

3. ADVANCED TOPICS ON SPACE-GROUP SYMMETRY

Table 3.2.1.6

Classes of general point groups in three dimensions ($N = \text{integer} \geq 0$)

Short general Hermann–Mauguin symbol, followed by full symbol where different	Schoenflies symbol	Order of group	General face form	General point form	Crystallographic groups
4 <i>N</i> -gonal system (single <i>n</i> -fold symmetry axis with $n = 4N$)					
<i>n</i>	<i>C_n</i>	<i>n</i>	<i>n</i> -gonal pyramid	Regular <i>n</i> -gon	4
\bar{n}	<i>S_n</i>	<i>n</i>	$\frac{1}{2}$ <i>n</i> -gonal streptohedron	$\frac{1}{2}$ <i>n</i> -gonal antiprism	$\bar{4}$
<i>n/m</i>	<i>C_nh</i>	2 <i>n</i>	<i>n</i> -gonal dipyrmaid	<i>n</i> -gonal prism	4/ <i>m</i>
<i>n</i> 22	<i>D_n</i>	2 <i>n</i>	<i>n</i> -gonal trapezohedron	Twisted <i>n</i> -gonal antiprism	422
<i>nmm</i>	<i>C_{nv}</i>	2 <i>n</i>	Di- <i>n</i> -gonal pyramid	Truncated <i>n</i> -gon	4 <i>mm</i>
$\bar{n}2m$	<i>D_{\frac{3}{2}nd}</i>	2 <i>n</i>	<i>n</i> -gonal scalenohedron	$\frac{1}{2}$ <i>n</i> -gonal antiprism sliced off by pinacoid	$\bar{4}2m$
$n/mmm, \frac{n}{m} 2 \frac{2}{m}$	<i>D_{nh}</i>	4 <i>n</i>	Di- <i>n</i> -gonal dipyrmaid	Edge-truncated <i>n</i> -gonal prism	4/ <i>mmm</i>
(2 <i>N</i> + 1)-gonal system (single <i>n</i> -fold symmetry axis with $n = 2N + 1$)					
<i>n</i>	<i>C_n</i>	<i>n</i>	<i>n</i> -gonal pyramid	Regular <i>n</i> -gon	1, 3
$\bar{n} = n \times \bar{1}$	<i>C_{ni}</i>	2 <i>n</i>	<i>n</i> -gonal streptohedron	<i>n</i> -gonal antiprism	$\bar{1}, \bar{3} = 3 \times \bar{1}$
<i>n</i> 2	<i>D_n</i>	2 <i>n</i>	<i>n</i> -gonal trapezohedron	Twisted <i>n</i> -gonal antiprism	32
<i>nm</i>	<i>C_{nv}</i>	2 <i>n</i>	Di- <i>n</i> -gonal pyramid	Truncated <i>n</i> -gon	3 <i>m</i>
$\bar{n}m, \bar{n} \frac{2}{m}$	<i>D_{nd}</i>	4 <i>n</i>	Di- <i>n</i> -gonal scalenohedron	<i>n</i> -gonal antiprism sliced off by pinacoid	$\bar{3}m$
(4 <i>N</i> + 2)-gonal system (single <i>n</i> -fold symmetry axis with $n = 4N + 2$)					
<i>n</i>	<i>C_n</i>	<i>n</i>	<i>n</i> -gonal pyramid	Regular <i>n</i> -gon	2, 6
$\bar{n} = \frac{1}{2}n/m$	<i>C_{\frac{3}{2}nh}</i>	<i>n</i>	$\frac{1}{2}$ <i>n</i> -gonal dipyrmaid	$\frac{1}{2}$ <i>n</i> -gonal prism	$\bar{2} \equiv m, \bar{6} \equiv 3/m$
<i>n/m</i>	<i>C_{nh}</i>	2 <i>n</i>	<i>n</i> -gonal dipyrmaid	<i>n</i> -gonal prism	2/ <i>m</i> , 6/ <i>m</i>
<i>n</i> 22	<i>D_n</i>	2 <i>n</i>	<i>n</i> -gonal trapezohedron	Twisted <i>n</i> -gonal antiprism	222, 622
<i>nmm</i>	<i>C_{nv}</i>	2 <i>n</i>	Di- <i>n</i> -gonal pyramid	Truncated <i>n</i> -gon	<i>mm</i> 2, 6 <i>mm</i>
$\bar{n}2m = \frac{1}{2}n/m2m$	<i>D_{\frac{3}{2}nh}</i>	2 <i>n</i>	Di- $\frac{1}{2}$ <i>n</i> -gonal dipyrmaid	Truncated $\frac{1}{2}$ <i>n</i> -gonal prism	$\bar{6}2m$
$n/mmm, \frac{n}{m} 2 \frac{2}{m}$	<i>D_{nh}</i>	4 <i>n</i>	Di- <i>n</i> -gonal dipyrmaid	Edge-truncated <i>n</i> -gonal prism	<i>mmm</i> , 6/ <i>mmm</i>
Cubic system (for details see Table 3.2.3.2)					
$\frac{23}{m\bar{3}}, \frac{2}{m}\bar{3}$	<i>T</i>	12	Pentagon-tritetrahedron	Snub tetrahedron	23
	<i>T_h</i>	24	Didodecahedron	Cube & octahedron & pentagon-dodecahedron	$m\bar{3}$
432	<i>O</i>	24	Pentagon-trioctahedron	Snub cube	432
$\bar{4}3m$	<i>T_d</i>	24	Hexatetrahedron	Cube truncated by two tetrahedra	$\bar{4}3m$
$m\bar{3}m, \frac{4}{m}\bar{3} \frac{2}{m}$	<i>O_h</i>	48	Hexaoctahedron	Cube truncated by octahedron and by rhomb-dodecahedron	$m\bar{3}m$
Icosahedral system† (for details see Table 3.2.3.3)					
235	<i>I</i>	60	Pentagon-hexacontahedron	Snub pentagon-dodecahedron	–
$m\bar{3}\bar{5}, \frac{2}{m}\bar{3}\bar{5}$	<i>I_h</i>	120	Hecatonicosahedron	Pentagon-dodecahedron truncated by icosahedron and by rhomb-triacontahedron	–
Cylindrical system‡					
∞	<i>C_{\infty}</i>	∞	Rotating cone	Rotating circle	–
$\infty/m \equiv \infty\bar{2}$	<i>C_{\infty h} \equiv S_{\infty} \equiv C_{\infty i}</i>	∞	Rotating double cone	Rotating finite cylinder	–
$\infty 2$	<i>D_{\infty}</i>	∞	‘Anti-rotating’ double cone	‘Anti-rotating’ finite cylinder	–
∞m	<i>C_{\infty v}</i>	∞	Stationary cone	Stationary circle	–
$\infty/mmm \equiv \infty\bar{6}m, \frac{\infty}{m} 2 \frac{2}{m} \equiv \infty\bar{6} \frac{2}{m}$	<i>D_{\infty h} \equiv D_{\infty d}</i>	∞	Stationary double cone	Stationary finite cylinder	–
Spherical system§					
$2\infty, \infty\infty$	<i>K</i>	∞	Rotating sphere	Rotating sphere	–
$m\bar{\infty}, \frac{2}{m}\bar{\infty}, \infty\infty m$	<i>K_h</i>	∞	Stationary sphere	Stationary sphere	–

