

1. INTRODUCTION TO SPACE-GROUP SYMMETRY

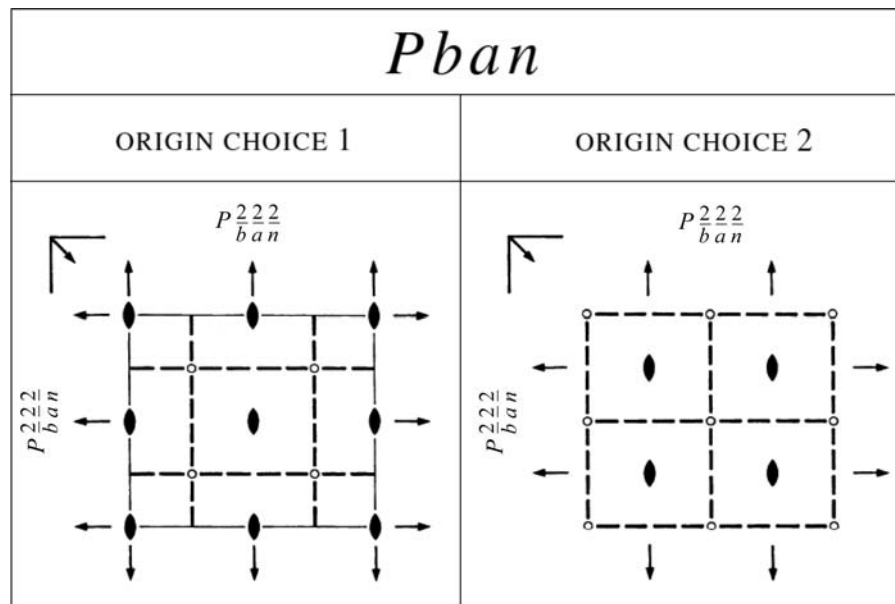


Figure 1.5.3.2 Two possible origin choices for the orthorhombic space group *Pban* (50). Origin choice 1 is on 222, whereas origin choice 2 is on $\bar{1}$.

hexagonal cell ($|\mathbf{a}_{\text{hex}}| = |\mathbf{b}_{\text{hex}}|$; $\mathbf{c}_{\text{hex}} \perp \mathbf{a}_{\text{hex}}, \mathbf{b}_{\text{hex}}$; $\gamma = 120^\circ$) with a volume three times larger than that of the primitive rhombohedral cell. The second presentation is given with a primitive rhombohedral cell with $a_{\text{rh}} = b_{\text{rh}} = c_{\text{rh}}$ and $\alpha_{\text{rh}} = \beta_{\text{rh}} = \gamma_{\text{rh}}$. The relation between the two types of cell is illustrated in Fig. 1.5.3.3 for the space group *R3m* (160). In the hexagonal cell, the coordinates of the special position with site symmetry $3m$ are 0, 0, z , whereas in the rhombohedral cell the same special position has coordinates x, x, x . If we refer to the transformations of the primitive rhombohedral cell cited in Table 1.5.1.1, we observe two different centring with three possible orientations R_1, R_2 and R_3 which are related by $\pm 120^\circ$ to each other. The two kinds of centring, called *obverse* and *reverse*, are illustrated in Fig. 1.5.1.6. A rotation of 180° around the rhombohedral axis relates the obverse and reverse descriptions of the rhombohedral lattice. The obverse triple R cells have lattice points at $0, 0, 0$; $\frac{2}{3}, \frac{1}{3}, \frac{1}{3}$; $\frac{1}{3}, \frac{2}{3}, \frac{2}{3}$, whereas the reverse R cells have lattice points at $0, 0, 0$; $\frac{1}{3}, \frac{2}{3}, \frac{1}{3}$; $\frac{2}{3}, \frac{1}{3}, \frac{2}{3}$. The triple hexagonal cell R_1 of the obverse setting (*i.e.* $\mathbf{a}_{\text{hex}} = \mathbf{a}_{\text{rh}} - \mathbf{b}_{\text{rh}}$, $\mathbf{b}_{\text{hex}} = \mathbf{b}_{\text{rh}} - \mathbf{c}_{\text{rh}}$, $\mathbf{c}_{\text{hex}} = \mathbf{a}_{\text{rh}} + \mathbf{b}_{\text{rh}} + \mathbf{c}_{\text{rh}}$) has been used in the description of the rhombohedral space groups in this volume (*cf.* Table 1.5.1.1 and Fig. 1.5.3.3).

