International Tables for Crystallography (2016). Vol. A, Section 3.3.1.4.3, pp. 778–779.

Table 3.3.1.2
Representatives for the sets of lattice symmetry directions in the various crystal families

	Crystal family	Anorthic (triclinic)	Monoclinic	Orthorhombic	Tetragonal	Hexagonal		Cubic
Lattice point group	Schoenflies	C_i	C_{2h}	D_{2h}	D_{4h}	D_{6h}	D_{3d}	O_h
	Hermann–Mauguin	$\bar{1}$	$\frac{2}{m}$	$\frac{2}{m}\frac{2}{m}\frac{2}{m}$	$\frac{4}{m}\frac{2}{m}\frac{2}{m}$	$\frac{6}{m}\frac{2}{m}\frac{2}{m}$	$\bar{3}\frac{2}{m}$†	$\frac{4}{m}\bar{3}\frac{2}{m}$
Set of lattice symmetry directions	Primary	–	[010] *b* unique [001] *c* unique	[100]	[001]	[001]	[001]	[001]
	Secondary	–	–	[010]	[100]	[100]	[100]	[111]
	Tertiary	–	–	[001]	[1$\bar{1}$0]	[1$\bar{1}$0]	–	[1$\bar{1}$0] [110]‡

† In this table, the directions refer to the hexagonal description. The use of the primitive rhombohedral cell brings out the relations between cubic and rhombohedral groups: the primary set is represented by [111] and the secondary by [1$\bar{1}$0]. ‡ Only for $\bar{4}3m$ and 432 [for reasons see text].

3.3.1.4.2. *Full Hermann–Mauguin symbols*

After the classification of the directions of rotation axes, the description of the seven maximal rotation subgroups of the lattice point groups is rather simple. For each representative direction, the rotational symmetry element is symbolized by an integer *n* for an *n*-fold axis, resulting in the symbols of the maximal rotation subgroups 1, 2, 222, 32, 422, 622, 432. The symbol 1 is used for the triclinic case. The complete lattice point group is constructed by multiplying the rotation group by the inversion $\bar{1}$. For the even-fold axes, 2, 4 and 6, this multiplication results in a mirror plane perpendicular to the rotation axis yielding the symbols $(2n)/m$ (n = 1, 2, 3). For the odd-fold axes 1 and 3, this product leads to the rotoinversion axes $\bar{1}$ and $\bar{3}$. Thus, for each representative of a set of lattice symmetry directions, the symmetry forms a point group that can be generated by one, or at most two, symmetry operations. The resulting symbols are called *full Hermann–Mauguin (or international) symbols*. For the lattice point groups they are shown in Table 3.3.1.2.

For the description of a point group of a crystal, we use its lattice symmetry directions. For the representative of each set of lattice symmetry directions, the remaining subgroup is symbolized; if only the primary symmetry direction contains symmetry higher than 1, the symbols '1' for the secondary and tertiary set (if present) can be omitted. For the cubic point groups T and T_h, the representative of the tertiary set would be '1', which is omitted. For the rotoinversion groups $\bar{1}$ and $\bar{3}$, the remaining subgroups can only be 1 and 3. If the supergroup is $(2n)/m$, five different types of subgroups can be derived: n/m, $2n$, $\overline{2n}$, n and m. In the cubic system, for instance, $4/m$, $2/m$, $\bar{4}$, 4 or 2 may occur in the primary set. In this case, the symbol m can only occur in the combinations $2/m$ or $4/m$ as can be seen from Table 3.3.1.3.

3.3.1.4.3. *Short symbols and generators*

If the symbols are not only used for the identification of a group but also for its construction, the symbol must contain a list of generating operations and additional relations, if necessary. Following this aspect, the Hermann–Mauguin symbols can be shortened. The choice of generators is not unique; two proposals were presented by Mauguin (1931). In the first proposal, in almost all cases the generators are the same as those of the Shubnikov symbols. In the second proposal, which, apart from some exceptions (see Section 3.3.4), is used for the international symbols, Mauguin selected a set of generators and thus a list of short symbols in which reflections have priority (Table 3.3.1.3, column 3). This selection makes the transition from the short point-group symbols to the space-group symbols fairly simple. These short symbols contain two kinds of notation components:

