

3.5. NORMALIZERS OF SPACE GROUPS

Table 3.5.2.4

Euclidean and chirality-preserving Euclidean normalizers of the orthorhombic space groups

The symbols in parentheses following a space-group symbol refer to the location of the origin ('origin choice' in Chapter 2.3).

Space group \mathcal{G}		Euclidean normalizer $\mathcal{N}_{\mathcal{E}}(\mathcal{G})$ and chirality-preserving normalizer $\mathcal{N}_{\mathcal{E}^+}(\mathcal{G})$		Additional generators of $\mathcal{N}_{\mathcal{E}}(\mathcal{G})$ and $\mathcal{N}_{\mathcal{E}^+}(\mathcal{G})$			Index of \mathcal{G} in $\mathcal{N}_{\mathcal{E}}(\mathcal{G})$ or $\mathcal{N}_{\mathcal{E}^+}(\mathcal{G})$		
No.	Hermann–Mauguin symbol	Cell metric	Symbol	Basis vectors	Translations	Inversion through a centre at		Further generators	
16	$P222$	$a \neq b \neq c \neq a$	$Pmmm$	$\frac{1}{2}\mathbf{a}, \frac{1}{2}\mathbf{b}, \frac{1}{2}\mathbf{c}$	$\frac{1}{2}, 0, 0; 0, \frac{1}{2}, 0; 0, 0, \frac{1}{2}$	0, 0, 0		8 · 2 · 1	
			$\mathcal{N}_{\mathcal{E}^+}(\mathcal{G}): P222$	$\frac{1}{2}\mathbf{a}, \frac{1}{2}\mathbf{b}, \frac{1}{2}\mathbf{c}$	$\frac{1}{2}, 0, 0; 0, \frac{1}{2}, 0; 0, 0, \frac{1}{2}$	/			8 · 1
		$a = b \neq c$	$P4/mmm$	$\frac{1}{2}\mathbf{a}, \frac{1}{2}\mathbf{b}, \frac{1}{2}\mathbf{c}$	$\frac{1}{2}, 0, 0; 0, \frac{1}{2}, 0; 0, 0, \frac{1}{2}$	$\frac{1}{2}, 0, 0; 0, \frac{1}{2}, 0; 0, 0, \frac{1}{2}$	0, 0, 0	y, x, z	8 · 2 · 2
			$\mathcal{N}_{\mathcal{E}^+}(\mathcal{G}): P422$	$\frac{1}{2}\mathbf{a}, \frac{1}{2}\mathbf{b}, \frac{1}{2}\mathbf{c}$	$\frac{1}{2}, 0, 0; 0, \frac{1}{2}, 0; 0, 0, \frac{1}{2}$	/		y, x, \bar{z}	8 · 2
		$a = b = c$	$Pm\bar{3}m$	$\frac{1}{2}\mathbf{a}, \frac{1}{2}\mathbf{b}, \frac{1}{2}\mathbf{c}$	$\frac{1}{2}, 0, 0; 0, \frac{1}{2}, 0; 0, 0, \frac{1}{2}$	$\frac{1}{2}, 0, 0; 0, \frac{1}{2}, 0; 0, 0, \frac{1}{2}$	0, 0, 0	$z, x, y; y, x, z$	8 · 2 · 6
			$\mathcal{N}_{\mathcal{E}^+}(\mathcal{G}): P432$	$\frac{1}{2}\mathbf{a}, \frac{1}{2}\mathbf{b}, \frac{1}{2}\mathbf{c}$	$\frac{1}{2}, 0, 0; 0, \frac{1}{2}, 0; 0, 0, \frac{1}{2}$	/		$z, x, y; y, x, \bar{z}$	8 · 6
17	$P222_1$	$a \neq b$	$Pmmm$	$\frac{1}{2}\mathbf{a}, \frac{1}{2}\mathbf{b}, \frac{1}{2}\mathbf{c}$	$\frac{1}{2}, 0, 0; 0, \frac{1}{2}, 0; 0, 0, \frac{1}{2}$	0, 0, 0		8 · 2 · 1	
			$\mathcal{N}_{\mathcal{E}^+}(\mathcal{G}): P222$	$\frac{1}{2}\mathbf{a}, \frac{1}{2}\mathbf{b}, \frac{1}{2}\mathbf{c}$	$\frac{1}{2}, 0, 0; 0, \frac{1}{2}, 0; 0, 0, \frac{1}{2}$	/		8 · 1	
