

1.5. TRANSFORMATIONS OF COORDINATE SYSTEMS

Table 1.5.1.1

 Selected 3×3 transformation matrices P and $Q = P^{-1}$

 For inverse transformations (against the arrow) replace P by Q and *vice versa*.

Transformation	P	$Q = P^{-1}$	Crystal system
Cell choice 1 \rightarrow cell choice 2: $\begin{cases} P \rightarrow P \\ C \rightarrow A \end{cases}$ Cell choice 2 \rightarrow cell choice 3: $\begin{cases} P \rightarrow P \\ A \rightarrow I \end{cases}$ Unique axis b invariant Cell choice 3 \rightarrow cell choice 1: $\begin{cases} P \rightarrow P \\ I \rightarrow C \end{cases}$ (Fig. 1.5.1.2a)	$\begin{pmatrix} \bar{1} & 0 & 1 \\ 0 & 1 & 0 \\ \bar{1} & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & \bar{1} \\ 0 & 1 & 0 \\ 1 & 0 & \bar{1} \end{pmatrix}$	Monoclinic (<i>cf.</i> Sections 1.5.4.3 and 2.1.3.15)
Cell choice 1 \rightarrow cell choice 2: $\begin{cases} P \rightarrow P \\ A \rightarrow B \end{cases}$ Cell choice 2 \rightarrow cell choice 3: $\begin{cases} P \rightarrow P \\ B \rightarrow I \end{cases}$ Unique axis c invariant Cell choice 3 \rightarrow cell choice 1: $\begin{cases} P \rightarrow P \\ I \rightarrow A \end{cases}$ (Fig. 1.5.1.2b)	$\begin{pmatrix} 0 & \bar{1} & 0 \\ 1 & \bar{1} & 0 \\ 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} \bar{1} & 1 & 0 \\ \bar{1} & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$	Monoclinic (<i>cf.</i> Sections 1.5.4.3 and 2.1.3.15)
Cell choice 1 \rightarrow cell choice 2: $\begin{cases} P \rightarrow P \\ B \rightarrow C \end{cases}$ Cell choice 2 \rightarrow cell choice 3: $\begin{cases} P \rightarrow P \\ C \rightarrow I \end{cases}$ Unique axis a invariant Cell choice 3 \rightarrow cell choice 1: $\begin{cases} P \rightarrow P \\ I \rightarrow B \end{cases}$ (Fig. 1.5.1.2c)	$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & \bar{1} \\ 0 & 1 & \bar{1} \end{pmatrix}$	$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \bar{1} & 1 \\ 0 & \bar{1} & 0 \end{pmatrix}$	Monoclinic (<i>cf.</i> Sections 1.5.4.3 and 2.1.3.15)
Unique axis b \rightarrow unique axis c Cell choice 1: $\begin{cases} P \rightarrow P \\ C \rightarrow A \end{cases}$ Cell choice 2: $\begin{cases} P \rightarrow P \\ A \rightarrow B \end{cases}$ Cell choice invariant Cell choice 3: $\begin{cases} P \rightarrow P \\ I \rightarrow I \end{cases}$	$\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$	Monoclinic (<i>cf.</i> Sections 1.5.4.3 and 2.1.3.15)
Unique axis b \rightarrow unique axis a Cell choice 1: $\begin{cases} P \rightarrow P \\ C \rightarrow B \end{cases}$ Cell choice 2: $\begin{cases} P \rightarrow P \\ A \rightarrow C \end{cases}$ Cell choice invariant Cell choice 3: $\begin{cases} P \rightarrow P \\ I \rightarrow I \end{cases}$	$\begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$	Monoclinic (<i>cf.</i> Sections 1.5.4.3 and 2.1.3.15)
Unique axis c \rightarrow unique axis a Cell choice 1: $\begin{cases} P \rightarrow P \\ A \rightarrow B \end{cases}$ Cell choice 2: $\begin{cases} P \rightarrow P \\ B \rightarrow C \end{cases}$ Cell choice invariant Cell choice 3: $\begin{cases} P \rightarrow P \\ I \rightarrow I \end{cases}$	$\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$	Monoclinic (<i>cf.</i> Sections 1.5.4.3 and 2.1.3.15)
$I \rightarrow P$ (Fig. 1.5.1.3)	$\begin{pmatrix} \bar{1} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \bar{1} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \bar{1} \end{pmatrix}$	$\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$	Orthorhombic Tetragonal Cubic
$F \rightarrow P$ (Fig. 1.5.1.4)	$\begin{pmatrix} 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & 0 \end{pmatrix}$	$\begin{pmatrix} \bar{1} & 1 & 1 \\ 1 & \bar{1} & 1 \\ 1 & 1 & \bar{1} \end{pmatrix}$	Orthorhombic Tetragonal Cubic

