

1. INTRODUCTION TO SPACE-GROUP SYMMETRY

These reflection conditions that are not related to space-group operations are given in Chapter 2.3 only for special positions. They may arise, however, also for different reasons. For example, a heavy atom at the origin of the space group $P2_12_12_1$ would generate F -centring with corresponding apparent absences (cf. the special position $4a$ of the space group $Pbca$ and the absences it generates).

We wish to point out that the most common ‘special-position absence’ in molecular structures is due to a heavy atom at the origin of the space group $P2_1/c$.

1.6.4. Tables of reflection conditions and possible space groups

BY H. D. FLACK AND U. SHMUELI

1.6.4.1. Introduction

The primary order of presentation of these tables of reflection conditions of space groups is the Bravais lattice. This order has been chosen because cell reduction on unit-cell dimensions leads to the Bravais lattice as described as stage 1 in Section 1.6.2.1. Within the space groups of a given Bravais lattice, the entries are arranged by Laue class, which may be obtained as described as stage 2 in Section 1.6.2.1. As a consequence of these decisions about the way the tables are structured, in the hexagonal family one finds for the Bravais lattice hP that the Laue classes $\bar{3}$, $\bar{3}m$, $\bar{3}1m$, $6/m$ and $6/mmm$ are grouped together.

As an aid in the study of naturally occurring macromolecules and compounds made by enantioselective synthesis, the space groups of enantiomerically pure compounds (Sohncke space groups) are typeset in bold.

The tables show, on the left, sets of reflection conditions and, on the right, those space groups that are compatible with the given set of reflection conditions. The reflection conditions, e.g. h or $k + l$, are to be understood as $h = 2n$ or $k + l = 2n$, respectively. All of the space groups in each table correspond to the same Patterson symmetry, which is indicated in the table header. This makes for easy comparison with the entries for the individual space groups in Chapter 2.3 of this volume, in which the Patterson symmetry is also very clearly shown. All space groups with a conventional choice of unit cell are included in Tables 1.6.4.2–1.6.4.30. All alternative settings displayed in Chapter 2.3 are thus included. The following further alternative settings, not displayed in Chapter 2.3, are also included: space group $Pb\bar{3}$ (205) and all the space groups with an hR Bravais lattice in the reverse setting with hexagonal axes.

Table 1.6.2.1 gives some relevant statistics drawn from Tables 1.6.4.2–1.6.4.30. The total number of space-group settings mentioned in these tables is 416. This number is considerably larger than the 230 space-group types described in Part 2 of this volume. The following example shows why the tables include data for several descriptions of the space-group types. At the stage of space-group determination for a crystal in the crystal class $mm2$, it is not yet known whether the twofold rotation axis lies along **a**, **b** or **c**. Consequently, space groups based on the three point groups $2mm$, $m2m$ and $mm2$ need to be considered.

In some texts dealing with space-group determination, a ‘diffraction symbol’ (sometimes also called an ‘extinction symbol’) in the form of a Hermann–Mauguin space-group symbol is used as a shorthand code for the reflection conditions and Laue class. These symbols were introduced by Buerger (1935, 1942,

1969) and a concise description is to be found in Looijenga-Vos & Buerger (2002). Nespolo *et al.* (2014) use them.

1.6.4.2. Examples of the use of the tables

- (1) If the Bravais lattice is oI and the Laue class is mmm , Table 1.6.4.1 directs us to Table 1.6.4.11. Given the observed reflection conditions

$$hkl: h + k + l = 2n, \quad 0kl: k = 2n, l = 2n, \quad h0l: h + l = 2n, \\ hk0: h + k = 2n, \quad h00: h = 2n, \quad 0k0: k = 2n, \quad 00l: l = 2n,$$

it is seen from Table 1.6.4.11 that the possible settings of the space groups are: $Ibm2$ (46), $Ic2m$ (46), $Ibmm$ (74) and $Icmm$ (74).

- (2) If the Bravais lattice is oP and the Laue class is mmm , Table 1.6.4.1 directs us to Table 1.6.4.7. If there are no conditions on $0kl$, the space groups $P222$ to $Pmnn$ should be searched. If the condition is $0kl: k = 2n$ or $l = 2n$, the space groups $Pbm2$ to $Pcnn$ should be searched. If the condition is $0kl: k + l = 2n$, the space groups $Pnm2_1$ to $Pnnn$ should be searched.
- (3) If the Bravais lattice is cP and the Laue class is $m\bar{3}$, Table 1.6.4.1 directs us to Table 1.6.4.25. If the conditions are $0kl: k = 2n$ and $h00: h = 2n$, it is readily seen that the space group is $Pa\bar{3}$.
- (4) If only the Bravais lattice is known or assumed, which is the case in powder-diffraction work (see Section 1.6.5.3), all tables of this section corresponding to this Bravais lattice need to be consulted. For example, if it is known that the Bravais lattice is of type cP , Table 1.6.4.1 tells us that the possible Laue classes are $m\bar{3}$ and $m\bar{3}m$, and the possible space groups can be found in Tables 1.6.4.25 and 1.6.4.26, respectively. The appropriate reflection conditions are of course given in these tables. All relevant tables can thus be located with the aid of Table 1.6.4.1 if the Bravais lattice is known.

1.6.5. Specialized methods of space-group determination

BY H. D. FLACK

1.6.5.1. Applications of resonant scattering to symmetry determination

1.6.5.1.1. Introduction

In small-molecule crystallography, it has been customary in crystal-structure analysis to make no use of the contribution of resonant scattering (otherwise called anomalous scattering and in older literature anomalous dispersion) other than in the specific area of absolute-structure and absolute-configuration determination. One may trace the causes of this situation to the weakness of the resonant-scattering contribution, to the high cost in time and labour of collecting intensity data sets containing measurements of all Friedel opposites and for a lack of any perceived or real need for the additional information that might be obtained from the effects of resonant scattering.