		$a = b$	$P4_2/mmc$	$\frac{1}{2}\mathbf{a}, \frac{1}{2}\mathbf{b}, \frac{1}{2}\mathbf{c}$	$\frac{1}{2}, 0, 0; 0, \frac{1}{2}, 0; 0, 0, \frac{1}{2}$	0, 0, 0	$y, x, z + \frac{1}{4}$	8 · 2 · 2	
	$\mathcal{N}_{\mathcal{E}^+}(\mathcal{G}): P4_222$	$\frac{1}{2}\mathbf{a}, \frac{1}{2}\mathbf{b}, \frac{1}{2}\mathbf{c}$	$\frac{1}{2}, 0, 0; 0, \frac{1}{2}, 0; 0, 0, \frac{1}{2}$	/		$y, x, \bar{z} + \frac{1}{4}$	8 · 2		
18	$P2_12_12$	$a \neq b$	$Pmmm$	$\frac{1}{2}\mathbf{a}, \frac{1}{2}\mathbf{b}, \frac{1}{2}\mathbf{c}$	$\frac{1}{2}, 0, 0; 0, \frac{1}{2}, 0; 0, 0, \frac{1}{2}$	0, 0, 0		8 · 2 · 1	
			$\mathcal{N}_{\mathcal{E}^+}(\mathcal{G}): P222$	$\frac{1}{2}\mathbf{a}, \frac{1}{2}\mathbf{b}, \frac{1}{2}\mathbf{c}$	$\frac{1}{2}, 0, 0; 0, \frac{1}{2}, 0; 0, 0, \frac{1}{2}$	/		8 · 1	
		$a = b$	$P4/mmm$	$\frac{1}{2}\mathbf{a}, \frac{1}{2}\mathbf{b}, \frac{1}{2}\mathbf{c}$	$\frac{1}{2}, 0, 0; 0, \frac{1}{2}, 0; 0, 0, \frac{1}{2}$	0, 0, 0	y, x, z	8 · 2 · 2	
	$\mathcal{N}_{\mathcal{E}^+}(\mathcal{G}): P422$	$\frac{1}{2}\mathbf{a}, \frac{1}{2}\mathbf{b}, \frac{1}{2}\mathbf{c}$	$\frac{1}{2}, 0, 0; 0, \frac{1}{2}, 0; 0, 0, \frac{1}{2}$	/		y, x, \bar{z}	8 · 2		
19	$P2_12_12_1$	$a \neq b \neq c \neq a$	$Pmmm$	$\frac{1}{2}\mathbf{a}, \frac{1}{2}\mathbf{b}, \frac{1}{2}\mathbf{c}$	$\frac{1}{2}, 0, 0; 0, \frac{1}{2}, 0; 0, 0, \frac{1}{2}$	0, 0, 0		8 · 2 · 1	
			$\mathcal{N}_{\mathcal{E}^+}(\mathcal{G}): P222$	$\frac{1}{2}\mathbf{a}, \frac{1}{2}\mathbf{b}, \frac{1}{2}\mathbf{c}$	$\frac{1}{2}, 0, 0; 0, \frac{1}{2}, 0; 0, 0, \frac{1}{2}$	/		8 · 1	
		$a = b \neq c$	$P4_2/mmc (2/m2/mn)$	$\frac{1}{2}\mathbf{a}, \frac{1}{2}\mathbf{b}, \frac{1}{2}\mathbf{c}$	$\frac{1}{2}, 0, 0; 0, \frac{1}{2}, 0; 0, 0, \frac{1}{2}$	0, 0, 0	$y + \frac{1}{4}, x + \frac{1}{4}, z + \frac{1}{4}$	8 · 2 · 2	
			$\mathcal{N}_{\mathcal{E}^+}(\mathcal{G}): P4_222 (222_1)$	$\frac{1}{2}\mathbf{a}, \frac{1}{2}\mathbf{b}, \frac{1}{2}\mathbf{c}$	$\frac{1}{2}, 0, 0; 0, \frac{1}{2}, 0; 0, 0, \frac{1}{2}$	/		$\bar{y} + \frac{1}{4}, \bar{x} + \frac{1}{4}, \bar{z} + \frac{1}{4}$	8 · 2
		$a = b = c$	$Pm\bar{3}n$	$\frac{1}{2}\mathbf{a}, \frac{1}{2}\mathbf{b}, \frac{1}{2}\mathbf{c}$	$\frac{1}{2}, 0, 0; 0, \frac{1}{2}, 0; 0, 0, \frac{1}{2}$	0, 0, 0	$z, x, y; y + \frac{1}{4}, x + \frac{1}{4}, z + \frac{1}{4}$	8 · 2 · 6	
			$\mathcal{N}_{\mathcal{E}^+}(\mathcal{G}): P4_232$	$\frac{1}{2}\mathbf{a}, \frac{1}{2}\mathbf{b}, \frac{1}{2}\mathbf{c}$	$\frac{1}{2}, 0, 0; 0, \frac{1}{2}, 0; 0, 0, \frac{1}{2}$	/		$z, x, y; \bar{y} + \frac{1}{4}, \bar{x} + \frac{1}{4}, \bar{z} + \frac{1}{4}$	8 · 6