1. INTRODUCTION TO SPACE-GROUP SYMMETRY

Table 1.5.1.1 (continued)

Transformation	P	$Q = P^{-1}$	Crystal system
$(\mathbf{b}, \mathbf{a}, \bar{\mathbf{c}}) \rightarrow (\mathbf{a}, \mathbf{b}, \mathbf{c})$	$\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & \bar{1} \end{pmatrix}$	$\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & \bar{1} \end{pmatrix}$	Unconventional orthorhombic setting
$(\mathbf{c}, \mathbf{a}, \mathbf{b}) \rightarrow (\mathbf{a}, \mathbf{b}, \mathbf{c})$	$\begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$	Unconventional orthorhombic setting
$(\bar{\mathbf{c}}, \mathbf{b}, \mathbf{a}) \rightarrow (\mathbf{a}, \mathbf{b}, \mathbf{c})$	$\begin{pmatrix} 0 & 0 & \bar{1} \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ \bar{1} & 0 & 0 \end{pmatrix}$	Unconventional orthorhombic setting
$(\mathbf{b}, \mathbf{c}, \mathbf{a}) \rightarrow (\mathbf{a}, \mathbf{b}, \mathbf{c})$	$\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$	Unconventional orthorhombic setting
$(\mathbf{a}, \bar{\mathbf{c}}, \mathbf{b}) \rightarrow (\mathbf{a}, \mathbf{b}, \mathbf{c})$	$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & \bar{1} \\ 0 & 1 & 0 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & \bar{1} & 0 \end{pmatrix}$	Unconventional orthorhombic setting
$\left. \begin{array}{l} P \rightarrow C_1 \\ I \rightarrow F_1 \end{array} \right\}$ (Fig. 1.5.1.5), \mathbf{c} axis invariant	$\begin{pmatrix} 1 & 1 & 0 \\ \bar{1} & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$	Tetragonal (cf. Section 1.5.4.3)
$\left. \begin{array}{l} P \rightarrow C_2 \\ I \rightarrow F_2 \end{array} \right\}$ (Fig. 1.5.1.5), \mathbf{c} axis invariant	$\begin{pmatrix} 1 & \bar{1} & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$	Tetragonal (cf. Section 1.5.4.3)
Primitive rhombohedral cell \rightarrow triple hexagonal cell R_1 , obverse setting (Fig. 1.5.1.6a,c)	$\begin{pmatrix} 1 & 0 & 1 \\ \bar{1} & 1 & 1 \\ 0 & \bar{1} & 1 \end{pmatrix}$	$\begin{pmatrix} \frac{2}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{2}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & \frac{2}{3} \end{pmatrix}$	Rhombohedral space groups (cf. Section 1.5.4.3)
Primitive rhombohedral cell \rightarrow triple hexagonal cell R_2 , obverse setting (Fig. 1.5.1.6c)	$\begin{pmatrix} 0 & \bar{1} & 1 \\ 1 & 0 & 1 \\ \bar{1} & 1 & 1 \end{pmatrix}$	$\begin{pmatrix} \frac{1}{3} & \frac{2}{3} & \frac{1}{3} \\ \frac{2}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & \frac{2}{3} \end{pmatrix}$	Rhombohedral space groups (cf. Section 1.5.4.3)
Primitive rhombohedral cell \rightarrow triple hexagonal cell R_3 , obverse setting (Fig. 1.5.1.6c)	$\begin{pmatrix} \bar{1} & 1 & 1 \\ 0 & \bar{1} & 1 \\ 1 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} \frac{1}{3} & \frac{1}{3} & \frac{2}{3} \\ \frac{1}{3} & \frac{2}{3} & \frac{1}{3} \\ \frac{2}{3} & \frac{1}{3} & \frac{1}{3} \end{pmatrix}$	Rhombohedral space groups (cf. Section 1.5.4.3)
Primitive rhombohedral cell \rightarrow triple hexagonal cell R_1 , reverse setting (Fig. 1.5.1.6d)	$\begin{pmatrix} \bar{1} & 0 & 1 \\ 1 & \bar{1} & 1 \\ 0 & 1 & 1 \end{pmatrix}$	$\begin{pmatrix} \frac{2}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{2}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & \frac{2}{3} \end{pmatrix}$	Rhombohedral space groups (cf. Section 1.5.4.3)
Primitive rhombohedral cell \rightarrow triple hexagonal cell R_2 , reverse setting (Fig. 1.5.1.6b,d)	$\begin{pmatrix} 0 & 1 & 1 \\ \bar{1} & 0 & 1 \\ 1 & \bar{1} & 1 \end{pmatrix}$	$\begin{pmatrix} \frac{1}{3} & \frac{2}{3} & \frac{1}{3} \\ \frac{2}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & \frac{2}{3} \end{pmatrix}$	Rhombohedral space groups (cf. Section 1.5.4.3)
Primitive rhombohedral cell \rightarrow triple hexagonal cell R_3 , reverse setting (Fig. 1.5.1.6d)	$\begin{pmatrix} 1 & \bar{1} & 1 \\ 0 & 1 & 1 \\ \bar{1} & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} \frac{1}{3} & \frac{1}{3} & \frac{2}{3} \\ \frac{1}{3} & \frac{2}{3} & \frac{1}{3} \\ \frac{2}{3} & \frac{1}{3} & \frac{1}{3} \end{pmatrix}$	Rhombohedral space groups (cf. Section 1.5.4.3)
Hexagonal cell $P \rightarrow$ orthohexagonal centred cell C_1 (Fig. 1.5.1.7)	$\begin{pmatrix} 1 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 1 & \frac{1}{2} & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$	Trigonal Hexagonal (cf. Section 1.5.4.3)
Hexagonal cell $P \rightarrow$ orthohexagonal centred cell C_2 (Fig. 1.5.1.7)	$\begin{pmatrix} 1 & \bar{1} & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$	Trigonal Hexagonal (cf. Section 1.5.4.3)

