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Table 3.5.2.5

Euclidean and chirality-preserving Euclidean normalizers of the tetragonal, trigonal, hexagonal and cubic space groups

The symbols in parentheses following a space-group symbol refer to the location of the origin ('origin choice' in Chapter 2.3).

Space group $\mathcal{G}$		Euclidean normalizer $\mathcal{N}_{\mathcal{E}}(\mathcal{G})$ and chirality-preserving normalizer $\mathcal{N}_{\mathcal{E}^+}(\mathcal{G})$		Additional generators of $\mathcal{N}_{\mathcal{E}}(\mathcal{G})$ and $\mathcal{N}_{\mathcal{E}^+}(\mathcal{G})$			
No.	Hermann–Mauguin symbol	Symbol	Basis vectors	Translations	Inversion through a centre at	Further generators	Index of $\mathcal{G}$ in $\mathcal{N}_{\mathcal{E}}(\mathcal{G})$ or $\mathcal{N}_{\mathcal{E}^+}(\mathcal{G})$
75	$P4$	P^14/mmm	$\frac{1}{2}(\mathbf{a}-\mathbf{b}), \frac{1}{2}(\mathbf{a}+\mathbf{b}), \varepsilon\mathbf{c}$	$\frac{1}{2}, \frac{1}{2}, 0;\ 0, 0, t$	0, 0, 0	y, x, z	$(2\cdot\infty)\cdot 2\cdot 2$
		$\mathcal{N}_{\mathcal{E}^+}(\mathcal{G})$: P^1422	$\frac{1}{2}(\mathbf{a}-\mathbf{b}), \frac{1}{2}(\mathbf{a}+\mathbf{b}), \varepsilon\mathbf{c}$	$\frac{1}{2}, \frac{1}{2}, 0;\ 0, 0, t$	/	$y, x, \bar{z}$	$(2\cdot\infty)\cdot 2$
76	$P4_1$	P^1422 [$\equiv \mathcal{N}_{\mathcal{E}^+}(\mathcal{G})$]	$\frac{1}{2}(\mathbf{a}-\mathbf{b}), \frac{1}{2}(\mathbf{a}+\mathbf{b}), \varepsilon\mathbf{c}$	$\frac{1}{2}, \frac{1}{2}, 0;\ 0, 0, t$	/	$y, x, \bar{z}$	$(2\cdot\infty)\cdot 2$
77	$P4_2$	P^14/mmm	$\frac{1}{2}(\mathbf{a}-\mathbf{b}), \frac{1}{2}(\mathbf{a}+\mathbf{b}), \varepsilon\mathbf{c}$	$\frac{1}{2}, \frac{1}{2}, 0;\ 0, 0, t$	0, 0, 0	y, x, z	$(2\cdot\infty)\cdot 2\cdot 2$
		$\mathcal{N}_{\mathcal{E}^+}(\mathcal{G})$: P^1422	$\frac{1}{2}(\mathbf{a}-\mathbf{b}), \frac{1}{2}(\mathbf{a}+\mathbf{b}), \varepsilon\mathbf{c}$	$\frac{1}{2}, \frac{1}{2}, 0;\ 0, 0, t$	/	$y, x, \bar{z}$	$(2\cdot\infty)\cdot 2$
78	$P4_3$	P^1422 [$\equiv \mathcal{N}_{\mathcal{E}^+}(\mathcal{G})$]	$\frac{1}{2}(\mathbf{a}-\mathbf{b}), \frac{1}{2}(\mathbf{a}+\mathbf{b}), \varepsilon\mathbf{c}$	$\frac{1}{2}, \frac{1}{2}, 0;\ 0, 0, t$	/	$y, x, \bar{z}$	$(2\cdot\infty)\cdot 2$