20	$C222_1$	$a \neq b$	$Pmmm$	$\frac{1}{2}\mathbf{a}, \frac{1}{2}\mathbf{b}, \frac{1}{2}\mathbf{c}$	$\frac{1}{2}, 0, 0; 0, 0, \frac{1}{2}$	0, 0, 0		4 · 2 · 1	
			$\mathcal{N}_{\mathcal{E}^+}(\mathcal{G}): P222$	$\frac{1}{2}\mathbf{a}, \frac{1}{2}\mathbf{b}, \frac{1}{2}\mathbf{c}$	$\frac{1}{2}, 0, 0; 0, 0, \frac{1}{2}$	/		4 · 1	
		$a = b$	$P4_2/mmc$	$\frac{1}{2}\mathbf{a}, \frac{1}{2}\mathbf{b}, \frac{1}{2}\mathbf{c}$	$\frac{1}{2}, 0, 0; 0, 0, \frac{1}{2}$	0, 0, 0	$y, x, z + \frac{1}{4}$	4 · 2 · 2	
	$\mathcal{N}_{\mathcal{E}^+}(\mathcal{G}): P4_222$	$\frac{1}{2}\mathbf{a}, \frac{1}{2}\mathbf{b}, \frac{1}{2}\mathbf{c}$	$\frac{1}{2}, 0, 0; 0, 0, \frac{1}{2}$	/		$y, x, \bar{z} + \frac{1}{4}$	4 · 2		
21	$C222$	$a \neq b$	$Pmmm$	$\frac{1}{2}\mathbf{a}, \frac{1}{2}\mathbf{b}, \frac{1}{2}\mathbf{c}$	$\frac{1}{2}, 0, 0; 0, 0, \frac{1}{2}$	0, 0, 0		4 · 2 · 1	
			$\mathcal{N}_{\mathcal{E}^+}(\mathcal{G}): P222$	$\frac{1}{2}\mathbf{a}, \frac{1}{2}\mathbf{b}, \frac{1}{2}\mathbf{c}$	$\frac{1}{2}, 0, 0; 0, 0, \frac{1}{2}$	/		4 · 1	
		$a = b$	$P4/mmm$	$\frac{1}{2}\mathbf{a}, \frac{1}{2}\mathbf{b}, \frac{1}{2}\mathbf{c}$	$\frac{1}{2}, 0, 0; 0, 0, \frac{1}{2}$	0, 0, 0	y, x, z	4 · 2 · 2	
	$\mathcal{N}_{\mathcal{E}^+}(\mathcal{G}): P422$	$\frac{1}{2}\mathbf{a}, \frac{1}{2}\mathbf{b}, \frac{1}{2}\mathbf{c}$	$\frac{1}{2}, 0, 0; 0, 0, \frac{1}{2}$	/		y, x, \bar{z}	4 · 2		
22	$F222$	$a \neq b \neq c \neq a$	$Immm$	$\frac{1}{2}\mathbf{a}, \frac{1}{2}\mathbf{b}, \frac{1}{2}\mathbf{c}$	$\frac{1}{4}, \frac{1}{4}, \frac{1}{4}$	0, 0, 0		4 · 2 · 1	
			$\mathcal{N}_{\mathcal{E}^+}(\mathcal{G}): I222$	$\frac{1}{2}\mathbf{a}, \frac{1}{2}\mathbf{b}, \frac{1}{2}\mathbf{c}$	$\frac{1}{4}, \frac{1}{4}, \frac{1}{4}$	/		4 · 1	
		$a = b \neq c$	$I4/mmm$	$\frac{1}{2}\mathbf{a}, \frac{1}{2}\mathbf{b}, \frac{1}{2}\mathbf{c}$	$\frac{1}{4}, \frac{1}{4}, \frac{1}{4}$	0, 0, 0	y, x, z	4 · 2 · 2	
			$\mathcal{N}_{\mathcal{E}^+}(\mathcal{G}): I422$	$\frac{1}{2}\mathbf{a}, \frac{1}{2}\mathbf{b}, \frac{1}{2}\mathbf{c}$	$\frac{1}{4}, \frac{1}{4}, \frac{1}{4}$	/		y, x, \bar{z}	4 · 2
		$a = b = c$	$Im\bar{3}m$	$\frac{1}{2}\mathbf{a}, \frac{1}{2}\mathbf{b}, \frac{1}{2}\mathbf{c}$	$\frac{1}{4}, \frac{1}{4}, \frac{1}{4}$	0, 0, 0	$z, x, y; y, x, z$	4 · 2 · 6	
			$\mathcal{N}_{\mathcal{E}^+}(\mathcal{G}): I432$	$\frac{1}{2}\mathbf{a}, \frac{1}{2}\mathbf{b}, \frac{1}{2}\mathbf{c}$	$\frac{1}{4}, \frac{1}{4}, \frac{1}{4}$	/		$z, x, y; y, x, \bar{z}$	4 · 6