On the experimental side, the turning point came with the widespread distribution of area detectors for small-molecule crystallography, giving the potential to measure, at no extra cost, full-sphere data sets leading to the intensity differences between Friedel opposites hkl and $\bar{h}\bar{k}\bar{l}$. In 2015, the new methods of data analysis briefly presented here are in the stage of development

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1.6. METHODS OF SPACE-GROUP DETERMINATION

Table 1.6.4.1

Summary of Tables 1.6.4.2–1.6.4.30

Table No.	Bravais lattice	Laue class	Patterson symmetry	Comment
1.6.4.2	<i>aP</i>	$\bar{1}$	$P\bar{1}$	
1.6.4.3	<i>mP</i>	$2/m$	$P12/m1$	Unique <i>b</i>
1.6.4.4	<i>mS</i> (<i>mC</i> , <i>mA</i> , <i>mI</i>)	$2/m$	$C12/m1$, $A12/m1$, $I12/m1$	Unique <i>b</i>
1.6.4.5	<i>mP</i>	$2/m$	$P112/m$	Unique <i>c</i>
1.6.4.6	<i>mS</i> (<i>mA</i> , <i>mB</i> , <i>mI</i>)	$2/m$	$A112/m$, $B112/m$, $I112/m$	Unique <i>c</i>
1.6.4.7	<i>oP</i>	<i>mmm</i>	<i>Pmmm</i>	
1.6.4.8	<i>oS</i> (<i>oC</i>)	<i>mmm</i>	<i>Cmmm</i>	
1.6.4.9	<i>oS</i> (<i>oB</i>)	<i>mmm</i>	<i>Bmmm</i>	
1.6.4.10	<i>oS</i> (<i>oA</i>)	<i>mmm</i>	<i>Ammm</i>	
1.6.4.11	<i>oI</i>	<i>mmm</i>	<i>Immm</i>	
1.6.4.12	<i>oF</i>	<i>mmm</i>	<i>Fmmm</i>	
1.6.4.13	<i>tP</i>	$4/m$	$P4/m$	
1.6.4.14	<i>tP</i>	$4/mmm$	$P4/mmm$	
1.6.4.15	<i>tI</i>	$4/m$	$I4/m$	
1.6.4.16	<i>tI</i>	$4/mmm$	$I4/mmm$	
1.6.4.17	<i>hP</i>	$\bar{3}$	$P\bar{3}$	
1.6.4.18	<i>hP</i>	$\bar{3}1m$ and $\bar{3}m1$	$P\bar{3}1m$ and $P\bar{3}m1$	
1.6.4.19	<i>hP</i>	$6/m$	$P6/m$	
1.6.4.20	<i>hP</i>	$6/mmm$	$P6/mmm$	
1.6.4.21	<i>hR</i>	$\bar{3}$	$R\bar{3}$	Hexagonal axes
1.6.4.22	<i>hR</i>	$\bar{3}m$	$R\bar{3}m$	Hexagonal axes
1.6.4.23	<i>hR</i>	$\bar{3}$	$R\bar{3}$	Rhombohedral axes
1.6.4.24	<i>hR</i>	$\bar{3}m$	$R\bar{3}m$	Rhombohedral axes
1.6.4.25	<i>cP</i>	$m\bar{3}$	$Pm\bar{3}$	
1.6.4.26	<i>cP</i>	$m\bar{3}m$	$Pm\bar{3}m$	
1.6.4.27	<i>cI</i>	$m\bar{3}$	$Im\bar{3}$	
1.6.4.28	<i>cI</i>	$m\bar{3}m$	$Im\bar{3}m$	
1.6.4.29	<i>cF</i>	$m\bar{3}$	$Fm\bar{3}$	
1.6.4.30	<i>cF</i>	$m\bar{3}m$	$Fm\bar{3}m$	

Table 1.6.4.2

Reflection conditions and possible space groups with Bravais lattice *aP* and Laue class $\bar{1}$; Patterson symmetry $P\bar{1}$

Reflection conditions	Space group		Space group	
	group	No.	group	No.
	P1	1	$P\bar{1}$	2

Table 1.6.4.3

Reflection conditions and possible space groups with Bravais lattice *mP* and Laue class $2/m$; (monoclinic, unique axis *b*); Patterson symmetry $P12/m1$

Reflection conditions						Space group		Space group		Space group	
<i>h0l</i>	<i>0kl</i>	<i>hk0</i>	<i>0k0</i>	<i>h00</i>	<i>00l</i>	group	No.	group	No.	group	No.
						P2	3	<i>Pm</i>	6	$P2/m$	10
			<i>k</i>			P2₁	4	$P2_1/m$	11		
<i>h</i>				<i>h</i>		<i>Pa</i>	7	$P2/a$	13		
<i>h</i>			<i>k</i>	<i>h</i>		$P2_1/a$	14				
<i>l</i>					<i>l</i>	<i>Pc</i>	7	$P2/c$	13		
<i>l</i>			<i>k</i>		<i>l</i>	$P2_1/c$	14				
<i>h + l</i>				<i>h</i>	<i>l</i>	<i>Pn</i>	7	$P2/n$	13		
<i>h + l</i>			<i>k</i>	<i>h</i>	<i>l</i>	$P2_1/n$	14				

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Table 1.6.4.4

Reflection conditions and possible space groups with Bravais lattice mS (mC , mA , mI) and Laue class $2/m$ (monoclinic, unique axis b); Patterson symmetry $C12/m1$, $A12/m1$, $I12/m1$

Reflection conditions							Space group	No.	Space group	No.	Space group	No.
hkl	$h0l$	$0kl$	$hk0$	$0k0$	$h00$	$00l$						
$h+k$	h	k	$h+k$	k	h		C2	5	Cm	8	$C2/m$	12
$h+k$	h, l	k	$h+k$	k	h	l	Cc	9	$C2/c$	15		
$k+l$	l	$k+l$	k	k		l	A2	5	Am	8	$A2/m$	12
$k+l$	h, l	$k+l$	k	k	h	l	An	9	$A2/n$	15		
$h+k+l$	$h+l$	$k+l$	$h+k$	k	h	l	I2	5	Im	8	$I2/m$	12
$h+k+l$	h, l	$k+l$	$h+k$	k	h	l	Ia	9	$I2/a$	15		

Table 1.6.4.5

Reflection conditions and possible space groups with Bravais lattice mP and Laue class $2/m$ (monoclinic, unique axis c); Patterson symmetry $P112/m$