79	$I4$	P^14/mmm	$\frac{1}{2}(\mathbf{a}-\mathbf{b}), \frac{1}{2}(\mathbf{a}+\mathbf{b}), \varepsilon\mathbf{c}$	$0, 0, t$	0, 0, 0	y, x, z	$\infty\cdot 2\cdot 2$
		$\mathcal{N}_{\mathcal{E}^+}(\mathcal{G})$: P^1422	$\frac{1}{2}(\mathbf{a}-\mathbf{b}), \frac{1}{2}(\mathbf{a}+\mathbf{b}), \varepsilon\mathbf{c}$	$0, 0, t$	/	$y, x, \bar{z}$	$\infty\cdot 2$
80	$I4_1$	P^14/nbm ($\bar{4}2m$)	$\frac{1}{2}(\mathbf{a}-\mathbf{b}), \frac{1}{2}(\mathbf{a}+\mathbf{b}), \varepsilon\mathbf{c}$	$0, 0, t$	$\frac{1}{4}, 0, 0$	$y, x, \bar{z}$	$\infty\cdot 2\cdot 2$
		$\mathcal{N}_{\mathcal{E}^+}(\mathcal{G})$: P^1422 (222)	$\frac{1}{2}(\mathbf{a}-\mathbf{b}), \frac{1}{2}(\mathbf{a}+\mathbf{b}), \varepsilon\mathbf{c}$	$0, 0, t$	/	$y, x, \bar{z}$	$\infty\cdot 2$
81	$P\bar{4}$	$P4/mmm$	$\frac{1}{2}(\mathbf{a}-\mathbf{b}), \frac{1}{2}(\mathbf{a}+\mathbf{b}), \frac{1}{2}\mathbf{c}$	$\frac{1}{2}, \frac{1}{2}, 0;\ 0, 0, \frac{1}{2}$	0, 0, 0	y, x, z	$4\cdot 2\cdot 2$
82	$I\bar{4}$	$I4/mmm$	$\frac{1}{2}(\mathbf{a}-\mathbf{b}), \frac{1}{2}(\mathbf{a}+\mathbf{b}), \frac{1}{2}\mathbf{c}$	$\frac{1}{2}, 0, \frac{1}{4}$	0, 0, 0	y, x, z	$4\cdot 2\cdot 2$
83	$P4/m$	$P4/mmm$	$\frac{1}{2}(\mathbf{a}-\mathbf{b}), \frac{1}{2}(\mathbf{a}+\mathbf{b}), \frac{1}{2}\mathbf{c}$	$\frac{1}{2}, \frac{1}{2}, 0;\ 0, 0, \frac{1}{2}$		y, x, z	$4\cdot 1\cdot 2$
84	$P4_2/m$	$P4/mmm$	$\frac{1}{2}(\mathbf{a}-\mathbf{b}), \frac{1}{2}(\mathbf{a}+\mathbf{b}), \frac{1}{2}\mathbf{c}$	$\frac{1}{2}, \frac{1}{2}, 0;\ 0, 0, \frac{1}{2}$		y, x, z	$4\cdot 1\cdot 2$
85	$P4/n$ ($\bar{4}$)	$P4/mmm$	$\frac{1}{2}(\mathbf{a}-\mathbf{b}), \frac{1}{2}(\mathbf{a}+\mathbf{b}), \frac{1}{2}\mathbf{c}$	$\frac{1}{2}, \frac{1}{2}, 0;\ 0, 0, \frac{1}{2}$		y, x, z	$4\cdot 1\cdot 2$
85	$P4/n$ ($\bar{1}$)	$P4/mmm$ (mmm)	$\frac{1}{2}(\mathbf{a}-\mathbf{b}), \frac{1}{2}(\mathbf{a}+\mathbf{b}), \frac{1}{2}\mathbf{c}$	$\frac{1}{2}, \frac{1}{2}, 0;\ 0, 0, \frac{1}{2}$		y, x, z	$4\cdot 1\cdot 2$