1.5. TRANSFORMATIONS OF COORDINATE SYSTEMS

Table 1.5.1.1 (continued)

Transformation	P	$Q = P^{-1}$	Crystal system
Hexagonal cell $P \rightarrow$ orthohexagonal centred cell C_3 (Fig. 1.5.1.7)	$\begin{pmatrix} 0 & \bar{2} & 0 \\ 1 & \bar{1} & 0 \\ 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} \bar{1} & 1 & 0 \\ \bar{1} & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$	Trigonal Hexagonal (cf. Section 1.5.4.3)
Hexagonal cell $P \rightarrow$ triple hexagonal cell H_1 (Fig. 1.5.1.8)	$\begin{pmatrix} 1 & 1 & 0 \\ \bar{1} & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} \frac{2}{3} & \frac{1}{3} & 0 \\ \frac{1}{3} & \frac{1}{3} & 0 \\ 0 & 0 & 1 \end{pmatrix}$	Trigonal Hexagonal (cf. Section 1.5.4.3)
Hexagonal cell $P \rightarrow$ triple hexagonal cell H_2 (Fig. 1.5.1.8)	$\begin{pmatrix} 2 & \bar{1} & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} \frac{1}{3} & \frac{1}{3} & 0 \\ \frac{1}{3} & \frac{2}{3} & 0 \\ 0 & 0 & 1 \end{pmatrix}$	Trigonal Hexagonal (cf. Section 1.5.4.3)
Hexagonal cell $P \rightarrow$ triple hexagonal cell H_3 (Fig. 1.5.1.8)	$\begin{pmatrix} 1 & \bar{2} & 0 \\ 2 & \bar{1} & 0 \\ 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} \frac{1}{3} & \frac{2}{3} & 0 \\ \frac{2}{3} & \frac{1}{3} & 0 \\ 0 & 0 & 1 \end{pmatrix}$	Trigonal Hexagonal (cf. Section 1.5.4.3)
Hexagonal cell $P \rightarrow$ triple rhombohedral cell D_1	$\begin{pmatrix} 1 & 0 & \bar{1} \\ 0 & 1 & \bar{1} \\ 1 & 1 & 1 \end{pmatrix}$	$\begin{pmatrix} \frac{2}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{2}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & \frac{2}{3} \end{pmatrix}$	Trigonal Hexagonal (cf. Section 1.5.4.3)
Hexagonal cell $P \rightarrow$ triple rhombohedral cell D_2	$\begin{pmatrix} \bar{1} & 0 & 1 \\ 0 & \bar{1} & 1 \\ 1 & 1 & 1 \end{pmatrix}$	$\begin{pmatrix} \frac{2}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{2}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & \frac{2}{3} \end{pmatrix}$	Trigonal Hexagonal (cf. Section 1.5.4.3)
Triple hexagonal cell R , obverse setting \rightarrow C -centred monoclinic cell, unique axis \mathbf{b} , cell choice 1 (Fig. 1.5.1.9a)	$\begin{pmatrix} \frac{2}{3} & 0 & 0 \\ \frac{1}{3} & 1 & 0 \\ \frac{2}{3} & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} \frac{2}{3} & 0 & 0 \\ \frac{1}{3} & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$	Rhombohedral space groups (cf. Section 1.5.4.3)