Reflection conditions						Space group	No.	Space group	No.	Space group	No.
$h0l$	$0kl$	$hk0$	$0k0$	$h00$	$00l$						
						P2	3	Pm	6	$P2/m$	10
					l	P2₁	4	$P2_1/m$	11		
		h		h		Pa	7	$P2/a$	13		
		h		h	l	$P2_1/a$	14				
		k	k			Pb	7	$P2/b$	13		
		k	k		l	$P2_1/b$	14				
		$h+k$	k	h		Pn	7	$P2/n$	13		
		$h+k$	k	h	l	$P2_1/n$	14				

Table 1.6.4.6

Reflection conditions and possible space groups with Bravais lattice mS (mA , mB , mI) and Laue class $2/m$ (monoclinic, unique axis c); Patterson symmetry $A112/m$, $B112/m1$, $I112/m$

Reflection conditions							Space group	No.	Space group	No.	Space group	No.
hkl	$h0l$	$0kl$	$hk0$	$0k0$	$h00$	$00l$						
$k+l$	l	$k+l$	k	k		l	A2	5	Am	8	$A2/m$	12
$k+l$	l	$k+l$	h, k	k	h	l	Aa	9	$A2/a$	15		
$h+l$	$h+l$	l	h		h	l	B2	5	Bm	8	$B2/m$	12
$h+l$	$h+l$	l	h, k	k	h	l	Bn	9	$B2/n$	15		
$h+k+l$	$h+l$	$k+l$	$h+k$	k	h	l	I2	5	Im	8	$I2/m$	12
$h+k+l$	$h+l$	$k+l$	h, k	k	h	l	Ib	9	$I2/b$	15		

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Table 1.6.4.7

Reflection conditions and possible space groups with Bravais lattice *oP* and Laue class *mmm*; Patterson symmetry *Pmmm*

Reflection conditions						Space group	No.	Space group	No.	Space group	No.		
<i>0kl</i>	<i>h0l</i>	<i>hk0</i>	<i>h00</i>	<i>0k0</i>	<i>00l</i>								
						<i>P222</i>	16	<i>Pmm2</i>	25	<i>Pm2m</i>	25		
						<i>P2mm</i>	25	<i>Pmmm</i>	47				
					<i>l</i>	<i>P222</i>₁	17						
				<i>k</i>		<i>P22</i>₁2	17						
				<i>k</i>	<i>l</i>	<i>P22</i>₁2₁	18						
			<i>h</i>			<i>P2</i>₁22	17						
			<i>h</i>		<i>l</i>	<i>P2</i>₁22₁	18						
			<i>h</i>	<i>k</i>		<i>P2</i>₁2₁2	18						
			<i>h</i>	<i>k</i>	<i>l</i>	<i>P2</i>₁2₁2₁	19						
		<i>h</i>	<i>h</i>			<i>P2</i> ₁ <i>ma</i>	26	<i>Pm2a</i>	28			<i>Pmma</i>	51
		<i>k</i>		<i>k</i>		<i>Pm2</i> ₁ <i>b</i>	26	<i>P2mb</i>	28			<i>Pmmb</i>	51
		<i>h + k</i>	<i>h</i>	<i>k</i>		<i>Pm2</i> ₁ <i>n</i>	31	<i>P2</i> ₁ <i>mn</i>	31			<i>Pmmn</i>	59
	<i>h</i>		<i>h</i>			<i>P2</i> ₁ <i>am</i>	26	<i>Pma2</i>	28			<i>Pmam</i>	51
	<i>h</i>	<i>h</i>	<i>h</i>			<i>P2aa</i>	27	<i>Pmaa</i>	49				
	<i>h</i>	<i>k</i>	<i>h</i>	<i>k</i>		<i>P2</i> ₁ <i>ab</i>	29	<i>Pmab</i>	57				
	<i>h</i>	<i>h + k</i>	<i>h</i>	<i>k</i>		<i>P2an</i>	30	<i>Pman</i>	53				
	<i>l</i>				<i>l</i>	<i>Pmc2</i> ₁	26	<i>P2cm</i>	28			<i>Pmcm</i>	51
	<i>l</i>	<i>h</i>	<i>h</i>		<i>l</i>	<i>P2</i> ₁ <i>ca</i>	29	<i>Pmca</i>	57				
	<i>l</i>	<i>k</i>		<i>k</i>	<i>l</i>	<i>P2cb</i>	32	<i>Pmcb</i>	55				
	<i>l</i>	<i>h + k</i>	<i>h</i>	<i>k</i>	<i>l</i>	<i>P2</i> ₁ <i>cn</i>	33	<i>Pmcn</i>	62				
	<i>h + l</i>		<i>h</i>		<i>l</i>	<i>Pmn2</i> ₁	31	<i>P2</i> ₁ <i>nm</i>	31	<i>Pmmm</i>	59		
	<i>h + l</i>	<i>h</i>	<i>h</i>		<i>l</i>	<i>P2na</i>	30	<i>Pmna</i>	53				
	<i>h + l</i>	<i>k</i>	<i>h</i>	<i>k</i>	<i>l</i>	<i>P2</i> ₁ <i>nb</i>	33	<i>Pmnb</i>	62				
	<i>h + l</i>	<i>h + k</i>	<i>h</i>	<i>k</i>	<i>l</i>	<i>P2nn</i>	34	<i>Pmnn</i>	58				
<i>k</i>				<i>k</i>		<i>Pb2</i> ₁ <i>m</i>	26	<i>Pbm2</i>	28	<i>Pbmm</i>	51		
<i>k</i>		<i>h</i>	<i>h</i>	<i>k</i>		<i>Pb2</i> ₁ <i>a</i>	29	<i>Pbma</i>	57				
<i>k</i>		<i>k</i>		<i>k</i>		<i>Pb2b</i>	27	<i>Pbmb</i>	49				
<i>k</i>		<i>h + k</i>	<i>h</i>	<i>k</i>		<i>Pb2n</i>	30	<i>Pbmn</i>	53				
<i>k</i>	<i>h</i>		<i>h</i>	<i>k</i>		<i>Pba2</i>	32	<i>Pbam</i>	55				
<i>k</i>	<i>h</i>	<i>h</i>	<i>h</i>	<i>k</i>		<i>Pbaa</i>	54						
<i>k</i>	<i>h</i>	<i>k</i>	<i>h</i>	<i>k</i>		<i>Pbab</i>	54						
<i>k</i>	<i>h</i>	<i>h + k</i>	<i>h</i>	<i>k</i>		<i>Pban</i>	50						
<i>k</i>	<i>l</i>			<i>k</i>	<i>l</i>	<i>Pbc2</i> ₁	29	<i>Pbcm</i>	57				
<i>k</i>	<i>l</i>	<i>h</i>	<i>h</i>	<i>k</i>	<i>l</i>	<i>Pbca</i>	61						
<i>k</i>	<i>l</i>	<i>k</i>		<i>k</i>	<i>l</i>	<i>Pbcb</i>	54						
<i>k</i>	<i>l</i>	<i>h + k</i>	<i>h</i>	<i>k</i>	<i>l</i>	<i>Pbcn</i>	60						