23	$I222$	$a \neq b \neq c \neq a$	$Pmmm$	$\frac{1}{2}\mathbf{a}, \frac{1}{2}\mathbf{b}, \frac{1}{2}\mathbf{c}$	$\frac{1}{2}, 0, 0; 0, \frac{1}{2}, 0$	0, 0, 0		4 · 2 · 1	
			$\mathcal{N}_{\mathcal{E}^+}(\mathcal{G}): P222$	$\frac{1}{2}\mathbf{a}, \frac{1}{2}\mathbf{b}, \frac{1}{2}\mathbf{c}$	$\frac{1}{2}, 0, 0; 0, \frac{1}{2}, 0$	/		4 · 1	
		$a = b \neq c$	$P4/mmm$	$\frac{1}{2}\mathbf{a}, \frac{1}{2}\mathbf{b}, \frac{1}{2}\mathbf{c}$	$\frac{1}{2}, 0, 0; 0, \frac{1}{2}, 0$	0, 0, 0	y, x, z	4 · 2 · 2	
			$\mathcal{N}_{\mathcal{E}^+}(\mathcal{G}): P422$	$\frac{1}{2}\mathbf{a}, \frac{1}{2}\mathbf{b}, \frac{1}{2}\mathbf{c}$	$\frac{1}{2}, 0, 0; 0, \frac{1}{2}, 0$	/		y, x, \bar{z}	4 · 2
		$a = b = c$	$Pm\bar{3}m$	$\frac{1}{2}\mathbf{a}, \frac{1}{2}\mathbf{b}, \frac{1}{2}\mathbf{c}$	$\frac{1}{2}, 0, 0; 0, \frac{1}{2}, 0$	0, 0, 0	$z, x, y; y, x, z$	4 · 2 · 6	
			$\mathcal{N}_{\mathcal{E}^+}(\mathcal{G}): P432$	$\frac{1}{2}\mathbf{a}, \frac{1}{2}\mathbf{b}, \frac{1}{2}\mathbf{c}$	$\frac{1}{2}, 0, 0; 0, \frac{1}{2}, 0$	/		$z, x, y; y, x, \bar{z}$	4 · 6
24	$I2_12_12_1$	$a \neq b \neq c \neq a$	$Pmmm$	$\frac{1}{2}\mathbf{a}, \frac{1}{2}\mathbf{b}, \frac{1}{2}\mathbf{c}$	$\frac{1}{2}, 0, 0; 0, \frac{1}{2}, 0$	0, 0, 0		4 · 2 · 1	
			$\mathcal{N}_{\mathcal{E}^+}(\mathcal{G}): P222$	$\frac{1}{2}\mathbf{a}, \frac{1}{2}\mathbf{b}, \frac{1}{2}\mathbf{c}$	$\frac{1}{2}, 0, 0; 0, \frac{1}{2}, 0$	/		4 · 1	
		$a = b \neq c$	$P4_2/mmc (2/m2/mn)$	$\frac{1}{2}\mathbf{a}, \frac{1}{2}\mathbf{b}, \frac{1}{2}\mathbf{c}$	$\frac{1}{2}, 0, 0; 0, \frac{1}{2}, 0$	0, 0, 0	$y + \frac{1}{4}, x + \frac{1}{4}, z + \frac{1}{4}$	4 · 2 · 2	
			$\mathcal{N}_{\mathcal{E}^+}(\mathcal{G}): P4_222 (222_1)$	$\frac{1}{2}\mathbf{a}, \frac{1}{2}\mathbf{b}, \frac{1}{2}\mathbf{c}$	$\frac{1}{2}, 0, 0; 0, \frac{1}{2}, 0$	/		$\bar{y} + \frac{1}{4}, \bar{x} + \frac{1}{4}, \bar{z} + \frac{1}{4}$	4 · 2
	$a = b = c$	$Pm\bar{3}n$	$\frac{1}{2}\mathbf{a}, \frac{1}{2}\mathbf{b}, \frac{1}{2}\mathbf{c}$	$\frac{1}{2}, 0, 0; 0, \frac{1}{2}, 0$	0, 0, 0	$z, x, y; y + \frac{1}{4}, x + \frac{1}{4}, z + \frac{1}{4}$	4 · 2 · 6		
	$\mathcal{N}_{\mathcal{E}^+}(\mathcal{G}): P4_232$	$\frac{1}{2}\mathbf{a}, \frac{1}{2}\mathbf{b}, \frac{1}{2}\mathbf{c}$	$\frac{1}{2}, 0, 0; 0, \frac{1}{2}, 0$	/		$z, x, y; \bar{y} + \frac{1}{4}, \bar{x} + \frac{1}{4}, \bar{z} + \frac{1}{4}$	4 · 6		
