

$I4_132$

$O^8$

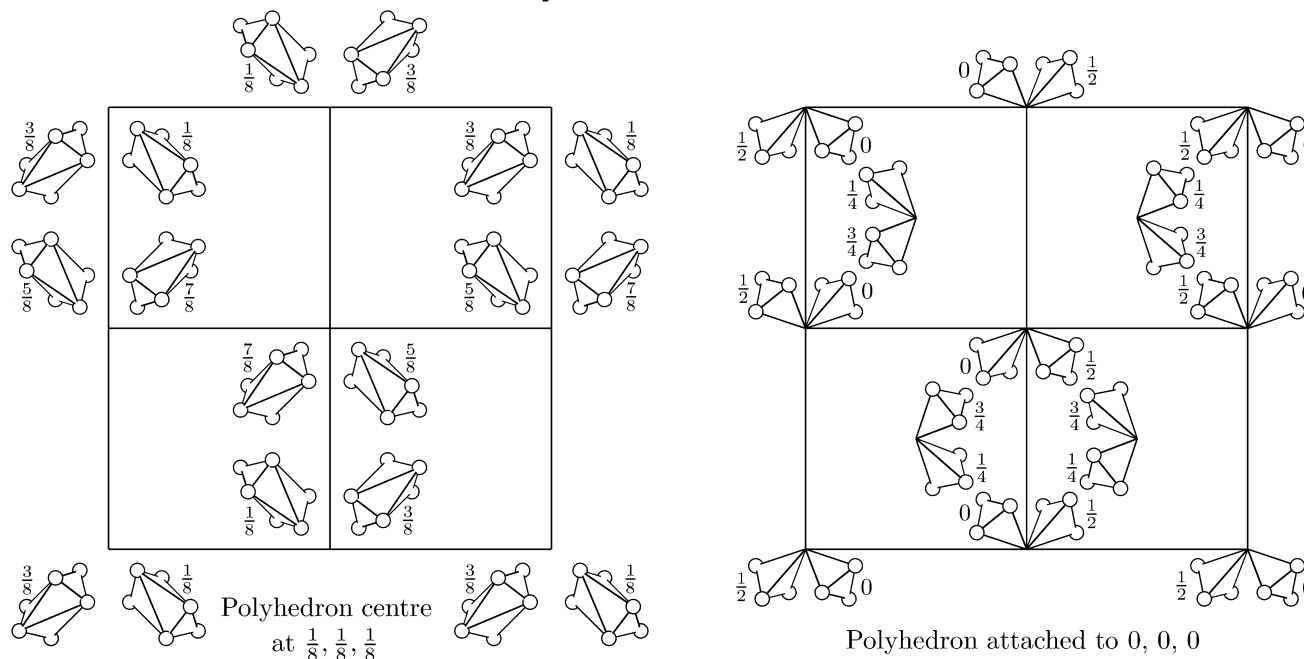
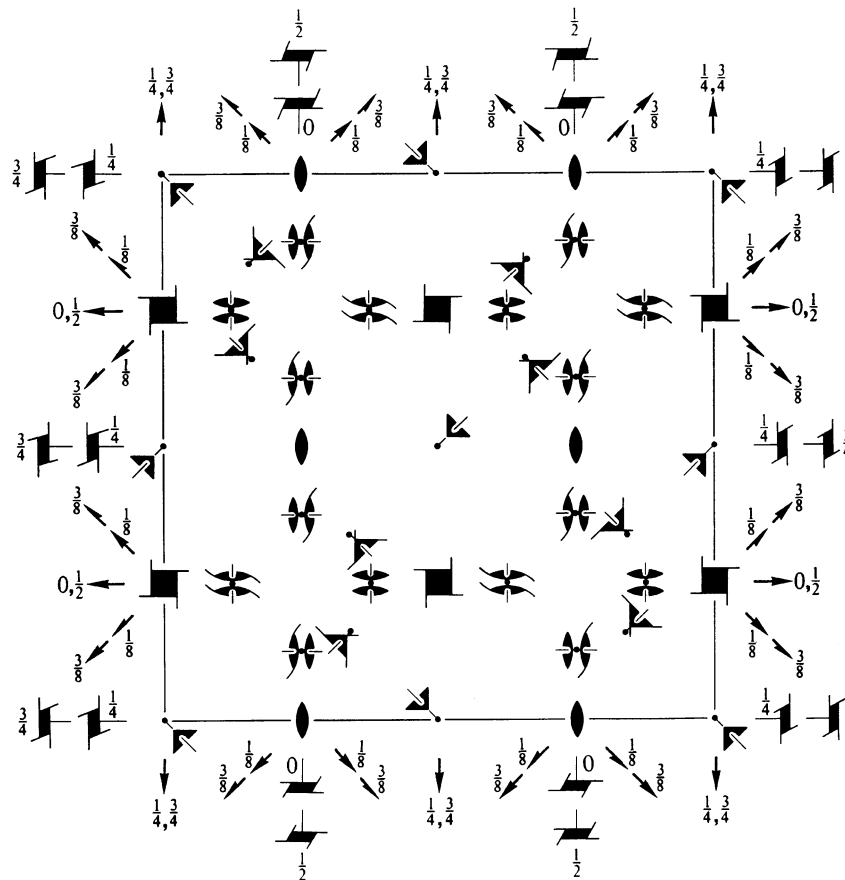
432