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Table 1.6.4.7 (continued)

Reflection conditions						Space group	No.	Space group	No.	Space group	No.
$0kl$	$h0l$	$hk0$	$h00$	$0k0$	$00l$						
k	$h+l$		h	k	l	$Pbn2_1$	33	$Pbnm$	62		
k	$h+l$	h	h	k	l	$Pbna$	60				
k	$h+l$	k	h	k	l	$Pbnb$	56				
k	$h+l$	$h+k$	h	k	l	$Pbnn$	52				
l					l	$Pcm2_1$	26	$Pc2m$	28	$Pcmm$	51
l		h	h		l	$Pc2a$	32	$Pcma$	55		
l		k		k	l	$Pc2_1b$	29	$Pcmb$	57		
l		$h+k$	h	k	l	$Pc2_1n$	33	$Pcmn$	62		
l	h		h		l	$Pca2_1$	29	$Pcam$	57		
l	h	h	h		l	$Pcaa$	54				
l	h	k	h	k	l	$Pcab$	61				
l	h	$h+k$	h	k	l	$Pcan$	60				
l	l				l	$Pcc2$	27	$Pccm$	49		
l	l	h	h		l	$Pcca$	54				
l	l	k		k	l	$Pccb$	54				
l	l	$h+k$	h	k	l	$Pccn$	56				
l	$h+l$		h		l	$Pcn2$	30	$Pcnm$	53		
l	$h+l$	h	h		l	$Pcna$	50				
l	$h+l$	k	h	k	l	$Pcnb$	60				
l	$h+l$	$h+k$	h	k	l	$Pcnn$	52				
$k+l$				k	l	$Pnm2_1$	31	$Pn2_1m$	31	$Pnmm$	59
$k+l$		h	h	k	l	$Pn2_1a$	33	$Pnma$	62		
$k+l$		k		k	l	$Pn2b$	30	$Pnmb$	53		
$k+l$		$h+k$	h	k	l	$Pn2n$	34	$Pnmn$	58		
$k+l$	h		h	k	l	$Pna2_1$	33	$Pnam$	62		
$k+l$	h	h	h	k	l	$Pnaa$	56				
$k+l$	h	k	h	k	l	$Pnab$	60				
$k+l$	h	$h+k$	h	k	l	$Pnan$	52				
$k+l$	l			k	l	$Pnc2$	30	$Pncm$	53		
$k+l$	l	h	h	k	l	$Pnca$	60				
$k+l$	l	k		k	l	$Pncb$	50				
$k+l$	l	$h+k$	h	k	l	$Pncn$	52				
$k+l$	$h+l$		h	k	l	$Pnn2$	34	$Pnmm$	58		
$k+l$	$h+l$	h	h	k	l	$Pnna$	52				
$k+l$	$h+l$	k	h	k	l	$Pnnb$	52				
$k+l$	$h+l$	$h+k$	h	k	l	$Pnnn$	48				

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Table 1.6.4.8

Reflection conditions and possible space groups with Bravais lattice oS (oC setting) and Laue class mmm ; Patterson symmetry $Cmmm$

Reflection conditions							Space group		Space group		Space group	
hkl	$0kl$	$h0l$	$hk0$	$h00$	$0k0$	$00l$	group	No.	group	No.	group	No.
$h+k$	k	h	$h+k$	h	k		C222 <i>C2mm</i>	21 38	<i>Cmm2</i> <i>Cmmm</i>	35 65	<i>Cm2m</i>	38
$h+k$	k	h	$h+k$	h	k	l	C222₁	20				
$h+k$	k	h	h,k	h	k		<i>Cm2e</i>	39	<i>C2me</i>	39	<i>Cmme</i>	67
$h+k$	k	h,l	$h+k$	h	k	l	<i>Cmc2₁</i>	36	<i>C2cm</i>	40	<i>Cmcm</i>	63
$h+k$	k	h,l	h,k	h	k	l	<i>C2ce</i>	41	<i>Cmce</i>	64		
$h+k$	k,l	h	$h+k$	h	k	l	<i>Ccm2₁</i>	36	<i>Cc2m</i>	40	<i>Ccmm</i>	63
$h+k$	k,l	h	h,k	h	k	l	<i>Cc2e</i>	41	<i>Ccme</i>	64		
$h+k$	k,l	h,l	$h+k$	h	k	l	<i>Ccc2</i>	37	<i>Cccm</i>	66		
$h+k$	k,l	h,l	h,k	h	k	l	<i>Ccce</i>	68				

Table 1.6.4.9

Reflection conditions and possible space groups with Bravais lattice oS (oB setting) and Laue class mmm ; Patterson symmetry $Bmmm$

Reflection conditions							Space group		Space group		Space group	
hkl	$0kl$	$h0l$	$hk0$	$h00$	$0k0$	$00l$	group	No.	group	No.	group	No.
$h+l$	l	$h+l$	h	h		l	B222 <i>B2mm</i>	21 38	<i>Bm2m</i> <i>Bmmm</i>	35 65	<i>Bmm2</i>	38
$h+l$	l	$h+l$	h	h	k	l	B22₁2	20				
$h+l$	l	$h+l$	h,k	h	k	l	<i>Bm2₁b</i>	36	<i>B2mb</i>	40	<i>Bmbm</i>	63
$h+l$	l	h,l	h	h		l	<i>Bme2</i>	39	<i>B2em</i>	39	<i>Bmem</i>	67
$h+l$	l	h,l	h,k	h	k	l	<i>B2eb</i>	41	<i>Bmeb</i>	64		
$h+l$	k,l	$h+l$	h	h	k	l	<i>Bb2₁m</i>	36	<i>Bbm2</i>	40	<i>Bbmm</i>	63
$h+l$	k,l	$h+l$	h,k	h	k	l	<i>Bb2b</i>	37	<i>Bbmb</i>	66		
$h+l$	k,l	h,l	h	h	k	l	<i>Bbe2</i>	41	<i>Bbem</i>	64		
$h+l$	k,l	h,l	h,k	h	k	l	<i>Bbeb</i>	68				