25	$Pmm2$	$a \neq b$	P^1mmm	$\frac{1}{2}\mathbf{a}, \frac{1}{2}\mathbf{b}, \varepsilon\mathbf{c}$	$\frac{1}{2}, 0, 0; 0, \frac{1}{2}, 0; 0, 0, t$	0, 0, 0		$(4 \cdot \infty) \cdot 2 \cdot 1$	
		$a = b$	P^14/mmm	$\frac{1}{2}\mathbf{a}, \frac{1}{2}\mathbf{b}, \varepsilon\mathbf{c}$	$\frac{1}{2}, 0, 0; 0, \frac{1}{2}, 0; 0, 0, t$	0, 0, 0	y, x, z	$(4 \cdot \infty) \cdot 2 \cdot 2$	
26	$Pmc2_1$		P^1mmm	$\frac{1}{2}\mathbf{a}, \frac{1}{2}\mathbf{b}, \varepsilon\mathbf{c}$	$\frac{1}{2}, 0, 0; 0, \frac{1}{2}, 0; 0, 0, t$	0, 0, 0		$(4 \cdot \infty) \cdot 2 \cdot 1$	
27	$Pcc2$	$a \neq b$	P^1mmm	$\frac{1}{2}\mathbf{a}, \frac{1}{2}\mathbf{b}, \varepsilon\mathbf{c}$	$\frac{1}{2}, 0, 0; 0, \frac{1}{2}, 0; 0, 0, t$	0, 0, 0		$(4 \cdot \infty) \cdot 2 \cdot 1$	
		$a = b$	P^14/mmm	$\frac{1}{2}\mathbf{a}, \frac{1}{2}\mathbf{b}, \varepsilon\mathbf{c}$	$\frac{1}{2}, 0, 0; 0, \frac{1}{2}, 0; 0, 0, t$	0, 0, 0	y, x, z	$(4 \cdot \infty) \cdot 2 \cdot 2$	

Table 3.5.2.4 (continued)

Space group \mathcal{G}		Euclidean normalizer $\mathcal{N}_{\mathcal{E}}(\mathcal{G})$ and chirality-preserving normalizer $\mathcal{N}_{\mathcal{E}^+}(\mathcal{G})$		Additional generators of $\mathcal{N}_{\mathcal{E}}(\mathcal{G})$ and $\mathcal{N}_{\mathcal{E}^+}(\mathcal{G})$			Index of \mathcal{G} in $\mathcal{N}_{\mathcal{E}}(\mathcal{G})$ or $\mathcal{N}_{\mathcal{E}^+}(\mathcal{G})$	
No.	Hermann–Mauguin symbol	Cell metric	Symbol	Basis vectors	Translations	Inversion through a centre at		Further generators
61	<i>Pbca</i>	$a \neq b$ or $b \neq c$ or $a \neq c$ $a = b = c$	<i>Pmmm</i>	$\frac{1}{2}\mathbf{a}, \frac{1}{2}\mathbf{b}, \frac{1}{2}\mathbf{c}$	$\frac{1}{2}, 0, 0; 0, \frac{1}{2}, 0; 0, 0, \frac{1}{2}$			$8 \cdot 1 \cdot 1$
			<i>Pm</i> $\bar{3}$	$\frac{1}{2}\mathbf{a}, \frac{1}{2}\mathbf{b}, \frac{1}{2}\mathbf{c}$	$\frac{1}{2}, 0, 0; 0, \frac{1}{2}, 0; 0, 0, \frac{1}{2}$		z, x, y	$8 \cdot 1 \cdot 3$
62	<i>Pnma</i>		<i>Pmmm</i>	$\frac{1}{2}\mathbf{a}, \frac{1}{2}\mathbf{b}, \frac{1}{2}\mathbf{c}$	$\frac{1}{2}, 0, 0; 0, \frac{1}{2}, 0; 0, 0, \frac{1}{2}$			$8 \cdot 1 \cdot 1$
63	<i>Cmcm</i>		<i>Pmmm</i>	$\frac{1}{2}\mathbf{a}, \frac{1}{2}\mathbf{b}, \frac{1}{2}\mathbf{c}$	$\frac{1}{2}, 0, 0; 0, 0, \frac{1}{2}$			$4 \cdot 1 \cdot 1$
64	<i>Cmce</i>		<i>Pmmm</i>	$\frac{1}{2}\mathbf{a}, \frac{1}{2}\mathbf{b}, \frac{1}{2}\mathbf{c}$	$\frac{1}{2}, 0, 0; 0, 0, \frac{1}{2}$			$4 \cdot 1 \cdot 1$
65	<i>Cmmm</i>	$a \neq b$ $a = b$	<i>Pmmm</i>	$\frac{1}{2}\mathbf{a}, \frac{1}{2}\mathbf{b}, \frac{1}{2}\mathbf{c}$	$\frac{1}{2}, 0, 0; 0, 0, \frac{1}{2}$			$4 \cdot 1 \cdot 1$