86	$P4_2/n$ ($\bar{4}$)	$P4/mmm$	$\frac{1}{2}(\mathbf{a}-\mathbf{b}), \frac{1}{2}(\mathbf{a}+\mathbf{b}), \frac{1}{2}\mathbf{c}$	$\frac{1}{2}, \frac{1}{2}, 0;\ 0, 0, \frac{1}{2}$		y, x, z	$4\cdot 1\cdot 2$
86	$P4_2/n$ ($\bar{1}$)	$P4/mmm$ (mmm)	$\frac{1}{2}(\mathbf{a}-\mathbf{b}), \frac{1}{2}(\mathbf{a}+\mathbf{b}), \frac{1}{2}\mathbf{c}$	$\frac{1}{2}, \frac{1}{2}, 0;\ 0, 0, \frac{1}{2}$		y, x, z	$4\cdot 1\cdot 2$
87	$I4/m$	$P4/mmm$	$\frac{1}{2}(\mathbf{a}-\mathbf{b}), \frac{1}{2}(\mathbf{a}+\mathbf{b}), \frac{1}{2}\mathbf{c}$	$0, 0, \frac{1}{2}$		y, x, z	$2\cdot 1\cdot 2$
88	$I4_1/a$ ($\bar{4}$)	$P4_2/nnm$ ($\bar{4}2m$)	$\frac{1}{2}(\mathbf{a}-\mathbf{b}), \frac{1}{2}(\mathbf{a}+\mathbf{b}), \frac{1}{2}\mathbf{c}$	$0, 0, \frac{1}{2}$		$y, x, \bar{z}$	$2\cdot 1\cdot 2$
88	$I4_1/a$ ($\bar{1}$)	$P4_2/nnm$ ($2/m$)	$\frac{1}{2}(\mathbf{a}-\mathbf{b}), \frac{1}{2}(\mathbf{a}+\mathbf{b}), \frac{1}{2}\mathbf{c}$	$0, 0, \frac{1}{2}$		$y+\frac{1}{4}, x+\frac{1}{4}, z+\frac{1}{4}$	$2\cdot 1\cdot 2$
89	$P422$	$P4/mmm$	$\frac{1}{2}(\mathbf{a}-\mathbf{b}), \frac{1}{2}(\mathbf{a}+\mathbf{b}), \frac{1}{2}\mathbf{c}$	$\frac{1}{2}, \frac{1}{2}, 0;\ 0, 0, \frac{1}{2}$	0, 0, 0		$4\cdot 2\cdot 1$
		$\mathcal{N}_{\mathcal{E}^+}(\mathcal{G})$: $P422$	$\frac{1}{2}(\mathbf{a}-\mathbf{b}), \frac{1}{2}(\mathbf{a}+\mathbf{b}), \frac{1}{2}\mathbf{c}$	$\frac{1}{2}, \frac{1}{2}, 0;\ 0, 0, \frac{1}{2}$	/		$4\cdot 1$
90	$P42_12$	$P4/mmm$	$\frac{1}{2}(\mathbf{a}-\mathbf{b}), \frac{1}{2}(\mathbf{a}+\mathbf{b}), \frac{1}{2}\mathbf{c}$	$\frac{1}{2}, \frac{1}{2}, 0;\ 0, 0, \frac{1}{2}$	0, 0, 0		$4\cdot 2\cdot 1$
		$\mathcal{N}_{\mathcal{E}^+}(\mathcal{G})$: $P422$	$\frac{1}{2}(\mathbf{a}-\mathbf{b}), \frac{1}{2}(\mathbf{a}+\mathbf{b}), \frac{1}{2}\mathbf{c}$	$\frac{1}{2}, \frac{1}{2}, 0;\ 0, 0, \frac{1}{2}$	/		$4\cdot 1$
91	$P4_122$	$P4_222$ (222 at 4_212) [$\equiv \mathcal{N}_{\mathcal{E}^+}(\mathcal{G})$]	$\frac{1}{2}(\mathbf{a}-\mathbf{b}), \frac{1}{2}(\mathbf{a}+\mathbf{b}), \frac{1}{2}\mathbf{c}$	$\frac{1}{2}, \frac{1}{2}, 0;\ 0, 0, \frac{1}{2}$	/		$4\cdot 1$