Table 1.6.4.10

Reflection conditions and possible space groups with Bravais lattice oS (oA setting) and Laue class mmm ; Patterson symmetry $Ammm$

Reflection conditions							Space group		Space group		Space group	
hkl	$0kl$	$h0l$	$hk0$	$h00$	$0k0$	$00l$	group	No.	group	No.	group	No.
$k+l$	$k+l$	l	k		k	l	A222 <i>Amm2</i>	21 38	<i>A2mm</i> <i>Ammm</i>	35 65	<i>Am2m</i>	38
$k+l$	$k+l$	l	k	h	k	l	A2₁22	20				
$k+l$	$k+l$	l	h,k	h	k	l	<i>A2₁ma</i>	36	<i>Am2a</i>	40	<i>Amma</i>	63
$k+l$	$k+l$	h,l	k	h	k	l	<i>A2₁am</i>	36	<i>Ama2</i>	40	<i>Amam</i>	63
$k+l$	$k+l$	h,l	h,k	h	k	l	<i>A2aa</i>	37	<i>Amaa</i>	66		
$k+l$	k,l	l	k		k	l	<i>Aem2</i>	39	<i>Ae2m</i>	39	<i>Aemm</i>	67
$k+l$	k,l	l	h,k	h	k	l	<i>Ae2a</i>	41	<i>Aema</i>	64		
$k+l$	k,l	h,l	k	h	k	l	<i>Aea2</i>	41	<i>Aeam</i>	64		
$k+l$	k,l	h,l	h,k	h	k	l	<i>Aeaa</i>	68				

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Table 1.6.4.11

Reflection conditions and possible space groups with Bravais lattice *oI* and Laue class *mmm*; Patterson symmetry *Immm*

Reflection conditions							Space group		Space group		Space group	
<i>hkl</i>	<i>0kl</i>	<i>h0l</i>	<i>hk0</i>	<i>h00</i>	<i>0k0</i>	<i>00l</i>	group	No.	group	No.	group	No.
$h + k + l$	$k + l$	$h + l$	$h + k$	h	k	l	I222	23	I₂₁2₁2₁	24	<i>Imm2</i>	44
							<i>Im2m</i>	44	<i>I2mm</i>	44	<i>Immm</i>	71
$h + k + l$	$k + l$	$h + l$	h, k	h	k	l	<i>Im2a</i>	46	<i>I2mb</i>	46	<i>Imma</i>	74
							<i>Immb</i>	74				
$h + k + l$	$k + l$	h, l	$h + k$	h	k	l	<i>Ima2</i>	46	<i>I2cm</i>	46	<i>Imam</i>	74
							<i>Imcm</i>	74				
$h + k + l$	$k + l$	h, l	h, k	h	k	l	<i>I2cb</i>	45	<i>Imcb</i>	72		
$h + k + l$	k, l	$h + l$	$h + k$	h	k	l	<i>Ibm2</i>	46	<i>Ic2m</i>	46	<i>Ibmm</i>	74
							<i>Icmm</i>	74				
$h + k + l$	k, l	$h + l$	h, k	h	k	l	<i>Ic2a</i>	45	<i>Icma</i>	72		
$h + k + l$	k, l	h, l	$h + k$	h	k	l	<i>Iba2</i>	45	<i>Ibam</i>	72		
$h + k + l$	k, l	h, l	h, k	h	k	l	<i>Ibca</i>	73	<i>Icab</i>	73		

Table 1.6.4.12

Reflection conditions and possible space groups with Bravais lattice *oF* and Laue class *mmm*; Patterson symmetry *Fmmm*

Reflection conditions							Space group	
<i>hkl</i>	<i>0kl</i>	<i>h0l</i>	<i>hk0</i>	<i>h00</i>	<i>0k0</i>	<i>00l</i>	group	No.
$h + k, h + l, k + l$	k, l	h, l	h, k	h	k	l	F222	22
							<i>Fmm2</i>	42
							<i>Fm2m</i>	42
							<i>F2mm</i>	42
							<i>Fmmm</i>	69
$h + k, h + l, k + l$	k, l	$h + l = 4n; h, l$	$h + k = 4n; h, k$	$h = 4n$	$k = 4n$	$l = 4n$	<i>F2dd</i>	43
$h + k, h + l, k + l$	$k + l = 4n; k, l$	h, l	$h + k = 4n; h, k$	$h = 4n$	$k = 4n$	$l = 4n$	<i>Fd2d</i>	43
$h + k, h + l, k + l$	$k + l = 4n; k, l$	$h + l = 4n; h, l$	h, k	$h = 4n$	$k = 4n$	$l = 4n$	<i>Fdd2</i>	43
$h + k, h + l, k + l$	$k + l = 4n; k, l$	$h + l = 4n; h, l$	$h + k = 4n; h, k$	$h = 4n$	$k = 4n$	$l = 4n$	<i>Fddd</i>	70

Table 1.6.4.13

Reflection conditions and possible space groups with Bravais lattice *tP* and Laue class *4/m*; *hk* are permutable; Patterson symmetry *P4/m*

Reflection conditions					Space group		Space group		Space group	
<i>hk0</i>	<i>0kl</i>	$h \pm hl$	<i>00l</i>	<i>h00</i>	group	No.	group	No.	group	No.
					P4	75	$P\bar{4}$	81	<i>P4/m</i>	83
			l		P4₂	77	<i>P4₂/m</i>	84		
			$l = 4n$		P4₁	76	P4₃	78		
$h + k$				h	<i>P4/n</i>	85				
$h + k$			l	h	<i>P4₂/n</i>	86				

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Table 1.6.4.14

Reflection conditions and possible space groups with Bravais lattice tP and Laue class $4/mmm$; hk are permutable; Patterson symmetry $P4/mmm$