Table 3.3.1.3
Point-group symbols

Schoenflies	Shubnikov	*International Tables*, short symbol	*International Tables*, full symbol
C_1	1	1	1
C_i	$\tilde{2}$	$\bar{1}$	$\bar{1}$
C_2	2	2	2
C_s	m	m	m
C_{2h}	$2:m$	$2/m$	$2/m$
D_2	$2:2$	222	222
C_{2v}	$2\cdot m$	$mm2$	$mm2$
D_{2h}	$m\cdot 2:m$	mmm	$2/m\ 2/m\ 2/m$
C_4	4	4	4
S_4	$\tilde{4}$	$\bar{4}$	$\bar{4}$
C_{4h}	$4:m$	$4/m$	$4/m$
D_4	$4:2$	422	422
C_{4v}	$4\cdot m$	$4mm$	$4mm$
D_{2d}	$\tilde{4}:2$	$\bar{4}2m$ or $\bar{4}m2$	$\bar{4}2m$ or $\bar{4}m2$
D_{4h}	$m\cdot 4:m$	$4/mmm$	$4/m\ 2/m\ 2/m$
C_3	3	3	3
C_{3i}	$\tilde{6}$	$\bar{3}$	$\bar{3}$
D_3	$3:2$	32 or 321 or 312	32 or 321 or 312
C_{3v}	$3\cdot m$	$3m$ or $3m1$ or $31m$	$3m$ or $3m1$ or $31m$
D_{3d}	$\tilde{6}\cdot m$	$\bar{3}m$ or $\bar{3}m1$ or $\bar{3}1m$	$\bar{3}\ 2/m$ or $\bar{3}\ 2/m1$ or $\bar{3}12/m$
C_6	6	6	6
C_{3h}	$3:m$	$\bar{6}$	$\bar{6}$
C_{6h}	$6:m$	$6/m$	$6/m$
D_6	$6:2$	622	622
C_{6v}	$6\cdot m$	$6mm$	$6mm$
D_{3h}	$m\cdot 3:m$	$\bar{6}m2$ or $\bar{6}2m$	$\bar{6}m2$ or $\bar{6}2m$
D_{6h}	$m\cdot 6:m$	$6/mmm$	$6/m\ 2/m\ 2/m$
T	$3/2$	23	23
T_h	$\tilde{6}/2$	$m\bar{3}$	$2/m\bar{3}$
O	$3/4$	432	432
T_d	$3/\tilde{4}$	$\bar{4}3m$	$\bar{4}3m$
O_h	$\tilde{6}/4$	$m\bar{3}m$	$4/m\ \bar{3}\ 2/m$

(i) components that represent the type of the generating operation, which are called *generators*;

(ii) components that are not used as generators but that serve to fix the directions of other symmetry elements (Hermann, 1931), and which are called *indicators*.

The generating matrices are uniquely defined by (i) and (ii) if it is assumed that they describe motions with counterclockwise rotational sense about the representative direction looked at end on by the observer. The symbols 2, 4, $\bar{4}$, 6 and $\bar{6}$ referring to direction [001] are indicators when the point-group symbol uses three sets of lattice symmetry directions. For instance, in 4*mm* the indicator 4 fixes the directions of the mirrors normal to [100] and [$1\bar{1}0$].

Note: The generation of (*a*) point group 432 by a rotation 3 around [111] and a rotation 2 and (*b*) point group $\bar{4}3m$ by 3 around [111] and a reflection *m* is only possible if the representative direction of the tertiary set is changed from [$1\bar{1}0$] to [110]; otherwise only the subgroup 32 or 3*m* of 432 or $\bar{4}3m$ will be generated.

Table 3.3.2.1
Symbols of glide planes in the Shubnikov and Hermann–Mauguin space-group symbols

Glide plane perpendicular to	Glide vector	Shubnikov symbol	Hermann–Mauguin symbol
b or **c**	$\frac{1}{2}\mathbf{a}$	$\tilde{a}$	*a*
a or **c**	$\frac{1}{2}\mathbf{b}$	$\tilde{b}$	*b*
a or **b** or **a** − **b**	$\frac{1}{2}\mathbf{c}$	$\tilde{c}$	*c*
c	$\frac{1}{2}(\mathbf{a}+\mathbf{b})$	$\widetilde{ab}$	*n*
a	$\frac{1}{2}(\mathbf{b}+\mathbf{c})$	$\widetilde{bc}$	*n*
b	$\frac{1}{2}(\mathbf{c}+\mathbf{a})$	$\widetilde{ac}$	*n*
a − **b**	$\frac{1}{2}(\mathbf{a}+\mathbf{b}+\mathbf{c})$	$\widetilde{abc}$	*n*
c	$\frac{1}{4}(\mathbf{a}+\mathbf{b})$	$\frac{1}{2}\widetilde{ab}$	*d*
a	$\frac{1}{4}(\mathbf{b}+\mathbf{c})$	$\frac{1}{2}\widetilde{bc}$	*d*
b	$\frac{1}{4}(\mathbf{c}+\mathbf{a})$	$\frac{1}{2}\widetilde{ac}$	*d*
a − **b**	$\frac{1}{4}(\mathbf{a}+\mathbf{b}+\mathbf{c})$	$\frac{1}{2}\widetilde{abc}$	*d*
a + **b**	$\frac{1}{4}(-\mathbf{a}+\mathbf{b}+\mathbf{c})$		*d*