92	$P4_12_12$	$P4_222$ [$\equiv \mathcal{N}_{\mathcal{E}^+}(\mathcal{G})$]	$\frac{1}{2}(\mathbf{a}-\mathbf{b}), \frac{1}{2}(\mathbf{a}+\mathbf{b}), \frac{1}{2}\mathbf{c}$	$\frac{1}{2}, \frac{1}{2}, 0;\ 0, 0, \frac{1}{2}$	/		$4\cdot 1$
93	$P4_222$	$P4/mmm$	$\frac{1}{2}(\mathbf{a}-\mathbf{b}), \frac{1}{2}(\mathbf{a}+\mathbf{b}), \frac{1}{2}\mathbf{c}$	$\frac{1}{2}, \frac{1}{2}, 0;\ 0, 0, \frac{1}{2}$	0, 0, 0		$4\cdot 2\cdot 1$
		$\mathcal{N}_{\mathcal{E}^+}(\mathcal{G})$: $P422$	$\frac{1}{2}(\mathbf{a}-\mathbf{b}), \frac{1}{2}(\mathbf{a}+\mathbf{b}), \frac{1}{2}\mathbf{c}$	$\frac{1}{2}, \frac{1}{2}, 0;\ 0, 0, \frac{1}{2}$	/		$4\cdot 1$
94	$P4_22_12$	$P4/mmm$	$\frac{1}{2}(\mathbf{a}-\mathbf{b}), \frac{1}{2}(\mathbf{a}+\mathbf{b}), \frac{1}{2}\mathbf{c}$	$\frac{1}{2}, \frac{1}{2}, 0;\ 0, 0, \frac{1}{2}$	0, 0, 0		$4\cdot 2\cdot 1$
		$\mathcal{N}_{\mathcal{E}^+}(\mathcal{G})$: $P422$	$\frac{1}{2}(\mathbf{a}-\mathbf{b}), \frac{1}{2}(\mathbf{a}+\mathbf{b}), \frac{1}{2}\mathbf{c}$	$\frac{1}{2}, \frac{1}{2}, 0;\ 0, 0, \frac{1}{2}$	/		$4\cdot 1$
95	$P4_322$	$P4_222$ (222 at 4_212) [$\equiv \mathcal{N}_{\mathcal{E}^+}(\mathcal{G})$]	$\frac{1}{2}(\mathbf{a}-\mathbf{b}), \frac{1}{2}(\mathbf{a}+\mathbf{b}), \frac{1}{2}\mathbf{c}$	$\frac{1}{2}, \frac{1}{2}, 0;\ 0, 0, \frac{1}{2}$	/		$4\cdot 1$
96	$P4_32_12$	$P4_222$ [$\equiv \mathcal{N}_{\mathcal{E}^+}(\mathcal{G})$]	$\frac{1}{2}(\mathbf{a}-\mathbf{b}), \frac{1}{2}(\mathbf{a}+\mathbf{b}), \frac{1}{2}\mathbf{c}$	$\frac{1}{2}, \frac{1}{2}, 0;\ 0, 0, \frac{1}{2}$	/		$4\cdot 1$
97	$I422$	$P4/mmm$	$\frac{1}{2}(\mathbf{a}-\mathbf{b}), \frac{1}{2}(\mathbf{a}+\mathbf{b}), \frac{1}{2}\mathbf{c}$	$0, 0, \frac{1}{2}$	0, 0, 0		$2\cdot 2\cdot 1$
		$\mathcal{N}_{\mathcal{E}^+}(\mathcal{G})$: $P422$	$\frac{1}{2}(\mathbf{a}-\mathbf{b}), \frac{1}{2}(\mathbf{a}+\mathbf{b}), \frac{1}{2}\mathbf{c}$	$0, 0, \frac{1}{2}$	/		$2\cdot 1$
98	$I4_122$	$P4_2/nnm$ ($\bar{4}2m$)	$\frac{1}{2}(\mathbf{a}-\mathbf{b}), \frac{1}{2}(\mathbf{a}+\mathbf{b}), \frac{1}{2}\mathbf{c}$	$0, 0, \frac{1}{2}$	$\frac{1}{4}, 0, \frac{1}{8}$		$2\cdot 2\cdot 1$