† The Hermann–Mauguin symbols of the two icosahedral point groups are often written as 532 and $\bar{5}3m$ (see text). ‡ Rotating and ‘anti-rotating’ forms in the cylindrical system have no ‘vertical’ mirror planes, whereas stationary forms have infinitely many vertical mirror planes. In classes ∞ and $\infty 2$, enantiomorphism occurs, *i.e.* forms with opposite senses of rotation. Class $\infty/m \equiv \infty\bar{2}$ exhibits no enantiomorphism due to the centre of symmetry, even though the double cone is rotating in one direction. This can be understood as follows: The handedness of a rotating cone depends on the sense of rotation with respect to the axial direction from the base to the tip of the cone. Thus, the rotating double cone consists of two cones with opposite handedness and opposite orientations related by the (single) horizontal mirror plane. In contrast, the ‘anti-rotating’ double cone in class $\infty 2$ consists of two cones of equal handedness and opposite orientations, which are related by the (infinitely many) twofold axes. The term ‘anti-rotating’ means that upper and lower halves of the forms rotate in opposite directions. § The spheres in class 2∞ of the spherical system must rotate around an axis with at least two different orientations, in order to suppress all mirror planes. This class exhibits enantiomorphism, *i.e.* it contains spheres with either right-handed or left-handed senses of rotation around the axes (*cf.* Section 3.2.2.4, *Optical properties*). The stationary spheres in class $m\bar{\infty}$ contain infinitely many mirror planes through the centres of the spheres. Group 2∞ is sometimes symbolized by $\infty\infty$; group $m\bar{\infty}$ by $\infty\bar{\infty}$ or $\infty\infty m$. The symbols used here indicate the minimal symmetry necessary to generate the groups; they show, furthermore, the relation to the cubic groups. The Schoenflies symbol *K* is derived from the German name *Kugelgruppe*.