3.3.2. Space-group symbols

3.3.2.1. Introduction

Each space group is related to a crystallographic point group. Space-group symbols, therefore, can be obtained by a modification of point-group symbols. The simplest modification which merely gives an enumeration of the space-group types (*cf.* Section 1.3.4.1) has been used by Schoenflies. The Shubnikov and Hermann–Mauguin symbols, however, reveal the glide or screw components of the symmetry operations and are designed in such a way that the nature of the symmetry elements and their relative locations can be deduced from the symbol. [A detailed discussion and listings of computer-adapted space-group symbols implemented in crystallographic software, such as the so-called *Hall symbols* (Hall, 1981*a*,*b*) or *explicit symbols* (Shmueli, 1984), can be found in Chapter 1.4 of *International Tables for Crystallography*, Volume B (2008).]

3.3.2.2. Schoenflies symbols

Space groups related to one point group are distinguished by adding a numerical superscript to the point-group symbol. Thus, the space groups related to the point group C_2 are called C_2^1, C_2^2, C_2^3.

3.3.2.3. The role of translation parts in the Shubnikov and Hermann–Mauguin symbols

A crystallographic symmetry operation *W* (*cf.* Chapter 1.2) is described by a pair of matrices

$$(\boldsymbol{W}, \boldsymbol{w}) = (\boldsymbol{I}, \boldsymbol{w})(\boldsymbol{W}, \boldsymbol{o}).$$

$\boldsymbol{W}$ is called the *rotation part*, $\boldsymbol{w}$ describes the *translation part* and determines the translation vector **w** of the operation. The translation part $\boldsymbol{w}$ can be decomposed into a *glide/screw part* $\boldsymbol{w}_g$ and a *location part* $\boldsymbol{w}_l$: $\boldsymbol{w} = \boldsymbol{w}_g + \boldsymbol{w}_l$; here, $\boldsymbol{w}_l$ determines the location of the corresponding symmetry element with respect to the origin. The glide/screw part $\boldsymbol{w}_g$ may be derived by projecting $\boldsymbol{w}$ on the space invariant under $\boldsymbol{W}$, *i.e.* for rotations and reflections $\boldsymbol{w}$ is projected on the corresponding rotation axis or mirror plane. With matrix notation, $\boldsymbol{w}_g$ is determined by $(\boldsymbol{W}, \boldsymbol{w})^k = (\boldsymbol{I}, \boldsymbol{t})$ and $\boldsymbol{w}_g = (m/k)\boldsymbol{t}_1$, where k is the order of $\boldsymbol{W}$, the integers m are restricted by $0 \le m < k$ and $\boldsymbol{t}_1$ is the shortest lattice vector in the direction of $\boldsymbol{t}$ (for details, *cf.* Sections 1.2.2.4 and 1.5.4.1). Space groups contain sets of screw and rotation axes or glide and mirror planes. A screw rotation is symbolized by k_m. The Shubnikov notation and the international notation use the same symbols for screw rotations. The symbols for glide reflections in both notations are listed in Table 3.3.2.1.

If the point-group symbol contains only one generator, the related space group is described completely by the Bravais lattice and a symbol corresponding to that of the point group in which rotations and reflections are replaced by screw rotations or glide reflections, if necessary. If, however, two or more operations generate the point group, it is necessary to have information on the mutual orientations and locations of the corresponding space-group symmetry elements, *i.e.* information on the location components $\boldsymbol{w}_l$. This is described in the following sections.

3.3.2.4. Shubnikov symbols

For the description of the mutual orientation of symmetry elements, the same symbols as for point groups are applied. In space groups, however, the symmetry elements need not intersect. In this case, the orientational symbols · (dot), : (colon), / (slash) are modified to ⨀, ⦂⃝, //. The space-group symbol starts with a description of the lattice defined by the basis **a**, **b**, **c**. For centred cells, the vectors to the centring points are given first. The same letters are used for basis vectors related by symmetry. The relative orientations of the vectors are denoted by the orientational symbols introduced above. The description of the lattice given in parentheses is followed by symbols of the generating elements of the related point group. If necessary, the symbols of the symmetry operations are modified to indicate their glide/screw parts. The first generator is separated from the lattice description by an orientation symbol. If this generator represents a mirror or glide plane, the dot connects the plane with the last two vectors whereas the colon refers only to the last vector. If the generator represents a rotation or a rotoreflection, the colon orients the related axis perpendicular to the plane given by the last two vectors whereas the dot refers only to the last vector. Two generators are separated by the symbols mentioned above to denote their relative orientations and sites. To make this description unique for space groups related to point group $O_h \equiv \tilde{6}/4$ with Bravais lattices *cP* and *cF*, it is necessary to use three generators instead of two: $4/\tilde{6} \cdot m$. For the sake of unifi-