Cubic

No. 214

$I4_132$

Patterson symmetry  $Im\bar{3}m$



**Origin** on  $3[111]$  at midpoint of three non-intersecting pairs of parallel screw axes  $4_1$  and  $4_3$  and of three non-intersecting pairs of parallel 2 axes

**Asymmetric unit**  $-\frac{3}{8} \leq x \leq \frac{1}{8}; -\frac{1}{8} \leq y \leq \frac{1}{8}; -\frac{1}{8} \leq z \leq \frac{1}{8}; \max(x, y, y-x-\frac{1}{8}) \leq z \leq y+\frac{1}{4}$   
**Vertices**  $\frac{1}{8}, \frac{1}{8}, \frac{1}{8}$   $\frac{1}{8}, \frac{1}{8}, \frac{3}{8}$   $\frac{1}{8}, -\frac{1}{8}, \frac{1}{8}$   $-\frac{1}{8}, \frac{1}{8}, \frac{1}{8}$   $-\frac{1}{8}, -\frac{1}{8}, -\frac{1}{8}$   $-\frac{3}{8}, \frac{1}{8}, \frac{3}{8}$   $-\frac{3}{8}, -\frac{1}{8}, \frac{1}{8}$

**Symmetry operations**

For  $(0,0,0)+$  set

- |   |   |   |   |
|---|---|---|---|
| (1) 1   | (2) $2(0,0,\frac{1}{2})$ $\frac{1}{4}, 0, z$  | (3) $2(0,\frac{1}{2},0)$ $0, y, \frac{1}{4}$  | (4) $2(\frac{1}{2},0,0)$ $x, \frac{1}{4}, 0$  |
| (5) $3^+ x, x, x$   | (6) $3^+ \bar{x}+\frac{1}{2}, x, \bar{x}$   | (7) $3^+ x+\frac{1}{2}, \bar{x}-\frac{1}{2}, \bar{x}$   | (8) $3^+ \bar{x}, \bar{x}+\frac{1}{2}, x$   |
| (9) $3^- x, x, x$   | (10) $3^- (-\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$ $x+\frac{1}{6}, \bar{x}+\frac{1}{6}, \bar{x}$ | (11) $3^- (\frac{1}{3}, \frac{1}{3}, -\frac{1}{3})$ $\bar{x}+\frac{1}{3}, \bar{x}+\frac{1}{6}, x$ | (12) $3^- (\frac{1}{3}, -\frac{1}{3}, \frac{1}{3})$ $\bar{x}-\frac{1}{6}, x+\frac{1}{3}, \bar{x}$ |
| (13) $2(\frac{1}{2}, \frac{1}{2}, 0)$ $x, x-\frac{1}{4}, \frac{1}{8}$ | (14) $2$ $x, \bar{x}+\frac{3}{4}, \frac{3}{8}$  | (15) $4^-(0,0,\frac{3}{4})$ $\frac{1}{4}, 0, z$   | (16) $4^+(0,0,\frac{1}{4})$ $-\frac{1}{4}, \frac{1}{2}, z$  |
| (17) $4^-(\frac{3}{4}, 0, 0)$ $x, \frac{1}{4}, 0$                     | (18) $2(0,\frac{1}{2}, \frac{1}{2})$ $\frac{1}{8}, y+\frac{1}{4}, y$                              | (19) $2$ $\frac{3}{8}, y+\frac{3}{4}, \bar{y}$  | (20) $4^+(\frac{1}{4}, 0, 0)$ $x, -\frac{1}{4}, \frac{1}{2}$                                      |
| (21) $4^+(0,\frac{1}{4}, 0)$ $\frac{1}{2}, y, -\frac{1}{4}$           | (22) $2(\frac{1}{2}, 0, \frac{1}{2})$ $x-\frac{1}{4}, \frac{3}{8}, x$                             | (23) $4^-(0,\frac{3}{4}, 0)$ $0, y, \frac{1}{4}$  | (24) $2$ $\bar{x}+\frac{3}{4}, \frac{3}{8}, x$  |