Reflection conditions					Space group	No.	Space group	No.	Space group	No.
$hk0$	$0kl$	$h \pm hl$	$00l$	$h00$						
					$P422$	89	$P4mm$	99	$P\bar{4}2m$	111
					$P4m2$	115	$P4/mmm$	123		
				h	$P42_12$	90	$P\bar{4}2_1m$	113		
			l		$P4_222$	93				
			l	h	$P4_22_12$	94				
			$l = 4n$		$P4_122$	91	$P4_322$	95		
			$l = 4n$	h	$P4_12_12$	92	$P4_32_12$	96		
		l	l		$P4_2mc$	105	$P\bar{4}2c$	112	$P4_2/mmc$	131
		l	l	h	$P\bar{4}2_1c$	114				
	k			h	$P4bm$	100	$P\bar{4}b2$	117	$P4/mbm$	127
	k	l	l	h	$P4_2bc$	106	$P4_2/mbc$	135		
	l		l		$P4_2cm$	101	$P\bar{4}c2$	116	$P4_2/mcm$	132
	l	l	l		$P4cc$	103	$P4/mcc$	124		
	$k + l$		l	h	$P4_2nm$	102	$P\bar{4}n2$	118	$P4_2/mnm$	136
	$k + l$	l	l	h	$P4nc$	104	$P4/mnc$	128		
$h + k$				h	$P4/nmm$	129				
$h + k$		l	l	h	$P4_2/nmc$	137				
$h + k$	k			h	$P4/nbm$	125				
$h + k$	k	l	l	h	$P4_2/nbc$	133				
$h + k$	l		l	h	$P4_2/ncm$	138				
$h + k$	l	l	l	h	$P4/ncc$	130				
$h + k$	$k + l$		l	h	$P4_2/nmm$	134				
$h + k$	$k + l$	l	l	h	$P4/nnc$	126				

Table 1.6.4.15

Reflection conditions and possible space groups with Bravais lattice tI and Laue class $4/m$; hk are permutable; Patterson symmetry $I4/m$

Reflection conditions							Space group	No.	Space group	No.	Space group	No.
hkl	$hk0$	$0kl$	$h \pm hl$	$00l$	$h00$	$h \pm h0$						
$h + k + l$	$h + k$	$k + l$	l	l	h		$I4$	79	$I\bar{4}$	82	$I4/m$	87
$h + k + l$	$h + k$	$k + l$	l	$l = 4n$	h		$I4_1$	80				
$h + k + l$	h, k	$k + l$	l	$l = 4n$	h	h	$I4_1/a$	88				

1. INTRODUCTION TO SPACE-GROUP SYMMETRY

Table 1.6.4.16

Reflection conditions and possible space groups with Bravais lattice tI and Laue class $4/mmm$; hk are permutable; Patterson symmetry $I4/mmm$

Reflection conditions							Space group	No.	Space group	No.	Space group	No.
hkl	$hk0$	$0kl$	$h \pm hl$	$00l$	$h00$	$h \pm h0$						
$h+k+l$	$h+k$	$k+l$	l	l	h		I422	97	$I4mm$	107	$I\bar{4}m2$	119
							$I\bar{4}2m$	121	$I4/mmm$	139		
$h+k+l$	$h+k$	$k+l$	l	$l=4n$	h		I4₁22	98				
$h+k+l$	$h+k$	$k+l$	$2h+l=4n$	$l=4n$	h	h	$I4_1md$	109	$I\bar{4}2d$	122		
$h+k+l$	$h+k$	k, l	l	l	h		$I4cm$	108	$I\bar{4}c2$	120	$I4/mcm$	140
$h+k+l$	$h+k$	k, l	$2h+l=4n$	$l=4n$	h	h	$I4_1cd$	110				
$h+k+l$	h, k	$k+l$	$2h+l=4n$	$l=4n$	h	h	$I4_1/amd$	141				
$h+k+l$	h, k	k, l	$2h+l=4n$	$l=4n$	h	h	$I4_1/acd$	142				

Table 1.6.4.17

Reflection conditions and possible space groups with Bravais lattice hP and Laue class $\bar{3}$; hki are permutable; Patterson symmetry $P\bar{3}$

Reflection conditions			Space group	No.	Space group	No.
$hh\bar{2}hl$	$h\bar{h}0l$	$000l$				
			P3	143	$P\bar{3}$	147
		$l=3n$	P3₁	144	P3₂	145

Table 1.6.4.18

Reflection conditions and possible space groups with Bravais lattice hP and Laue classes $\bar{3}1m$ and $\bar{3}m1$; hki are permutable; Patterson symmetry $P\bar{3}1m$ and $P\bar{3}m1$

Reflection conditions			Class $\bar{3}1m$		Class $\bar{3}m1$	
$hh\bar{2}hl$	$h\bar{h}0l$	$000l$	Space group	No.	Space group	No.
			P312	149	P321	150
			$P31m$	157	$P3m1$	156
			$P\bar{3}1m$	162	$P\bar{3}m1$	164
		$l=3n$	P3₁12	151	P3₁21	152
			P3₂12	153	P3₂21	154
l		l	$P31c$	159		
			$P\bar{3}1c$	163		
	l	l			$P3c1$	158
					$P\bar{3}c1$	165

Table 1.6.4.19

Reflection conditions and possible space groups with Bravais lattice hP and Laue class $6/m$; hki are permutable; Patterson symmetry $P6/m$

Reflection conditions			Space group	No.	Space group	No.	Space group	No.
$hh\bar{2}hl$	$h\bar{h}0l$	$000l$						
			P6	168	$P\bar{6}$	174	$P6/m$	175
		l	P6₃	173	$P6_3/m$	176		
		$l=3n$	P6₂	171	P6₄	172		
		$l=6n$	P6₁	169	P6₅	170		

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Table 1.6.4.20

Reflection conditions and possible space groups with Bravais lattice hP and Laue class $6/mmm$; hki are permutable; Patterson symmetry $P6/mmm$

Reflection conditions			Space group	No.	Space group	No.	Space group	No.
$hh\bar{2}hl$	$h\bar{h}0l$	$000l$						
l			P622	177	$P6mm$	183	$P\bar{6}m2$	187
			$P62m$	189	$P6/mmm$	191		
		l	P6₃22	182				
		$l = 3n$	P6₂22	180	P6₄22	181		
		$l = 6n$	P6₁22	178	P6₅22	179		
		l	$P6_3mc$	186	$P\bar{6}2c$	190	$P6_3/mmc$	194
	l	l	$P6_3cm$	185	$P\bar{6}c2$	188	$P6_3/mcm$	193
l	l	l	$P6cc$	184	$P6/mcc$	192		