			<i>P4/mmm</i>	$\frac{1}{2}\mathbf{a}, \frac{1}{2}\mathbf{b}, \frac{1}{2}\mathbf{c}$	$\frac{1}{2}, 0, 0; 0, 0, \frac{1}{2}$		y, x, z	$4 \cdot 1 \cdot 2$
66	<i>Cccm</i>	$a \neq b$ $a = b$	<i>Pmmm</i>	$\frac{1}{2}\mathbf{a}, \frac{1}{2}\mathbf{b}, \frac{1}{2}\mathbf{c}$	$\frac{1}{2}, 0, 0; 0, 0, \frac{1}{2}$			$4 \cdot 1 \cdot 1$
			<i>P4/mmm</i>	$\frac{1}{2}\mathbf{a}, \frac{1}{2}\mathbf{b}, \frac{1}{2}\mathbf{c}$	$\frac{1}{2}, 0, 0; 0, 0, \frac{1}{2}$		y, x, z	$4 \cdot 1 \cdot 2$
67	<i>Cmme</i>	$a \neq b$ $a = b$	<i>Pmmm</i>	$\frac{1}{2}\mathbf{a}, \frac{1}{2}\mathbf{b}, \frac{1}{2}\mathbf{c}$	$\frac{1}{2}, 0, 0; 0, 0, \frac{1}{2}$			$4 \cdot 1 \cdot 1$
			<i>P4/mmm (mmm)</i>	$\frac{1}{2}\mathbf{a}, \frac{1}{2}\mathbf{b}, \frac{1}{2}\mathbf{c}$	$\frac{1}{2}, 0, 0; 0, 0, \frac{1}{2}$		$y + \frac{1}{4}, x - \frac{1}{4}, z$	$4 \cdot 1 \cdot 2$
68	<i>Ccce (222)</i>	$a \neq b$ $a = b$	<i>Pmmm</i>	$\frac{1}{2}\mathbf{a}, \frac{1}{2}\mathbf{b}, \frac{1}{2}\mathbf{c}$	$\frac{1}{2}, 0, 0; 0, 0, \frac{1}{2}$			$4 \cdot 1 \cdot 1$
			<i>P4/mmm</i>	$\frac{1}{2}\mathbf{a}, \frac{1}{2}\mathbf{b}, \frac{1}{2}\mathbf{c}$	$\frac{1}{2}, 0, 0; 0, 0, \frac{1}{2}$		y, x, z	$4 \cdot 1 \cdot 2$
68	<i>Ccce ($\bar{1}$)</i>	$a \neq b$ $a = b$	<i>Pmmm</i>	$\frac{1}{2}\mathbf{a}, \frac{1}{2}\mathbf{b}, \frac{1}{2}\mathbf{c}$	$\frac{1}{2}, 0, 0; 0, 0, \frac{1}{2}$			$4 \cdot 1 \cdot 1$
			<i>P4/mmm (mmm)</i>	$\frac{1}{2}\mathbf{a}, \frac{1}{2}\mathbf{b}, \frac{1}{2}\mathbf{c}$	$\frac{1}{2}, 0, 0; 0, 0, \frac{1}{2}$		$y + \frac{1}{4}, x - \frac{1}{4}, z$	$4 \cdot 1 \cdot 2$
69	<i>Fmmm</i>	$a \neq b \neq c \neq a$ $a = b \neq c$ $a = b = c$	<i>Pmmm</i>	$\frac{1}{2}\mathbf{a}, \frac{1}{2}\mathbf{b}, \frac{1}{2}\mathbf{c}$	$\frac{1}{2}, 0, 0$			$2 \cdot 1 \cdot 1$
			<i>P4/mmm</i>	$\frac{1}{2}\mathbf{a}, \frac{1}{2}\mathbf{b}, \frac{1}{2}\mathbf{c}$	$\frac{1}{2}, 0, 0$		y, x, z	$2 \cdot 1 \cdot 2$
			<i>Pm</i> $\bar{3}m$	$\frac{1}{2}\mathbf{a}, \frac{1}{2}\mathbf{b}, \frac{1}{2}\mathbf{c}$	$\frac{1}{2}, 0, 0$		$z, x, y; y, x, z$	$2 \cdot 1 \cdot 6$
70	<i>Fddd (222)</i>	$a \neq b \neq c \neq a$ $a = b \neq c$ $a = b = c$	<i>Pnnn (222)</i>	$\frac{1}{2}\mathbf{a}, \frac{1}{2}\mathbf{b}, \frac{1}{2}\mathbf{c}$	$\frac{1}{2}, 0, 0$			$2 \cdot 1 \cdot 1$
			<i>P4₂/nnm ($\bar{4}2m$)</i>	$\frac{1}{2}\mathbf{a}, \frac{1}{2}\mathbf{b}, \frac{1}{2}\mathbf{c}$	$\frac{1}{2}, 0, 0$		\bar{y}, \bar{x}, z	$2 \cdot 1 \cdot 2$