**Symmetry operations (continued)**

For  $(\frac{1}{2}, \frac{1}{2}, \frac{1}{2})+$  set

- |   |  |   |   |
|---|--|---|---|
| (1) $t(\frac{1}{2}, \frac{1}{2}, \frac{1}{2})$                        | (2) $2\ 0, \frac{1}{4}, z$   | (3) $2\ \frac{1}{4}, y, 0$  | (4) $2\ x, 0, \frac{1}{4}$  |
| (5) $3^+(\frac{1}{2}, \frac{1}{2}, \frac{1}{2})\ x, x, x$             | (6) $3^+(\frac{1}{6}, -\frac{1}{6}, \frac{1}{6})\ \bar{x}-\frac{1}{6}, x+\frac{1}{3}, \bar{x}$   | (7) $3^+(\frac{1}{6}, \frac{1}{6}, \frac{1}{6})\ x+\frac{1}{6}, \bar{x}+\frac{1}{6}, \bar{x}$   | (8) $3^+(\frac{1}{6}, \frac{1}{6}, -\frac{1}{6})\ \bar{x}+\frac{1}{3}, \bar{x}+\frac{1}{6}, x$  |
| (9) $3^-(\frac{1}{2}, \frac{1}{2}, \frac{1}{2})\ x, x, x$             | (10) $3^-(\frac{1}{6}, -\frac{1}{6}, -\frac{1}{6})\ x+\frac{1}{6}, \bar{x}+\frac{1}{6}, \bar{x}$ | (11) $3^-(\frac{1}{6}, -\frac{1}{6}, \frac{1}{6})\ \bar{x}+\frac{1}{3}, \bar{x}+\frac{1}{6}, x$ | (12) $3^-(\frac{1}{6}, \frac{1}{6}, -\frac{1}{6})\ \bar{x}-\frac{1}{6}, x+\frac{1}{3}, \bar{x}$ |
| (13) $2(\frac{1}{2}, \frac{1}{2}, 0)\ x, x+\frac{1}{4}, \frac{3}{8}z$ | (14) $2\ x, \bar{x}+\frac{1}{4}, \frac{1}{8}z$   | (15) $4^-(0, 0, \frac{1}{4})\ \frac{3}{4}, 0, z$  | (16) $4^+(0, 0, \frac{3}{4})\ \frac{1}{4}, \frac{1}{2}, z$                                      |
| (17) $4^-(\frac{1}{4}, 0, 0)\ x, \frac{3}{4}, 0$                      | (18) $2(0, \frac{1}{2}, \frac{1}{2})\ \frac{3}{8}z, y-\frac{1}{4}, y$                            | (19) $2\ \frac{1}{8}z, y+\frac{1}{4}, \bar{y}$  | (20) $4^+(\frac{3}{4}, 0, 0)\ x, \frac{1}{4}, \frac{1}{2}$                                      |
| (21) $4^+(0, \frac{3}{4}, 0)\ \frac{1}{2}, y, \frac{1}{4}$            | (22) $2(\frac{1}{2}, 0, \frac{1}{2})\ x+\frac{1}{4}, \frac{3}{8}z, x$                            | (23) $4^-(0, \frac{1}{4}, 0)\ 0, y, \frac{3}{4}$  | (24) $2\ \bar{x}+\frac{1}{4}, \frac{1}{8}z, x$  |

