

3. ADVANCED TOPICS ON SPACE-GROUP SYMMETRY

Table 3.2.1.3

The 47 crystallographic face and point forms, their names, eigensymmetries, and their occurrence in the crystallographic point groups (generating point groups)

The oriented face (site) symmetries of the forms are given in parentheses after the Hermann–Mauguin symbol (column 6); a symbol such as $mm2(m., m.)$ indicates that the form occurs in point group $mm2$ twice, with face (site) symmetries $m.$ and $m..$ Basic (general and special) forms are printed in bold face, limiting (general and special) forms in normal type. The various settings of point groups 32 , $3m$, $\bar{3}m$, $\bar{4}2m$ and $\bar{6}m2$ are connected by braces. The 47 crystal forms are shown in Fig. 3.2.1.1. (Note that the numbering of the forms in column 1 does not correspond to the numbering used in Fig. 3.2.1.1.)

No.	Crystal form	Point form	Number of faces or points	Eigensymmetry	Generating point groups with oriented face (site) symmetries between parentheses
1	Pedion or monohedron	Single point	1	∞m	1(1); 2(2); $m(m)$; 3(3); 4(4); 6(6); $mm2(mm2)$; $4mm(4mm)$; $3m(3m)$; $6mm(6mm)$
2	Pinacoid or parallelohedron	Line segment through origin	2	$\frac{\infty}{m}m$	$\bar{1}(1)$; 2(1); $m(1)$; $\frac{2}{m}(2, m)$; 222(2.., 2., ..2) ; $mm2(m., m.)$; $mmm(2mm, m2m, mm2)$; $\bar{4}(2..)$; $\frac{4}{m}(4..)$; 422(4..) ; $\left\{ \begin{array}{l} \bar{4}2m(2.mm) \\ \bar{4}m2(2mm.) \end{array} \right.$; $\frac{4}{m}mm(4mm)$; $\bar{3}(3)$; $\left\{ \begin{array}{l} 321(3.) \\ 312(3..) \end{array} \right.$; $\left\{ \begin{array}{l} \bar{3}m1(3m.) \\ \bar{3}m(3m) \end{array} \right.$; $\bar{6}(3..)$; $\frac{6}{m}(6..)$; 622(6..) ; $\left\{ \begin{array}{l} \bar{6}m2(3m.) \\ \bar{6}2m(3m) \end{array} \right.$; $\frac{6}{m}mm(6mm)$