Triple hexagonal cell R , obverse setting \rightarrow C -centred monoclinic cell, unique axis \mathbf{b} , cell choice 2 (Fig. 1.5.1.9a)	$\begin{pmatrix} \frac{1}{3} & \bar{1} & 0 \\ \frac{1}{3} & \bar{1} & 0 \\ \frac{2}{3} & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} \frac{1}{3} & \frac{2}{3} & 0 \\ \frac{1}{3} & \frac{1}{3} & 0 \\ 1 & 1 & 1 \end{pmatrix}$	Rhombohedral space groups (cf. Section 1.5.4.3)
Triple hexagonal cell R , obverse setting \rightarrow C -centred monoclinic cell, unique axis \mathbf{b} , cell choice 3 (Fig. 1.5.1.9a)	$\begin{pmatrix} \frac{1}{3} & 1 & 0 \\ \frac{2}{3} & 0 & 0 \\ \frac{2}{3} & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 & \frac{1}{3} & 0 \\ 1 & \frac{1}{3} & 0 \\ 0 & 1 & 1 \end{pmatrix}$	Rhombohedral space groups (cf. Section 1.5.4.3)
Triple hexagonal cell R , obverse setting \rightarrow A -centred monoclinic cell, unique axis \mathbf{c} , cell choice 1 (Fig. 1.5.1.9b)	$\begin{pmatrix} 0 & \frac{2}{3} & 0 \\ 0 & \frac{1}{3} & 1 \\ 1 & \frac{2}{3} & 0 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 & 1 \\ \frac{2}{3} & 0 & 0 \\ \frac{1}{3} & 1 & 0 \end{pmatrix}$	Rhombohedral space groups (cf. Section 1.5.4.3)
Triple hexagonal cell R , obverse setting \rightarrow A -centred monoclinic cell, unique axis \mathbf{c} , cell choice 2 (Fig. 1.5.1.9b)	$\begin{pmatrix} 0 & \frac{1}{3} & \bar{1} \\ 0 & \frac{1}{3} & \bar{1} \\ 1 & \frac{2}{3} & 0 \end{pmatrix}$	$\begin{pmatrix} \bar{1} & 1 & 1 \\ \frac{2}{3} & \frac{2}{3} & 0 \\ \frac{1}{3} & \frac{1}{3} & 0 \end{pmatrix}$	Rhombohedral space groups (cf. Section 1.5.4.3)
Triple hexagonal cell R , obverse setting \rightarrow A -centred monoclinic cell, unique axis \mathbf{c} , cell choice 3 (Fig. 1.5.1.9b)	$\begin{pmatrix} 0 & \frac{1}{3} & 1 \\ 0 & \frac{2}{3} & 0 \\ 1 & \frac{2}{3} & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & \bar{1} & 1 \\ 0 & \frac{2}{3} & 0 \\ 1 & \frac{1}{3} & 0 \end{pmatrix}$	Rhombohedral space groups (cf. Section 1.5.4.3)
Primitive rhombohedral cell \rightarrow C -centred monoclinic cell, unique axis \mathbf{b} , cell choice 1 (Fig. 1.5.1.10a)	$\begin{pmatrix} 0 & 0 & 1 \\ \bar{1} & 1 & 1 \\ \bar{1} & \bar{1} & 1 \end{pmatrix}$	$\begin{pmatrix} 1 & \frac{1}{2} & \frac{1}{2} \\ 0 & \frac{1}{2} & \frac{1}{2} \\ 1 & 0 & 0 \end{pmatrix}$	Rhombohedral space groups (cf. Section 1.5.4.3)
Primitive rhombohedral cell \rightarrow C -centred monoclinic cell, unique axis \mathbf{b} , cell choice 2 (Fig. 1.5.1.10a)	$\begin{pmatrix} \bar{1} & \bar{1} & 1 \\ 0 & 0 & 1 \\ \bar{1} & 1 & 1 \end{pmatrix}$	$\begin{pmatrix} \frac{1}{2} & 1 & \frac{1}{2} \\ \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 1 & 0 \end{pmatrix}$	Rhombohedral space groups (cf. Section 1.5.4.3)

