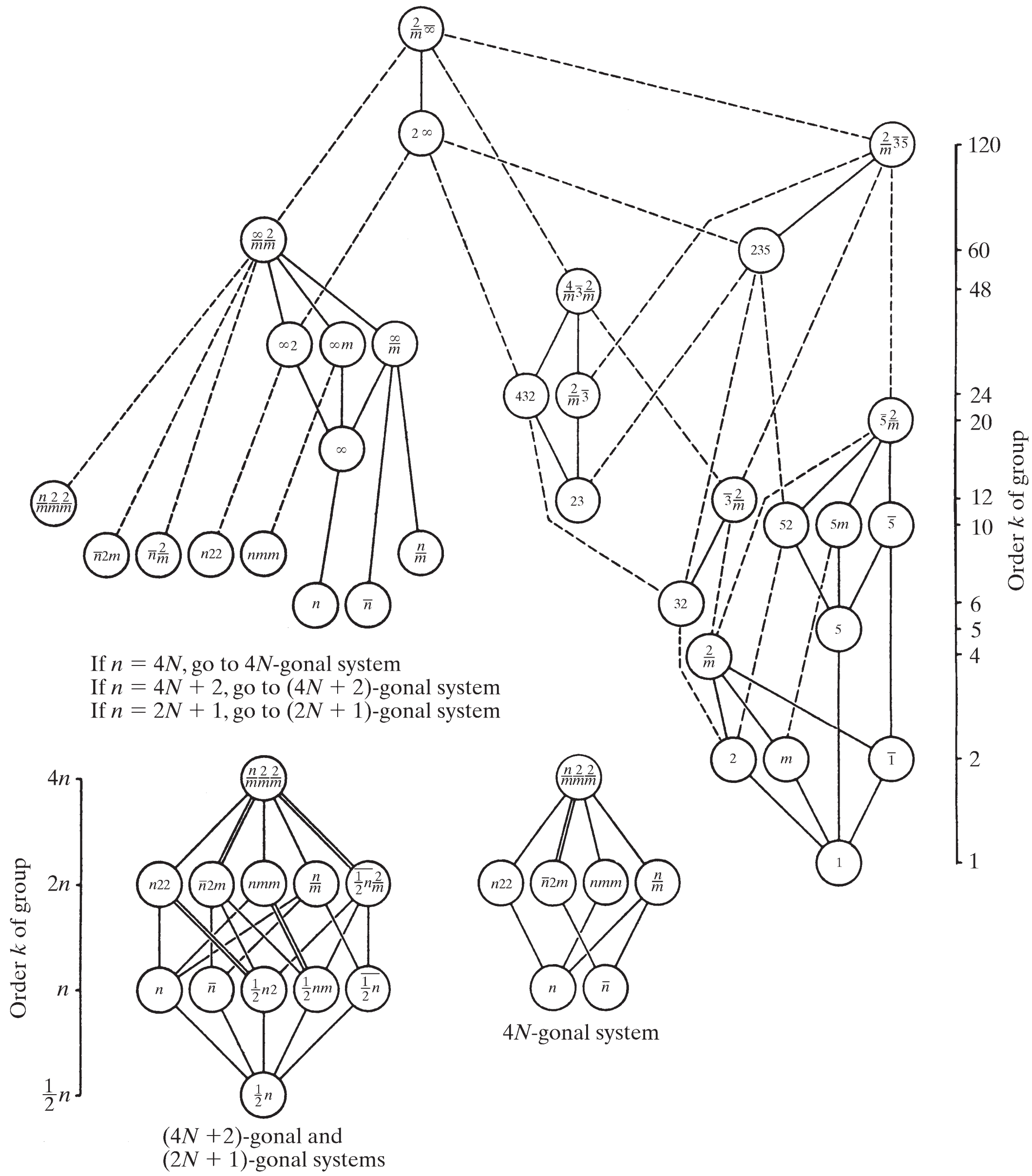

Figure 3.2.1.6

Subgroups and supergroups of the three-dimensional general point groups. Solid lines indicate maximal normal subgroups, double solid lines mean that there are two maximal normal subgroups with the same symbol. Dashed lines refer to sets of maximal conjugate subgroups. For the finite groups, the orders are given on the left and on the right. Note that the subgroups of the five cylindrical groups are not maximal and that the diagram applies only to a specified value of N (see text). Only those crystallographic point groups are included that are maximal subgroups of noncrystallographic point groups, *cf.* Fig. 3.2.1.3. Full Hermann–Mauguin symbols are used.

respectively, each of index $[\infty]$. The order of the rotation point may be $n = 4N$, $n = 4N + 2$ or $n = 2N + 1$. In the first case, the subgroups belong to the $4N$-gonal system, in the latter two cases, they belong to the $(4N + 2)$-gonal system. [In the diagram of the $(4N + 2)$-gonal system, the $(2N + 1)$-gonal groups appear with the symbols $\frac{1}{2}nm$ and $\frac{1}{2}n$.] The subgroups of the circular groups are not maximal because for any given value of N there exists a group with $N' = 2N$ which is both a subgroup of the circular group and a supergroup of the initial group.

The subgroup relations, for a specified value of N, within the $4N$-gonal and the $(4N + 2)$-gonal system, are shown in the lower part of the figure. They correspond to those of the crystallographic groups. A finite number of further maximal subgroups is obtained for lower values of N, until the crystallographic groups (Fig. 3.2.1.2) are reached. This is illustrated for both systems in Fig. 3.2.1.5.

Fig. 3.2.1.6 for *three dimensions* illustrates that the two spherical groups $2/m\overline{\infty}$ and 2∞ each have one infinite set of cylindrical maximal conjugate subgroups, as well as one infinite set of cubic and one infinite set of icosahedral maximal finite conjugate subgroups, all of index $[\infty]$.

Each of the two icosahedral groups 235 and $2/m\,\bar{3}\,\bar{5}$ has one set of five cubic, one set of six pentagonal and one set of ten trigonal maximal conjugate subgroups of indices [5], [6] and [10], respectively (*cf.* Section 3.2.1.4.2, *The two icosahedral groups*); they are listed on the right of Fig. 3.2.1.6. For the crystallographic groups, Fig. 3.2.1.3 applies. The subgroup types of the five cylindrical point groups are shown on the upper left part of Fig. 3.2.1.6. As explained above for two dimensions, these subgroups are *not maximal* and of index $[\infty]$. Depending upon whether the main symmetry axis has the multiplicity $4N$, $4N + 2$ or $2N + 1$, the subgroups belong to the $4N$-gonal, $(4N + 2)$-gonal or $(2N + 1)$-gonal system.

The subgroup and supergroup relations within these three systems are displayed in the lower left part of Fig. 3.2.1.6. They are analogous to the crystallographic groups. To facilitate the