Table 1.6.4.21

Reflection conditions and possible space groups with Bravais lattice hR and Laue class $\bar{3}$ (hexagonal axes); hki are permutable; Patterson symmetry $R\bar{3}$; Ov = obverse setting; Rv = reverse setting

Reflection conditions						Space group	No.	Space group	No.	
$hkil$	$hki0$	$hh\bar{2}hl$	$h\bar{h}0l$	$000l$	$h\bar{h}00$					
$-h + k + l = 3n$	$-h + k = 3n$	$l = 3n$	$h + l = 3n$	$l = 3n$	$h = 3n$	R3	146	$R\bar{3}$	148	Ov
$h - k + l = 3n$	$h - k = 3n$	$l = 3n$	$-h + l = 3n$	$l = 3n$	$h = 3n$	R3	146	$R\bar{3}$	148	Rv

Table 1.6.4.22

Reflection conditions and possible space groups with Bravais lattice hR and Laue class $\bar{3}m$ (hexagonal axes); hki are permutable; Patterson symmetry $R\bar{3}m$; Ov = obverse setting; Rv = reverse setting

Reflection conditions						Space group	No.	Space group	No.	Space group	No.	
$hkil$	$hki0$	$hh\bar{2}hl$	$h\bar{h}0l$	$000l$	$h\bar{h}00$							
$-h + k + l = 3n$	$-h + k = 3n$	$l = 3n$	$h + l = 3n$	$l = 3n$	$h = 3n$	R32	155	$R3m$	160	$R\bar{3}m$	166	Ov
$-h + k + l = 3n$	$-h + k = 3n$	$l = 3n$	$h + l = 3n, l = 2m$	$l = 6n$	$h = 3n$	$R3c$	161	$R\bar{3}c$	167			Ov
$h - k + l = 3n$	$h - k = 3n$	$l = 3n$	$-h + l = 3n$	$l = 3n$	$h = 3n$	R32	155	$R3m$	160	$R\bar{3}m$	166	Rv
$h - k + l = 3n$	$h - k = 3n$	$l = 3n$	$-h + l = 3n, l = 2m$	$l = 6n$	$h = 3n$	$R3c$	161	$R\bar{3}c$	167			Rv

Table 1.6.4.23

Reflection conditions and possible space groups with Bravais lattice hR and Laue class $\bar{3}$ (rhombohedral axes); hkl are permutable; Patterson symmetry $R\bar{3}$

Reflection conditions		Space group	No.	Space group	No.
hhl	hhh				
		R3	146	$R\bar{3}$	148

Table 1.6.4.24

Reflection conditions and possible space groups with Bravais lattice hR and Laue class $\bar{3}m$ (rhombohedral axes); hkl are permutable; Patterson symmetry $R\bar{3}m$

Reflection conditions		Space group	No.	Space group	No.	Space group	No.
hhl	hhh						
		R32	155	$R3m$	160	$R\bar{3}m$	166
l	h	$R3c$	161	$R\bar{3}c$	167		

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Table 1.6.4.25

Reflection conditions and possible space groups with Bravais lattice cP and Laue class $m\bar{3}$; hkl are cyclically permutable; Patterson symmetry $Pm\bar{3}$

Reflection conditions			Space group		Space group	
$0kl$	$h\pm hl$	$h00$	group	No.	group	No.
			$P23$	195	$Pm\bar{3}$	200
k l $k+l$		h	$P2_13$	198		
		h	$Pa\bar{3}$	205		
		h	$Pb\bar{3}$	205		
		h	$Pn\bar{3}$	201		

Table 1.6.4.26

Reflection conditions and possible space groups with Bravais lattice cP and Laue class $m\bar{3}m$; hkl are permutable; Patterson symmetry $Pm\bar{3}m$

Reflection conditions			Space group		Space group		Space group	
$0kl$	$h\pm hl$	$h00$	group	No.	group	No.	group	No.
			$P432$	207	$P\bar{4}3m$	215	$Pm\bar{3}m$	221
$k+l$ $k+l$		h	$P4_232$	208				
		$h=4n$	$P4_332$	212	$P4_132$	213		
	l	h	$P\bar{4}3n$	218	$Pm\bar{3}n$	223		
		h	$Pn\bar{3}m$	224				
	l	h	$Pn\bar{3}n$	222				

Table 1.6.4.27

Reflection conditions and possible space groups with Bravais lattice cI and Laue class $m\bar{3}$; hkl are cyclically permutable; Patterson symmetry $Im\bar{3}$

Reflection conditions				Space group		Space group		Space group	
hkl	$0kl$	$h\pm hl$	$h00$	group	No.	group	No.	group	No.
$h+k+l$	$k+l$	l	h	$I23$	197	$I2_13$	199	$Im\bar{3}$	204
$h+k+l$	k, l	l	h	$Ia\bar{3}$	206				

Table 1.6.4.28

Reflection conditions and possible space groups with Bravais lattice cI and Laue class $m\bar{3}m$; hkl are permutable; Patterson symmetry $Im\bar{3}m$

Reflection conditions				Space group		Space group		Space group	
hkl	$0kl$	$h\pm hl$	$h00$	group	No.	group	No.	group	No.
$h+k+l$	$k+l$	l	h	$I432$	211	$I\bar{4}3m$	217	$Im\bar{3}m$	229
$h+k+l$	$k+l$	l	$h=4n$	$I4_132$	214				
$h+k+l$	$k+l$	$2h+l=4n$	$h=4n$	$I\bar{4}3d$	220				
$h+k+l$	k, l	$2h+l=4n$	$h=4n$	$Ia\bar{3}d$	230				

Table 1.6.4.29

Reflection conditions and possible space groups with Bravais lattice cF and Laue class $m\bar{3}$; hkl are cyclically permutable; Patterson symmetry $Fm\bar{3}$

Reflection conditions				Space group		Space group	
hkl	$0kl$	$h\pm hl$	$h00$	group	No.	group	No.
$h+k, h+l, k+l$	k, l	$h+l$	h	$F23$	196	$Fm\bar{3}$	202
$h+k, h+l, k+l$	$k+l=4n; k, l$	$h+l$	$h=4n$	$Fd\bar{3}$	203		