		$\mathcal{N}_{\mathcal{E}^+}(\mathcal{G})$: $P4_222$	$\frac{1}{2}(\mathbf{a}-\mathbf{b}), \frac{1}{2}(\mathbf{a}+\mathbf{b}), \frac{1}{2}\mathbf{c}$	$0, 0, \frac{1}{2}$	/		$2\cdot 1$
99	$P4mm$	P^14/mmm	$\frac{1}{2}(\mathbf{a}-\mathbf{b}), \frac{1}{2}(\mathbf{a}+\mathbf{b}), \varepsilon\mathbf{c}$	$\frac{1}{2}, \frac{1}{2}, 0;\ 0, 0, t$	0, 0, 0		$(2\cdot\infty)\cdot 2\cdot 1$
100	$P4bm$	P^14/mmm	$\frac{1}{2}(\mathbf{a}-\mathbf{b}), \frac{1}{2}(\mathbf{a}+\mathbf{b}), \varepsilon\mathbf{c}$	$\frac{1}{2}, \frac{1}{2}, 0;\ 0, 0, t$	0, 0, 0		$(2\cdot\infty)\cdot 2\cdot 1$
101	$P4_2cm$	P^14/mmm	$\frac{1}{2}(\mathbf{a}-\mathbf{b}), \frac{1}{2}(\mathbf{a}+\mathbf{b}), \varepsilon\mathbf{c}$	$\frac{1}{2}, \frac{1}{2}, 0;\ 0, 0, t$	0, 0, 0		$(2\cdot\infty)\cdot 2\cdot 1$
102	$P4_2nm$	P^14/mmm	$\frac{1}{2}(\mathbf{a}-\mathbf{b}), \frac{1}{2}(\mathbf{a}+\mathbf{b}), \varepsilon\mathbf{c}$	$\frac{1}{2}, \frac{1}{2}, 0;\ 0, 0, t$	0, 0, 0		$(2\cdot\infty)\cdot 2\cdot 1$
103	$P4cc$	P^14/mmm	$\frac{1}{2}(\mathbf{a}-\mathbf{b}), \frac{1}{2}(\mathbf{a}+\mathbf{b}), \varepsilon\mathbf{c}$	$\frac{1}{2}, \frac{1}{2}, 0;\ 0, 0, t$	0, 0, 0		$(2\cdot\infty)\cdot 2\cdot 1$
104	$P4nc$	P^14/mmm	$\frac{1}{2}(\mathbf{a}-\mathbf{b}), \frac{1}{2}(\mathbf{a}+\mathbf{b}), \varepsilon\mathbf{c}$	$\frac{1}{2}, \frac{1}{2}, 0;\ 0, 0, t$	0, 0, 0		$(2\cdot\infty)\cdot 2\cdot 1$
105	$P4_2mc$	P^14/mmm	$\frac{1}{2}(\mathbf{a}-\mathbf{b}), \frac{1}{2}(\mathbf{a}+\mathbf{b}), \varepsilon\mathbf{c}$	$\frac{1}{2}, \frac{1}{2}, 0;\ 0, 0, t$	0, 0, 0		$(2\cdot\infty)\cdot 2\cdot 1$
106	$P4_2bc$	P^14/mmm	$\frac{1}{2}(\mathbf{a}-\mathbf{b}), \frac{1}{2}(\mathbf{a}+\mathbf{b}), \varepsilon\mathbf{c}$	$\frac{1}{2}, \frac{1}{2}, 0;\ 0, 0, t$	0, 0, 0		$(2\cdot\infty)\cdot 2\cdot 1$
107	$I4mm$	P^14/mmm	$\frac{1}{2}(\mathbf{a}-\mathbf{b}), \frac{1}{2}(\mathbf{a}+\mathbf{b}), \varepsilon\mathbf{c}$	$0, 0, t$	0, 0, 0		$\infty\cdot 2\cdot 1$
108	$I4cm$	P^14/mmm	$\frac{1}{2}(\mathbf{a}-\mathbf{b}), \frac{1}{2}(\mathbf{a}+\mathbf{b}), \varepsilon\mathbf{c}$	$0, 0, t$	0, 0, 0		$\infty\cdot 2\cdot 1$