**Generators selected** (1);  $t(1, 0, 0)$ ;  $t(0, 1, 0)$ ;  $t(0, 0, 1)$ ;  $t(\frac{1}{2}, \frac{1}{2}, \frac{1}{2})$ ; (2); (3); (5); (13)

**Positions**

Multiplicity,  
Wyckoff letter,  
Site symmetry

Coordinates

$(0, 0, 0)+\ (\frac{1}{2}, \frac{1}{2}, \frac{1}{2})+$

Reflection conditions

$h, k, l$  permutable

General:

- |    |          |   |  |  |  |  |                 |
|----|----------|---|--|--|--|--|-----------------|
| 48 | <i>i</i> | 1 | (1) $x, y, z$  | (2) $\bar{x}+\frac{1}{2}, \bar{y}, z+\frac{1}{2}$                    | (3) $\bar{x}, y+\frac{1}{2}, \bar{z}+\frac{1}{2}$                    | (4) $x+\frac{1}{2}, \bar{y}+\frac{1}{2}, \bar{z}$                    | $hkl: h+k+l=2n$ |
|    |          |   | (5) $z, x, y$  | (6) $z+\frac{1}{2}, \bar{x}+\frac{1}{2}, \bar{y}$                    | (7) $\bar{z}+\frac{1}{2}, \bar{x}, y+\frac{1}{2}$                    | (8) $\bar{z}, x+\frac{1}{2}, \bar{y}+\frac{1}{2}$                    | $Ok: k+l=2n$    |
|    |          |   | (9) $y, z, x$  | (10) $\bar{y}, z+\frac{1}{2}, \bar{x}+\frac{1}{2}$                   | (11) $y+\frac{1}{2}, \bar{z}+\frac{1}{2}, \bar{x}$                   | (12) $\bar{y}+\frac{1}{2}, \bar{z}, x+\frac{1}{2}$                   | $hhl: l=2n$     |
|    |          |   | (13) $y+\frac{3}{4}, x+\frac{1}{4}, \bar{z}+\frac{1}{4}$ | (14) $\bar{y}+\frac{3}{4}, \bar{x}+\frac{3}{4}, \bar{z}+\frac{3}{4}$ | (15) $y+\frac{1}{4}, \bar{x}+\frac{1}{4}, z+\frac{3}{4}$             | (16) $\bar{y}+\frac{1}{4}, x+\frac{3}{4}, z+\frac{1}{4}$             | $h00: h=4n$     |
|    |          |   | (17) $x+\frac{3}{4}, z+\frac{1}{4}, \bar{y}+\frac{1}{4}$ | (18) $\bar{x}+\frac{1}{4}, z+\frac{3}{4}, y+\frac{1}{4}$             | (19) $\bar{x}+\frac{3}{4}, \bar{z}+\frac{3}{4}, \bar{y}+\frac{3}{4}$ | (20) $x+\frac{1}{4}, \bar{z}+\frac{1}{4}, y+\frac{3}{4}$             |                 |
|    |          |   | (21) $z+\frac{3}{4}, y+\frac{1}{4}, \bar{x}+\frac{1}{4}$ | (22) $z+\frac{1}{4}, \bar{y}+\frac{1}{4}, x+\frac{3}{4}$             | (23) $\bar{z}+\frac{1}{4}, y+\frac{3}{4}, x+\frac{1}{4}$             | (24) $\bar{z}+\frac{3}{4}, \bar{y}+\frac{3}{4}, \bar{x}+\frac{3}{4}$ |                 |