			<i>Pn</i> $\bar{3}m$ ($\bar{4}3m$)	$\frac{1}{2}\mathbf{a}, \frac{1}{2}\mathbf{b}, \frac{1}{2}\mathbf{c}$	$\frac{1}{2}, 0, 0$		$z, x, y; y, x, z$	$2 \cdot 1 \cdot 6$
70	<i>Fddd ($\bar{1}$)</i>	$a \neq b \neq c \neq a$ $a = b \neq c$ $a = b = c$	<i>Pnnn ($\bar{1}$)</i>	$\frac{1}{2}\mathbf{a}, \frac{1}{2}\mathbf{b}, \frac{1}{2}\mathbf{c}$	$\frac{1}{2}, 0, 0$			$2 \cdot 1 \cdot 1$
			<i>P4₂/nnm (2/m at $0, \frac{1}{2}, 0$)</i>	$\frac{1}{2}\mathbf{a}, \frac{1}{2}\mathbf{b}, \frac{1}{2}\mathbf{c}$	$\frac{1}{2}, 0, 0$		y, x, z	$2 \cdot 1 \cdot 2$
			<i>Pn</i> $\bar{3}m$ ($\bar{3}m$)	$\frac{1}{2}\mathbf{a}, \frac{1}{2}\mathbf{b}, \frac{1}{2}\mathbf{c}$	$\frac{1}{2}, 0, 0$		$z, x, y; y, x, z$	$2 \cdot 1 \cdot 6$
71	<i>Immm</i>	$a \neq b \neq c \neq a$ $a = b \neq c$ $a = b = c$	<i>Pmmm</i>	$\frac{1}{2}\mathbf{a}, \frac{1}{2}\mathbf{b}, \frac{1}{2}\mathbf{c}$	$\frac{1}{2}, 0, 0; 0, \frac{1}{2}, 0$			$4 \cdot 1 \cdot 1$
			<i>P4/mmm</i>	$\frac{1}{2}\mathbf{a}, \frac{1}{2}\mathbf{b}, \frac{1}{2}\mathbf{c}$	$\frac{1}{2}, 0, 0; 0, \frac{1}{2}, 0$		y, x, z	$4 \cdot 1 \cdot 2$
			<i>Pm</i> $\bar{3}m$	$\frac{1}{2}\mathbf{a}, \frac{1}{2}\mathbf{b}, \frac{1}{2}\mathbf{c}$	$\frac{1}{2}, 0, 0; 0, \frac{1}{2}, 0$		$z, x, y; y, x, z$	$4 \cdot 1 \cdot 6$
72	<i>Ibam</i>	$a \neq b$ $a = b$	<i>Pmmm</i>	$\frac{1}{2}\mathbf{a}, \frac{1}{2}\mathbf{b}, \frac{1}{2}\mathbf{c}$	$\frac{1}{2}, 0, 0; 0, \frac{1}{2}, 0$			$4 \cdot 1 \cdot 1$
			<i>P4/mmm</i>	$\frac{1}{2}\mathbf{a}, \frac{1}{2}\mathbf{b}, \frac{1}{2}\mathbf{c}$	$\frac{1}{2}, 0, 0; 0, \frac{1}{2}, 0$		y, x, z	$4 \cdot 1 \cdot 2$
73	<i>Ibca</i>	$a \neq b \neq c \neq a$ $a = b \neq c$ $a = b = c$	<i>Pmmm</i>	$\frac{1}{2}\mathbf{a}, \frac{1}{2}\mathbf{b}, \frac{1}{2}\mathbf{c}$	$\frac{1}{2}, 0, 0; 0, \frac{1}{2}, 0$			$4 \cdot 1 \cdot 1$
			<i>P4₂/nmc (2/m 2/m n)</i>	$\frac{1}{2}\mathbf{a}, \frac{1}{2}\mathbf{b}, \frac{1}{2}\mathbf{c}$	$\frac{1}{2}, 0, 0; 0, \frac{1}{2}, 0$		$y + \frac{1}{4}, x + \frac{1}{4}, z + \frac{1}{4}$	$4 \cdot 1 \cdot 2$
			<i>Pm</i> $\bar{3}n$	$\frac{1}{2}\mathbf{a}, \frac{1}{2}\mathbf{b}, \frac{1}{2}\mathbf{c}$	$\frac{1}{2}, 0, 0; 0, \frac{1}{2}, 0$		$z, x, y; y + \frac{1}{4}, x + \frac{1}{4}, z + \frac{1}{4}$	$4 \cdot 1 \cdot 6$
74	<i>Imma</i>	$a \neq b$ $a = b$	<i>Pmmm</i>	$\frac{1}{2}\mathbf{a}, \frac{1}{2}\mathbf{b}, \frac{1}{2}\mathbf{c}$	$\frac{1}{2}, 0, 0; 0, \frac{1}{2}, 0$			$4 \cdot 1 \cdot 1$