109	$I4_1md$	P^14/nbm ($\bar{4}2m$)	$\frac{1}{2}(\mathbf{a}-\mathbf{b}), \frac{1}{2}(\mathbf{a}+\mathbf{b}), \varepsilon\mathbf{c}$	$0, 0, t$	$\frac{1}{4}, 0, 0$		$\infty\cdot 2\cdot 1$
110	$I4_1cd$	P^14/nbm ($\bar{4}2m$)	$\frac{1}{2}(\mathbf{a}-\mathbf{b}), \frac{1}{2}(\mathbf{a}+\mathbf{b}), \varepsilon\mathbf{c}$	$0, 0, t$	$\frac{1}{4}, 0, 0$		$\infty\cdot 2\cdot 1$
111	$P\bar{4}2m$	$P4/mmm$	$\frac{1}{2}(\mathbf{a}-\mathbf{b}), \frac{1}{2}(\mathbf{a}+\mathbf{b}), \frac{1}{2}\mathbf{c}$	$\frac{1}{2}, \frac{1}{2}, 0;\ 0, 0, \frac{1}{2}$	0, 0, 0		$4\cdot 2\cdot 1$
112	$P\bar{4}2c$	$P4/mmm$	$\frac{1}{2}(\mathbf{a}-\mathbf{b}), \frac{1}{2}(\mathbf{a}+\mathbf{b}), \frac{1}{2}\mathbf{c}$	$\frac{1}{2}, \frac{1}{2}, 0;\ 0, 0, \frac{1}{2}$	0, 0, 0		$4\cdot 2\cdot 1$
113	$P\bar{4}2_1m$	$P4/mmm$	$\frac{1}{2}(\mathbf{a}-\mathbf{b}), \frac{1}{2}(\mathbf{a}+\mathbf{b}), \frac{1}{2}\mathbf{c}$	$\frac{1}{2}, \frac{1}{2}, 0;\ 0, 0, \frac{1}{2}$	0, 0, 0		$4\cdot 2\cdot 1$

Table 3.5.2.5 (continued)

Space group $\mathcal{G}$		Euclidean normalizer $\mathcal{N}_{\mathcal{E}}(\mathcal{G})$ and chirality-preserving normalizer $\mathcal{N}_{\mathcal{E}^+}(\mathcal{G})$		Additional generators of $\mathcal{N}_{\mathcal{E}}(\mathcal{G})$ and $\mathcal{N}_{\mathcal{E}^+}(\mathcal{G})$			Index of $\mathcal{G}$ in $\mathcal{N}_{\mathcal{E}}(\mathcal{G})$ or $\mathcal{N}_{\mathcal{E}^+}(\mathcal{G})$
No.	Hermann–Mauguin symbol	Symbol	Basis vectors	Translations	Inversion through a centre at	Further generators	
114	$P\bar{4}2_1c$	$P4/mmm$	$\frac{1}{2}(\mathbf{a}-\mathbf{b}), \frac{1}{2}(\mathbf{a}+\mathbf{b}), \frac{1}{2}\mathbf{c}$	$\frac{1}{2},\frac{1}{2},0;\ 0,0,\frac{1}{2}$	0, 0, 0		$4\cdot2\cdot1$
115	$P\bar{4}m2$	$P4/mmm$	$\frac{1}{2}(\mathbf{a}-\mathbf{b}), \frac{1}{2}(\mathbf{a}+\mathbf{b}), \frac{1}{2}\mathbf{c}$	$\frac{1}{2},\frac{1}{2},0;\ 0,0,\frac{1}{2}$	0, 0, 0		$4\cdot2\cdot1$
116	$P\bar{4}c2$	$P4/mmm$	$\frac{1}{2}(\mathbf{a}-\mathbf{b}), \frac{1}{2}(\mathbf{a}+\mathbf{b}), \frac{1}{2}\mathbf{c}$	$\frac{1}{2},\frac{1}{2},0;\ 0,0,\frac{1}{2}$	0, 0, 0		$4\cdot2\cdot1$