Special: as above, plus

- |    |          |     |                                       |   |   |   |                     |
|----|----------|-----|---------------------------------------|---|---|---|---------------------|
| 24 | <i>h</i> | ..2 | $\frac{1}{8}, y, \bar{y}+\frac{1}{4}$ | $\frac{3}{8}, \bar{y}, \bar{y}+\frac{3}{4}$ | $\frac{7}{8}, y+\frac{1}{2}, y+\frac{1}{4}$ | $\frac{5}{8}, \bar{y}+\frac{1}{2}, y+\frac{3}{4}$ | no extra conditions |
|    |          |     | $\bar{y}+\frac{1}{4}, \frac{1}{8}, y$ | $\bar{y}+\frac{3}{4}, \frac{3}{8}, \bar{y}$ | $y+\frac{1}{4}, \frac{7}{8}, y+\frac{1}{2}$ | $y+\frac{3}{4}, \frac{5}{8}, \bar{y}+\frac{1}{2}$ |                     |
|    |          |     | $y, \bar{y}+\frac{1}{4}, \frac{1}{8}$ | $\bar{y}, \bar{y}+\frac{3}{4}, \frac{3}{8}$ | $y+\frac{1}{2}, y+\frac{1}{4}, \frac{7}{8}$ | $\bar{y}+\frac{1}{2}, y+\frac{3}{4}, \frac{5}{8}$ |                     |

- |    |          |     |                                 |                                       |   |   |                     |
|----|----------|-----|---------------------------------|---------------------------------------|---|---|---------------------|
| 24 | <i>g</i> | ..2 | $\frac{1}{8}, y, y+\frac{1}{4}$ | $\frac{3}{8}, \bar{y}, y+\frac{3}{4}$ | $\frac{7}{8}, y+\frac{1}{2}, \bar{y}+\frac{1}{4}$ | $\frac{5}{8}, \bar{y}+\frac{1}{2}, \bar{y}+\frac{3}{4}$ | no extra conditions |
|    |          |     | $y+\frac{1}{4}, \frac{1}{8}, y$ | $y+\frac{3}{4}, \frac{3}{8}, \bar{y}$ | $\bar{y}+\frac{1}{4}, \frac{7}{8}, y+\frac{1}{2}$ | $\bar{y}+\frac{3}{4}, \frac{5}{8}, \bar{y}+\frac{1}{2}$ |                     |
|    |          |     | $y, y+\frac{1}{4}, \frac{1}{8}$ | $\bar{y}, y+\frac{3}{4}, \frac{3}{8}$ | $y+\frac{1}{2}, \bar{y}+\frac{1}{4}, \frac{7}{8}$ | $\bar{y}+\frac{1}{2}, \bar{y}+\frac{3}{4}, \frac{5}{8}$ |                     |

- |    |          |     |                                 |   |   |                                       |                                       |   |               |
|----|----------|-----|---------------------------------|---|---|---------------------------------------|---------------------------------------|---|---------------|
| 24 | <i>f</i> | 2.. | $x, 0, \frac{1}{4}$             | $\bar{x}+\frac{1}{2}, 0, \frac{3}{4}$           | $\frac{1}{4}, x, 0$                       | $\frac{3}{4}, \bar{x}+\frac{1}{2}, 0$ | $0, \frac{1}{4}, x$                   | $0, \frac{3}{4}, \bar{x}+\frac{1}{2}$     | $hkl: h=2n+1$ |
|    |          |     | $\frac{3}{4}, x+\frac{1}{4}, 0$ | $\frac{3}{4}, \bar{x}+\frac{3}{4}, \frac{1}{2}$ | $x+\frac{3}{4}, \frac{1}{2}, \frac{1}{4}$ | $\bar{x}+\frac{1}{4}, 0, \frac{1}{4}$ | $0, \frac{1}{4}, \bar{x}+\frac{1}{4}$ | $\frac{1}{2}, \frac{1}{4}, x+\frac{3}{4}$ | or $h=4n$     |
|    |          |     |                                 |   |   |                                       |                                       |   | $hhl: h=2n+1$ |
|    |          |     |                                 |   |   |                                       |                                       |   | or $h+k+l=4n$ |