3	Sphenoid, dome, or dihedron	Line segment	2	$mm2$	2(1); $m(1)$; $mm2(m., m.)$
4	Rhombic disphenoid or rhombic tetrahedron	Rhombic tetrahedron	4	222	222(1)
5	Rhombic pyramid	Rectangle	4	$mm2$	$mm2(1)$
6	Rhombic prism	Rectangle through origin	4	mmm	2/m(1); 222(1)†; $mm2(1)$; $mmm(m., m., ..m)$
7	Rhombic dipyramid	Rectangular prism	8	mmm	$mmm(1)$
8	Tetragonal pyramid	Square	4	$4mm$	4(1); $4mm(..m, .m.)$
9	Tetragonal disphenoid or tetragonal tetrahedron	Tetragonal tetrahedron	4	$\bar{4}2m$	$\bar{4}(1)$; $\left\{ \begin{array}{l} \bar{4}2m(..m) \\ \bar{4}m2(.m.) \end{array} \right.$
10	Tetragonal prism	Square through origin	4	$\frac{4}{m}mm$	4(1); $\bar{4}(1)$; $\frac{4}{m}(m..)$; 422(..2, .2.) ; $4mm(..m, .m.)$; $\ddagger \left\{ \begin{array}{l} \bar{4}2m(.2.) \text{ and } \bar{4}2m(.m.) \\ \bar{4}m2(..2) \text{ and } \bar{4}m2(.m.) \end{array} \right.$; $\frac{4}{m}mm(m.m2, m2m.)$
11	Tetragonal trapezohedron	Twisted tetragonal antiprism	8	422	422(1)
12	Ditetragonal pyramid	Truncated square	8	$4mm$	$4mm(1)$
13	Tetragonal scalenohedron	Tetragonal tetrahedron cut off by pinacoid	8	$\bar{4}2m$	$\left\{ \begin{array}{l} \bar{4}2m(1) \\ \bar{4}m2(1) \end{array} \right.$
14	Tetragonal dipyramid	Tetragonal prism	8	$\frac{4}{m}mm$	$\frac{4}{m}(1)$; 422(1)† ; $\left\{ \begin{array}{l} \bar{4}2m(1) \\ \bar{4}m2(1) \end{array} \right.$; $\frac{4}{m}mm(m., .m.)$
15	Ditetragonal prism	Truncated square through origin	8	$\frac{4}{m}mm$	422(1); $4mm(1)$; $\left\{ \begin{array}{l} \bar{4}2m(1) \\ \bar{4}m2(1) \end{array} \right.$; $\frac{4}{m}mm(m..)$
16	Ditetragonal dipyramid	Edge-truncated tetragonal prism	16	$\frac{4}{m}mm$	$\frac{4}{m}mm(1)$
17	Trigonal pyramid	Trigon	3	$3m$	3(1); $\left\{ \begin{array}{l} 3m1(.m.) \\ 31m(..m) \\ 3m(.m) \end{array} \right.$