117	$P\bar{4}b2$	$P4/mmm$	$\frac{1}{2}(\mathbf{a}-\mathbf{b}), \frac{1}{2}(\mathbf{a}+\mathbf{b}), \frac{1}{2}\mathbf{c}$	$\frac{1}{2},\frac{1}{2},0;\ 0,0,\frac{1}{2}$	0, 0, 0		$4\cdot2\cdot1$
118	$P\bar{4}n2$	$P4/mmm$	$\frac{1}{2}(\mathbf{a}-\mathbf{b}), \frac{1}{2}(\mathbf{a}+\mathbf{b}), \frac{1}{2}\mathbf{c}$	$\frac{1}{2},\frac{1}{2},0;\ 0,0,\frac{1}{2}$	0, 0, 0		$4\cdot2\cdot1$
119	$I\bar{4}m2$	$I4/mmm$	$\frac{1}{2}(\mathbf{a}-\mathbf{b}), \frac{1}{2}(\mathbf{a}+\mathbf{b}), \frac{1}{2}\mathbf{c}$	$\frac{1}{2},0,\frac{1}{4}$	0, 0, 0		$4\cdot2\cdot1$
120	$I\bar{4}c2$	$I4/mmm$	$\frac{1}{2}(\mathbf{a}-\mathbf{b}), \frac{1}{2}(\mathbf{a}+\mathbf{b}), \frac{1}{2}\mathbf{c}$	$\frac{1}{2},0,\frac{1}{4}$	0, 0, 0		$4\cdot2\cdot1$
121	$I\bar{4}2m$	$P4/mmm$	$\frac{1}{2}(\mathbf{a}-\mathbf{b}), \frac{1}{2}(\mathbf{a}+\mathbf{b}), \frac{1}{2}\mathbf{c}$	$0,0,\frac{1}{2}$	0, 0, 0		$2\cdot2\cdot1$
122	$I\bar{4}2d$	$P4_2/nnm$ ($\bar{4}2m$)	$\frac{1}{2}(\mathbf{a}-\mathbf{b}), \frac{1}{2}(\mathbf{a}+\mathbf{b}), \frac{1}{2}\mathbf{c}$	$0,0,\frac{1}{2}$	$\frac{1}{4},0,\frac{1}{8}$		$2\cdot2\cdot1$
123	$P4/mmm$	$P4/mmm$	$\frac{1}{2}(\mathbf{a}-\mathbf{b}), \frac{1}{2}(\mathbf{a}+\mathbf{b}), \frac{1}{2}\mathbf{c}$	$\frac{1}{2},\frac{1}{2},0;\ 0,0,\frac{1}{2}$			$4\cdot1\cdot1$
124	$P4/mcc$	$P4/mmm$	$\frac{1}{2}(\mathbf{a}-\mathbf{b}), \frac{1}{2}(\mathbf{a}+\mathbf{b}), \frac{1}{2}\mathbf{c}$	$\frac{1}{2},\frac{1}{2},0;\ 0,0,\frac{1}{2}$			$4\cdot1\cdot1$
125	$P4/nbm$ (422)	$P4/mmm$	$\frac{1}{2}(\mathbf{a}-\mathbf{b}), \frac{1}{2}(\mathbf{a}+\mathbf{b}), \frac{1}{2}\mathbf{c}$	$\frac{1}{2},\frac{1}{2},0;\ 0,0,\frac{1}{2}$			$4\cdot1\cdot1$
125	$P4/nbm$ ($2/m$)	$P4/mmm$ (mmm)	$\frac{1}{2}(\mathbf{a}-\mathbf{b}), \frac{1}{2}(\mathbf{a}+\mathbf{b}), \frac{1}{2}\mathbf{c}$	$\frac{1}{2},\frac{1}{2},0;\ 0,0,\frac{1}{2}$			$4\cdot1\cdot1$
126	$P4/nnc$ (422)	$P4/mmm$	$\frac{1}{2}(\mathbf{a}-\mathbf{b}), \frac{1}{2}(\mathbf{a}+\mathbf{b}), \frac{1}{2}\mathbf{c}$	$\frac{1}{2},\frac{1}{2},0;\ 0,0,\frac{1}{2}$			$4\cdot1\cdot1$