Table 1.5.1.1 (continued)

Transformation	P	$Q = P^{-1}$	Crystal system
Primitive rhombohedral cell \rightarrow C -centred monoclinic cell, unique axis \mathbf{b} , cell choice 3 (Fig. 1.5.1.10a)	$\begin{pmatrix} \bar{1} & 1 & 1 \\ \bar{1} & \bar{1} & 1 \\ 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} \frac{1}{2} & \frac{1}{2} & 1 \\ \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$	Rhombohedral space groups (cf. Section 1.5.4.3)
Primitive rhombohedral cell \rightarrow A -centred monoclinic cell, unique axis \mathbf{c} , cell choice 1 (Fig. 1.5.1.10b)	$\begin{pmatrix} 1 & 0 & 0 \\ 1 & \bar{1} & 1 \\ 1 & \bar{1} & \bar{1} \end{pmatrix}$	$\begin{pmatrix} 1 & 0 & 0 \\ 1 & \frac{1}{2} & \frac{1}{2} \\ 0 & \frac{1}{2} & \frac{1}{2} \end{pmatrix}$	Rhombohedral space groups (cf. Section 1.5.4.3)
Primitive rhombohedral cell \rightarrow A -centred monoclinic cell, unique axis \mathbf{c} , cell choice 2 (Fig. 1.5.1.10b)	$\begin{pmatrix} 1 & \bar{1} & \bar{1} \\ 1 & 0 & 0 \\ 1 & \bar{1} & 1 \end{pmatrix}$	$\begin{pmatrix} 0 & 1 & 0 \\ \frac{1}{2} & 1 & \frac{1}{2} \\ \frac{1}{2} & 0 & \frac{1}{2} \end{pmatrix}$	Rhombohedral space groups (cf. Section 1.5.4.3)
Primitive rhombohedral cell \rightarrow A -centred monoclinic cell, unique axis \mathbf{c} , cell choice 3 (Fig. 1.5.1.10b)	$\begin{pmatrix} 1 & \bar{1} & 1 \\ 1 & \bar{1} & \bar{1} \\ 1 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 1 \\ \frac{1}{2} & \frac{1}{2} & 1 \\ \frac{1}{2} & \frac{1}{2} & 0 \end{pmatrix}$	Rhombohedral space groups (cf. Section 1.5.4.3)