3.2. POINT GROUPS AND CRYSTAL CLASSES

Table 3.2.1.3 (continued)

No.	Crystal form	Point form	Number of faces or points	Eigensymmetry	Generating point groups with oriented face (site) symmetries between parentheses
18	Trigonal prism	Trigon through origin	3	$\bar{6}2m$	$3(1); \left\{ \begin{array}{l} 321(.2) \\ 312(..2) \\ 32 (.2) \end{array} \right\}; \left\{ \begin{array}{l} 3m1(.m.) \\ 31m(..m) \\ 3m (.m) \end{array} \right\};$ $\bar{6}(m.); \left\{ \begin{array}{l} \bar{6}m2(mm2) \\ \bar{6}2m(m2m) \end{array} \right\}$
19	Trigonal trapezohedron	Twisted trigonal antiprism	6	32	$\left\{ \begin{array}{l} 321(1) \\ 312(1) \\ 32 (1) \end{array} \right\}$
20	Ditrigonal pyramid	Truncated trigon	6	$3m$	$\left\{ \begin{array}{l} 3m1(1) \\ 31m(1) \\ 3m (1) \end{array} \right\}$
21	Rhombohedral	Trigonal antiprism	6	$\bar{3}m$	$\bar{3}(1); \left\{ \begin{array}{l} 321(1) \\ 312(1) \\ 32 (1) \end{array} \right\}; \left\{ \begin{array}{l} \bar{3}m1(.m.) \\ \bar{3}1m(..m) \\ \bar{3}m (.m) \end{array} \right\}$
22	Ditrigonal prism	Truncated trigon through origin	6	$\bar{6}2m$	$\left\{ \begin{array}{l} 321(1) \\ 312(1) \\ 32 (1) \end{array} \right\}; \left\{ \begin{array}{l} 3m1(1) \\ 31m(1) \\ 3m (1) \end{array} \right\};$ $\left\{ \begin{array}{l} \bar{6}m2(m..) \\ \bar{6}2m(m..) \end{array} \right\}$
23	Hexagonal pyramid	Hexagon	6	$6mm$	$\left\{ \begin{array}{l} 3m1(1) \\ 31m(1); 6(1); 6mm(..m, .m.) \\ 3m (1) \end{array} \right\}$
24	Trigonal dipyrmaid	Trigonal prism	6	$\bar{6}2m$	$\left\{ \begin{array}{l} 321(1) \\ 312(1); \bar{6}(1); \left\{ \begin{array}{l} \bar{6}m2(.m.) \\ \bar{6}2m(..m) \end{array} \right\} \\ 32 (1) \end{array} \right\}$
25	Hexagonal prism	Hexagon through origin	6	$\frac{6}{m}mm$	$\bar{3}(1); \left\{ \begin{array}{l} 321(1) \\ 312(1); \left\{ \begin{array}{l} 3m1(1) \\ 31m(1) \\ 3m (1) \end{array} \right\} \\ 32 (1) \end{array} \right\};$ $\left\{ \begin{array}{l} \bar{3}m1(.2) \text{ and } \bar{3}m1(.m.) \\ \bar{3}1m(..2) \text{ and } \bar{3}1m(..m) \\ \bar{3}m(.2) \text{ and } \bar{3}m(.m) \end{array} \right\};$ $6(1); \frac{6}{m}(m.); 622(.2., ..2);$ $6mm(..m, .m.); \left\{ \begin{array}{l} \bar{6}m2(m..) \\ \bar{6}2m(m..) \end{array} \right\};$ $\frac{6}{m}mm(m2m, mm2)$
26	Ditrigonal scalenohedron or hexagonal scalenohedron	Trigonal antiprism sliced off by pinacoid	12	$\bar{3}m$	$\left\{ \begin{array}{l} \bar{3}m1(1) \\ \bar{3}1m(1) \\ \bar{3}m (1) \end{array} \right\}$
27	Hexagonal trapezohedron	Twisted hexagonal antiprism	12	622	622(1)
28	Dihexagonal pyramid	Truncated hexagon	12	$6mm$	6mm(1)
29	Ditrigonal dipyrmaid	Edge-truncated trigonal prism	12	$\bar{6}2m$	$\left\{ \begin{array}{l} \bar{6}m2(1) \\ \bar{6}2m(1) \end{array} \right\}$
30	Dihexagonal prism	Truncated hexagon	12	$\frac{6}{m}mm$	$\left\{ \begin{array}{l} \bar{3}m1(1) \\ \bar{3}1m(1); 622(1); 6mm(1); \\ \bar{3}m (1) \end{array} \right\};$ $\frac{6}{m}mm(m..)$
31	Hexagonal dipyrmaid	Hexagonal prism	12	$\frac{6}{m}mm$	$\left\{ \begin{array}{l} \bar{3}m1(1) \\ \bar{3}1m(1); \frac{6}{m}(1); 622(1)\ddagger; \\ \bar{3}m (1) \end{array} \right\};$ $\left\{ \begin{array}{l} \bar{6}m2(1); \frac{6}{m}mm(..m, .m.) \\ \bar{6}2m(1) \end{array} \right\};$

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Table 3.2.1.3 (continued)