Table 1.6.4.30Reflection conditions and possible space groups with Bravais lattice cF and Laue class $m\bar{3}m$; hkl are permutable; Patterson symmetry $Fm\bar{3}m$

Reflection conditions				Space group		Space group		Space group	
hkl	$0kl$	$h \pm hl$	$h00$	group	No.	group	No.	group	No.
$h+k, h+l, k+l$	k, l	$h+l$	h	F432	209	$F\bar{4}3m$	216	$Fm\bar{3}m$	225
$h+k, h+l, k+l$	k, l	$h+l$	$h=4n$	F4₁32	210				
$h+k, h+l, k+l$	k, l	h, l	h	$F\bar{4}3c$	219	$Fm\bar{3}c$	226		
$h+k, h+l, k+l$	$k+l=4n; k, l$	$h+l$	$h=4n$	$Fd\bar{3}m$	227				
$h+k, h+l, k+l$	$k+l=4n; k, l$	h, l	$h=4n$	$Fd\bar{3}c$	228				

and have not yet enjoyed widespread distribution, use and acceptance by the community. Flack *et al.* (2011) and Parsons *et al.* (2012) give detailed information on these calculations.

1.6.5.1.2. Status of centrosymmetry and resonant scattering

The basic starting point in this analysis is the following linear transformation of $|F(hkl)|^2$ and $|F(\bar{h}\bar{k}\bar{l})|^2$, applicable to both observed and model values, to give the average (A) and difference (D) intensities:

$$A(hkl) = \frac{1}{2}[|F(hkl)|^2 + |F(\bar{h}\bar{k}\bar{l})|^2],$$

$$D(hkl) = |F(hkl)|^2 - |F(\bar{h}\bar{k}\bar{l})|^2.$$

In equation (1.6.2.1), $A(hkl)$ was denoted by $|F_{av}(hkl)|^2$. The expression for $D(hkl)$ corresponding to that for $A(hkl)$ given in equation (1.6.2.1) and using the same nomenclature is

$$D(\mathbf{h}) = \sum_{i,j} [(f_i + f'_i)f''_j - (f_j + f'_j)f''_i] \sin[2\pi\mathbf{h}(\mathbf{r}_i - \mathbf{r}_j)].$$

In general $|D(hkl)|$ is small compared to $A(hkl)$. A compound with an appreciable resonant-scattering contribution has $|D(hkl)| \approx 0.01A(hkl)$, whereas a compound with a small resonant-scattering contribution has $|D(hkl)| \approx 0.0001A(hkl)$. For centric reflections, $D_{model} = 0$, and so the values of $D_{obs}(hkl)$ of these are entirely due to random uncertainties and systematic errors in the intensity measurements. $D_{obs}(hkl)$ of acentric reflections contains contributions both from the random uncertainties and the systematic errors of the data measurements, and from the differences between $|F(hkl)|^2$ and $|F(\bar{h}\bar{k}\bar{l})|^2$ which arise through the effect of resonant scattering. A slight experimental limitation is that a data set of intensities needs to contain both reflections hkl and $\bar{h}\bar{k}\bar{l}$ in order to obtain $A_{obs}(hkl)$ and $D_{obs}(hkl)$.

The Bijvoet ratio, defined by

$$\chi = \frac{\langle D^2 \rangle^{1/2}}{\langle A \rangle},$$

is the ratio of the root-mean-square value of D to the mean value of A . In a structure analysis, two independent estimates of the Bijvoet ratio are available and their comparison leads to useful information as to whether the crystal structure is centrosymmetric or not.

The first estimate arises from considerations of intensity statistics leading to the definition of the Bijvoet ratio as a value called $Friedif_{stat}$, whose functional form was derived by Flack & Shmueli (2007) and Shmueli & Flack (2009). One needs only to know the chemical composition of the compound and the

wavelength of the X-radiation to calculate $Friedif_{stat}$ using various available software.

The second estimate of the Bijvoet ratio, $Friedif_{obs}$, is obtained from the observed diffraction intensities. One problematic point in the evaluation of $Friedif_{obs}$ arises because A and D do not have the same dependence on $\sin\theta/\lambda$ and it is necessary to eliminate this difference as far as possible. A second problematic point in the calculation is to make sure that only acentric reflections of any of the noncentrosymmetric point groups in the chosen Laue class are selected for the calculation of $Friedif_{obs}$. In this way one is sure that if the point group of the crystal is centrosymmetric, all of the chosen reflections are centric, and if the point group of the crystal is noncentrosymmetric, all of the chosen reflections are acentric. The necessary selection is achieved by taking only those reflections that are general in the Laue group. To date (2015), the calculation of $Friedif_{obs}$ is not available in distributed software. On comparison of $Friedif_{stat}$ with $Friedif_{obs}$, one is able to state with some confidence that:

- (1) if $Friedif_{obs}$ is much lower than $Friedif_{stat}$, then the crystal structure is either centrosymmetric, and random uncertainties and systematic errors in the data set are minor, or noncentrosymmetric with the crystal twinned by inversion in a proportion close to 50:50;
- (2) if $Friedif_{obs}$ is close in value to $Friedif_{stat}$, then the crystal is probably noncentrosymmetric and random uncertainties and systematic errors in the data set are minor. However, data from a centrosymmetric crystal with large random uncertainties and systematic errors may also produce this result; and
- (3) if $Friedif_{obs}$ is much larger than $Friedif_{stat}$ then either the data set is dominated by random uncertainties and systematic errors or the chemical formula is erroneous.

Example 1

The crystal of compound Ex1 (Udupa & Krebs, 1979) is known to be centrosymmetric (space group $P2_1/c$) and has a significant resonant-scattering contribution, $Friedif_{stat} = 498$ and $Friedif_{obs} = 164$. The comparison of $Friedif_{stat}$ and $Friedif_{obs}$ indicates that the crystal structure is centrosymmetric.

Example 2

The crystal of compound Ex2, potassium hydrogen (2R,3R) tartrate, is known to be enantiomerically pure and appears in space group $P2_12_12_1$. The value of $Friedif_{obs}$ is 217 compared to a $Friedif_{stat}$ value of 174. The agreement is good and allows the deduction that the crystal is neither centrosymmetric, nor twinned by inversion in a proportion near to 50:50, nor that the