The hexagonal lattice can be referred to a centred rhombohedral cell, called the *D* cell (*cf.* Table 1.5.1.1). The centring points of this cell are $0, 0, 0$, $\frac{1}{3}, \frac{1}{3}, \frac{1}{3}$ and $\frac{2}{3}, \frac{2}{3}, \frac{2}{3}$. However, the *D* cell is rarely used in crystallography.

1.5.3.2. Examples

1.5.3.2.1. Transformations between different settings of $P2_1/c$

In the space-group tables of this volume, the monoclinic space group $P2_1/c$ (14) is described in six different settings: for each of the ‘unique axis *b*’ and ‘unique axis *c*’ settings there are three descriptions specified by different cell choices (*cf.* Section 2.1.3.15). The different settings are identified by the appropriate full Hermann–Mauguin symbols. The basis transfor-

mations (\mathbf{P}, \mathbf{p}) between the different settings are completely specified by the linear part of the transformation, the 3×3 matrix \mathbf{P} [*cf.* equation (1.5.1.4)], as all settings of $P2_1/c$ refer to the same origin, *i.e.* $\mathbf{p} = \mathbf{o}$. The transformation matrices \mathbf{P} necessary for switching between the different descriptions of $P2_1/c$ can either be read off directly or constructed from the transformation-matrix data listed in Table 1.5.1.1.

(A) Transformation from $P12_1/c1$ (unique axis *b*, cell choice 1) to $P112_1/a$ (unique axis *c*, cell choice 1). The change of the direction of the screw axis 2_1 indicates that the unique direction \mathbf{b}

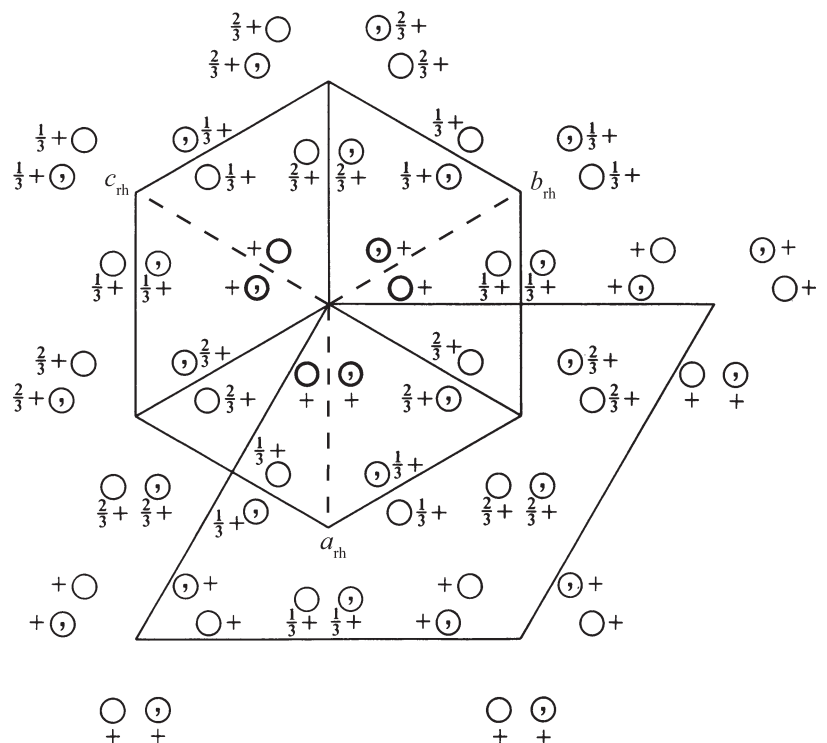


Figure 1.5.3.3 General-position diagram of the space group *R3m* (160) showing the relation between the hexagonal and rhombohedral axes in the obverse setting: $\mathbf{a}_{\text{rh}} = \frac{1}{3}(2\mathbf{a}_{\text{hex}} + \mathbf{b}_{\text{hex}} + \mathbf{c}_{\text{hex}})$, $\mathbf{b}_{\text{rh}} = \frac{1}{3}(-\mathbf{a}_{\text{hex}} + \mathbf{b}_{\text{hex}} + \mathbf{c}_{\text{hex}})$, $\mathbf{c}_{\text{rh}} = \frac{1}{3}(-\mathbf{a}_{\text{hex}} - 2\mathbf{b}_{\text{hex}} + \mathbf{c}_{\text{hex}})$.