- |    |          |     |   |   |   |   |              |
|----|----------|-----|---|---|---|---|--------------|
| 16 | <i>e</i> | .3. | $x, x, x$   | $\bar{x}+\frac{1}{2}, \bar{x}, x+\frac{1}{2}$                   | $\bar{x}, x+\frac{1}{2}, \bar{x}+\frac{1}{2}$       | $x+\frac{1}{2}, \bar{x}+\frac{1}{2}, \bar{x}$       | $Ok: k=2n+1$ |
|    |          |     | $x+\frac{3}{4}, x+\frac{1}{4}, \bar{x}+\frac{1}{4}$ | $\bar{x}+\frac{3}{4}, \bar{x}+\frac{3}{4}, \bar{x}+\frac{3}{4}$ | $x+\frac{1}{4}, \bar{x}+\frac{1}{4}, x+\frac{3}{4}$ | $\bar{x}+\frac{1}{4}, x+\frac{3}{4}, x+\frac{1}{4}$ | or $k+l=4n$  |

- |    |          |      |                               |                               |                               |                               |                               |                               |   |
|----|----------|------|-------------------------------|-------------------------------|-------------------------------|-------------------------------|-------------------------------|-------------------------------|---|
| 12 | <i>d</i> | 2.22 | $\frac{5}{8}, 0, \frac{1}{4}$ | $\frac{7}{8}, 0, \frac{3}{4}$ | $\frac{1}{4}, \frac{5}{8}, 0$ | $\frac{3}{4}, \frac{7}{8}, 0$ | $0, \frac{1}{4}, \frac{5}{8}$ | $0, \frac{3}{4}, \frac{7}{8}$ | $hkl: h, k=2n, h+k+l=4n$<br>or $h, k=2n+1, l=4n+2$<br>or $h=8n, k=8n+4$ and<br>$h+k+l=4n+2$<br>or $h, k=8n+1, l=4n$<br>or $h=8n+1$ and<br>$k=8n-1, l=4n$<br>or $h, k=8n+3, l=4n$<br>or $h=8n+3$ and<br>$k=8n-3, l=4n$ |
| 12 | <i>c</i> | 2.22 | $\frac{1}{8}, 0, \frac{1}{4}$ | $\frac{3}{8}, 0, \frac{3}{4}$ | $\frac{1}{4}, \frac{1}{8}, 0$ | $\frac{3}{4}, \frac{3}{8}, 0$ | $0, \frac{1}{4}, \frac{1}{8}$ | $0, \frac{3}{4}, \frac{3}{8}$ |   |

- |   |          |     |   |   |   |   |   |
|---|----------|-----|---|---|---|---|---|
| 8 | <i>b</i> | .32 | $\frac{7}{8}, \frac{7}{8}, \frac{7}{8}$ | $\frac{5}{8}, \frac{1}{8}, \frac{3}{8}$ | $\frac{1}{8}, \frac{3}{8}, \frac{5}{8}$ | $\frac{3}{8}, \frac{5}{8}, \frac{1}{8}$ | $hkl: h=2n+1$<br>or $h, k, l=4n+2$<br>or $h, k, l=4n$ |
| 8 | <i>a</i> | .32 | $\frac{1}{8}, \frac{1}{8}, \frac{1}{8}$ | $\frac{3}{8}, \frac{7}{8}, \frac{5}{8}$ | $\frac{7}{8}, \frac{5}{8}, \frac{3}{8}$ | $\frac{5}{8}, \frac{3}{8}, \frac{7}{8}$ |   |

**Symmetry of special projections**

Along [001]  $p4mm$

$\mathbf{a}' = \frac{1}{2}(\mathbf{a}-\mathbf{b})$      $\mathbf{b}' = \frac{1}{2}(\mathbf{a}+\mathbf{b})$

Origin at  $\frac{1}{4}, 0, z$

Along [111]  $p3m1$

$\mathbf{a}' = \frac{1}{3}(2\mathbf{a}-\mathbf{b}-\mathbf{c})$      $\mathbf{b}' = \frac{1}{3}(-\mathbf{a}+2\mathbf{b}-\mathbf{c})$

Origin at  $x, x, x$

Along [110]  $p2mm$

$\mathbf{a}' = \frac{1}{2}(-\mathbf{a}+\mathbf{b})$      $\mathbf{b}' = \frac{1}{2}\mathbf{c}$

Origin at  $x, x+\frac{1}{4}, \frac{1}{8}$