}_j$ fulfil $0 \le w_{j,k} < 1$ (by subtracting integers). To save space, each matrix–column pair $(\boldsymbol{W}_j,\boldsymbol{w}_j)$ is represented by the corresponding *coordinate triplet* (*cf.* Section 1.2.2.3 for the shorthand notation of matrix–column pairs).

Example

The right coset decomposition of $P2_1/c$, No. 14 (unique axis b, cell choice 1) with respect to its translation subgroup is shown in Table 1.4.2.6. All possible symmetry operations of $P2_1/c$ are distributed into four cosets:

The first column represents the infinitely many translations $t = (\boldsymbol{I},\boldsymbol{t}) = x+u_1, y+u_2, z+u_3 = \{1|u_1,u_2,u_3\}$ of the translation subgroup $\mathcal{T}$ of $P2_1/c$. The numbers u_1, u_2 and u_3 are positive or negative integers. The identity operation $(\boldsymbol{I},\boldsymbol{o})$ is usually chosen as a coset representative.

The third coset of the decomposition $(\mathcal{G}:\mathcal{T}_{\mathcal{G}})$ represents the infinite set of inversions $(-\boldsymbol{I},\boldsymbol{t}) = \bar{x}+u_1, \bar{y}+u_2, \bar{z}+u_3 = \{\bar{1}|u_1,u_2,u_3\}$ of the space group $P2_1/c$ with inversion centres located at $u_1/2, u_2/2, u_3/2$ (*cf.* Section 1.2.2.4 for the determination of the location of the inversion centres). The inversion in the origin, *i.e.* $\bar{x},\bar{y},\bar{z} = \{\bar{1}|0\}$, is taken as a coset representative.

The coset representative of the second coset is the twofold screw rotation $\{2_{010}|0,\frac{1}{2},\frac{1}{2}\}$ around the line $0, y, \frac{1}{4}$, followed by its infinite combinations with all lattice translations: $\bar{x}+u_1$, $y+\frac{1}{2}+u_2, \bar{z}+\frac{1}{2}+u_3 = \{2_{010}|u_1,\frac{1}{2}+u_2,\frac{1}{2}+u_3\}$. These are twofold screw rotations around the lines $u_1/2, y, u_3/2+\frac{1}{4}$ with screw components $\begin{pmatrix} 0 \\ \frac{1}{2}+u_2 \\ 0 \end{pmatrix}$.

The symmetry operations of the fourth column represented by $x+u_1,\ \bar{y}+\frac{1}{2}+u_2,\ z+\frac{1}{2}+u_3 = \{m_{010}|u_1,\frac{1}{2}+u_2,\frac{1}{2}+u_3\}$ correspond to glide reflections with glide components $\begin{pmatrix} u_1 \\ 0 \\ \frac{1}{2}+u_3 \end{pmatrix}$ through the (infinite) set of glide planes at $x, \frac{1}{4}, z$;