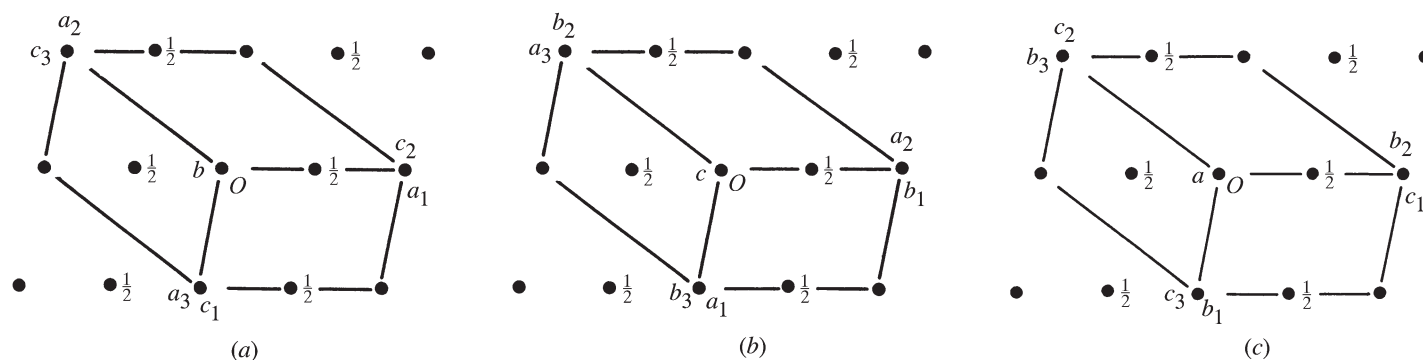


Figure 1.5.1.2

Monoclinic centred lattice, projected along the unique axis. The origin for all the cells is the same. The fractions $\frac{1}{2}$ indicate the height of the lattice points along the axis of projection.

(a) Unique axis b :

Cell choice 1: C -centred cell a_1, b, c_1 .

Cell choice 2: A -centred cell a_2, b, c_2 .

Cell choice 3: I -centred cell a_3, b, c_3 .

(b) Unique axis c :

Cell choice 1: A -centred cell a_1, b_1, c .

Cell choice 2: B -centred cell a_2, b_2, c .

Cell choice 3: I -centred cell a_3, b_3, c .

(c) Unique axis a :

Cell choice 1: B -centred cell a, b_1, c_1 .

Cell choice 2: C -centred cell a, b_2, c_2 .

Cell choice 3: I -centred cell a, b_3, c_3 .

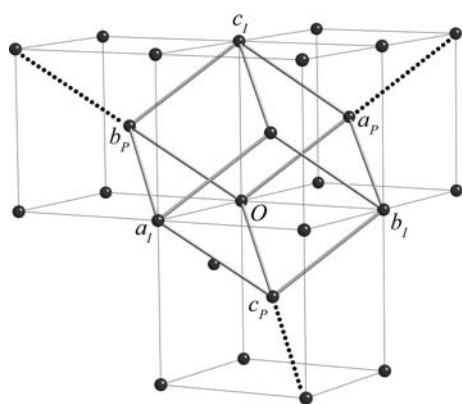


Figure 1.5.1.3

Body-centred cell I with a_I, b_I, c_I and a corresponding primitive cell P with a_P, b_P, c_P . The origin for both cells is O . A cubic I cell with lattice constant a_c can be considered as a primitive rhombohedral cell with $a_{rh} = a_c \frac{1}{2} \sqrt{3}$ and $\alpha = 109.47^\circ$ (rhombohedral axes) or a triple hexagonal cell with $a_{hex} = a_c \sqrt{2}$ and $c_{hex} = a_c \frac{1}{2} \sqrt{3}$ (hexagonal axes).

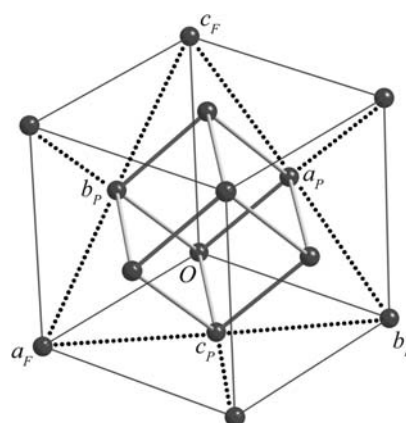


Figure 1.5.1.4

Face-centred cell F with a_F, b_F, c_F and a corresponding primitive cell P with a_P, b_P, c_P . The origin for both cells is O . A cubic F cell with lattice constant a_c can be considered as a primitive rhombohedral cell with $a_{rh} = a_c \frac{1}{2} \sqrt{2}$ and $\alpha = 60^\circ$ (rhombohedral axes) or a triple hexagonal cell with $a_{hex} = a_c \frac{1}{2} \sqrt{2}$ and $c_{hex} = a_c \sqrt{3}$ (hexagonal axes).