			<i>P4₂/nmc (2/m 2/m n)</i>	$\frac{1}{2}\mathbf{a}, \frac{1}{2}\mathbf{b}, \frac{1}{2}\mathbf{c}$	$\frac{1}{2}, 0, 0; 0, \frac{1}{2}, 0$		$y + \frac{1}{4}, x - \frac{1}{4}, z + \frac{1}{4}$	$4 \cdot 1 \cdot 2$

Starting from any given point configuration of a space group \mathcal{G} , one may derive all Euclidean-equivalent point configurations and – except for monoclinic and triclinic space groups – all affine-equivalent ones by successive application of the ‘additional generators’ of the normalizer as given in Tables 3.5.2.3, 3.5.2.4 and 3.5.2.5.

Examples

- (1) A point configuration $F\bar{4}3m$ $16e$ x, x, x with $x_1 = 0.10$ may be visualized as a set of parallel tetrahedra arranged in a cubic face-centred lattice. The Euclidean and affine normalizer of $F\bar{4}3m$ is $Im\bar{3}m$ with $a' = \frac{1}{2}a$ (cf. Table 3.5.2.5). Since the index k_g of \mathcal{G} in $\mathcal{K}(\mathcal{G})$ is 4, three additional equivalent point configurations exist, which follow from the original one by repeated application of the tabulated translation $t(\frac{1}{4}, \frac{1}{4}, \frac{1}{4})$: $16e$ x, x, x with $x_2 = 0.35, x_3 = 0.60, x_4 = 0.85$. $\mathcal{L}(\mathcal{G})$ differs from $\mathcal{K}(\mathcal{G})$ and an additional centre of symmetry is located at $0, 0, 0$. Accordingly, the following four equivalent point configurations may be

derived from the first four: $16e$ x, x, x with $x_5 = -0.10, x_6 = -0.35, x_7 = -0.60, x_8 = -0.85$. In this case, the index 8 of \mathcal{G} in $\mathcal{N}_{\mathcal{E}}(\mathcal{G})$ equals the number of Euclidean-equivalent point configurations.

- (2) $F\bar{4}3m$ $4a$ $0, 0, 0$ represents a face-centred cubic lattice. The additional translations of $\mathcal{K}(F\bar{4}3m)$ generate three equivalent point configurations: $4c$ $\frac{1}{4}, \frac{1}{4}, \frac{1}{4}$, $4b$ $\frac{1}{2}, \frac{1}{2}, \frac{1}{2}$ and $4d$ $\frac{3}{4}, \frac{3}{4}, \frac{3}{4}$. Inversion through $0, 0, 0$ maps $4a$ and $4b$ each onto itself and interchanges $4c$ and $4d$. Therefore, here the number of equivalent point configurations is four, i.e. only half the index of \mathcal{G} in $\mathcal{N}_{\mathcal{E}}(\mathcal{G})$.

The difference between the two examples is the following: The reference point $0.1, 0.1, 0.1$ of the first example does not change its site symmetry $.3m$ when passing from $F\bar{4}3m$ to $Im\bar{3}m$. Point $0, 0, 0$ of the second example, however, has site symmetry $\bar{4}3m$ in $F\bar{4}3m$, but $m\bar{3}m$ in $Im\bar{3}m$.

(continued on page 846)