No.	Crystal form	Point form	Number of faces or points	Eigensymmetry	Generating point groups with oriented face (site) symmetries between parentheses
32	Dihexagonal dipyramid	Edge-truncated hexagonal prism	24	$\frac{6}{m}mm$	$\frac{6}{m}mm(1)$
33	Tetrahedron	Tetrahedron	4	$\bar{4}3m$	$23(.3.); \bar{4}3m(.3m)$
34	Cube or hexahedron	Octahedron	6	$m\bar{3}m$	$23(2.); m\bar{3}(2mm.); 432(4.); \bar{4}3m(2.mm); m\bar{3}m(4m.m)$
35	Octahedron	Cube	8	$m\bar{3}m$	$m\bar{3}(.3.); 432(.3.); m\bar{3}m(.3m)$
36	Pentagon-tritetrahedron or tetartoid or tetrahedral pentagon-dodecahedron	Snub tetrahedron (= pentagon-tritetrahedron + two tetrahedra)	12	23	$23(1)$
37	Pentagon-dodecahedron or dihexahedron or pyritohedron	Irregular icosahedron (= pentagon-dodecahedron + octahedron)	12	$m\bar{3}$	$23(1); m\bar{3}(m.)$
38	Tetragon-tritetrahedron or deltohedron or deltoid-dodecahedron	Cube and two tetrahedra	12	$\bar{4}3m$	$23(1); \bar{4}3m(..m)$
39	Trigon-tritetrahedron or tristetrahedron	Tetrahedron truncated by tetrahedron	12	$\bar{4}3m$	$23(1); \bar{4}3m(..m)$
40	Rhomb-dodecahedron	Cuboctahedron	12	$m\bar{3}m$	$23(1); m\bar{3}(m.); 432(..2); \bar{4}3m(..m); m\bar{3}m(m.m2)$
41	Didodecahedron or diploid or dyakisdodecahedron	Cube & octahedron & pentagon-dodecahedron	24	$m\bar{3}$	$m\bar{3}(1)$
42	Trigon-trioctahedron or trisoctahedron	Cube truncated by octahedron	24	$m\bar{3}m$	$m\bar{3}(1); 432(1); m\bar{3}m(..m)$
43	Tetragon-trioctahedron or trapezohedron or deltoid-icositetrahedron	Cube & octahedron & rhomb-dodecahedron	24	$m\bar{3}m$	$m\bar{3}(1); 432(1); m\bar{3}m(..m)$
44	Pentagon-trioctahedron or gyroid	Cube + octahedron + pentagon-trioctahedron	24	432	$432(1)$
45	Hexatetrahedron or hexakistetrahedron	Cube truncated by two tetrahedra	24	$\bar{4}3m$	$\bar{4}3m(1)$
46	Tetrahexahedron or tetrakisohedron	Octahedron truncated by cube	24	$m\bar{3}m$	$432(1); \bar{4}3m(1); m\bar{3}m(m.)$
47	Hexaoctahedron or hexakisohedron	Cube truncated by octahedron and by rhomb-dodecahedron	48	$m\bar{3}m$	$m\bar{3}m(1)$

† These limiting forms occur in three or two non-equivalent orientations (different types of limiting forms); cf. Table 3.2.3.2. ‡ In point groups $\bar{4}2m$ and $\bar{3}m$, the tetragonal prism and the hexagonal prism occur twice, as a 'basic special form' and as a 'limiting special form'. In these cases, the point groups are listed twice, as ' $\bar{4}2m(2.)$ ' and ' $\bar{4}2m(..m)$ ' and as ' $\bar{3}m(2.)$ ' and ' $\bar{3}m(m.)$ '.

hedry $6/mmm$ has to be taken). Different types of limiting forms may have the same eigensymmetry and the same topology, as shown by the examples below. The occurrence of two topologically different polyhedra as two 'realizations' of one type of limiting form in point groups 23, $m\bar{3}$ and 432 is explained below in Section 3.2.1.2.4, *Notes on crystal and point forms*, item (viii).

Examples

- (1) In point group 32, the limiting general crystal forms are of four types:
 - (i) ditrigonal prisms, eigensymmetry $\bar{6}2m$ (face poles on horizontal mirror plane of holohedry $6/mmm$);
 - (ii) trigonal dipyramids, eigensymmetry $\bar{6}2m$ (face poles on one kind of vertical mirror plane);

- (iii) rhombohedra, eigensymmetry $\bar{3}m$ (face poles on second kind of vertical mirror plane);
- (iv) hexagonal prisms, eigensymmetry $6/mmm$ (face poles on horizontal twofold axes).

Types (i) and (ii) have the same eigensymmetry but different topologies; types (i) and (iv) have the same topology but different eigensymmetries; type (iii) differs from the other three types in both eigensymmetry and topology.

- (2) In point group 222, the face poles of the three types of general limiting forms, rhombic prisms, are located on the three (non-equivalent) symmetry planes of the holohedry mmm . Geometrically, the axes of the prisms are directed along the three non-equivalent orthorhombic symmetry directions. The three types of limiting forms have the same