126	$P4/nnc$ ($\bar{1}$)	$P4/mmm$ (mmm)	$\frac{1}{2}(\mathbf{a}-\mathbf{b}), \frac{1}{2}(\mathbf{a}+\mathbf{b}), \frac{1}{2}\mathbf{c}$	$\frac{1}{2},\frac{1}{2},0;\ 0,0,\frac{1}{2}$			$4\cdot1\cdot1$
127	$P4/mbm$	$P4/mmm$	$\frac{1}{2}(\mathbf{a}-\mathbf{b}), \frac{1}{2}(\mathbf{a}+\mathbf{b}), \frac{1}{2}\mathbf{c}$	$\frac{1}{2},\frac{1}{2},0;\ 0,0,\frac{1}{2}$			$4\cdot1\cdot1$
128	$P4/mnc$	$P4/mmm$	$\frac{1}{2}(\mathbf{a}-\mathbf{b}), \frac{1}{2}(\mathbf{a}+\mathbf{b}), \frac{1}{2}\mathbf{c}$	$\frac{1}{2},\frac{1}{2},0;\ 0,0,\frac{1}{2}$			$4\cdot1\cdot1$
129	$P4/nmm$ ($\bar{4}m2$)	$P4/mmm$	$\frac{1}{2}(\mathbf{a}-\mathbf{b}), \frac{1}{2}(\mathbf{a}+\mathbf{b}), \frac{1}{2}\mathbf{c}$	$\frac{1}{2},\frac{1}{2},0;\ 0,0,\frac{1}{2}$			$4\cdot1\cdot1$
129	$P4/nmm$ ($2/m$)	$P4/mmm$ (mmm)	$\frac{1}{2}(\mathbf{a}-\mathbf{b}), \frac{1}{2}(\mathbf{a}+\mathbf{b}), \frac{1}{2}\mathbf{c}$	$\frac{1}{2},\frac{1}{2},0;\ 0,0,\frac{1}{2}$			$4\cdot1\cdot1$
130	$P4/ncc$ ($\bar{4}$)	$P4/mmm$	$\frac{1}{2}(\mathbf{a}-\mathbf{b}), \frac{1}{2}(\mathbf{a}+\mathbf{b}), \frac{1}{2}\mathbf{c}$	$\frac{1}{2},\frac{1}{2},0;\ 0,0,\frac{1}{2}$			$4\cdot1\cdot1$
130	$P4/ncc$ ($\bar{1}$)	$P4/mmm$ (mmm)	$\frac{1}{2}(\mathbf{a}-\mathbf{b}), \frac{1}{2}(\mathbf{a}+\mathbf{b}), \frac{1}{2}\mathbf{c}$	$\frac{1}{2},\frac{1}{2},0;\ 0,0,\frac{1}{2}$			$4\cdot1\cdot1$
131	$P4_2/mmc$	$P4/mmm$	$\frac{1}{2}(\mathbf{a}-\mathbf{b}), \frac{1}{2}(\mathbf{a}+\mathbf{b}), \frac{1}{2}\mathbf{c}$	$\frac{1}{2},\frac{1}{2},0;\ 0,0,\frac{1}{2}$			$4\cdot1\cdot1$
132	$P4_2/mcm$	$P4/mmm$	$\frac{1}{2}(\mathbf{a}-\mathbf{b}), \frac{1}{2}(\mathbf{a}+\mathbf{b}), \frac{1}{2}\mathbf{c}$	$\frac{1}{2},\frac{1}{2},0;\ 0,0,\frac{1}{2}$			$4\cdot1\cdot1$
133	$P4_2/nbc$ ($\bar{4}$)	$P4/mmm$	$\frac{1}{2}(\mathbf{a}-\mathbf{b}), \frac{1}{2}(\mathbf{a}+\mathbf{b}), \frac{1}{2}\mathbf{c}$	$\frac{1}{2},\frac{1}{2},0;\ 0,0,\frac{1}{2}$			$4\cdot1\cdot1$
133	$P4_2/nbc$ ($\bar{1}$)	$P4/mmm$ (mmm)	$\frac{1}{2}(\mathbf{a}-\mathbf{b}), \frac{1}{2}(\mathbf{a}+\mathbf{b}), \frac{1}{2}\mathbf{c}$	$\frac{1}{2},\frac{1}{2},0;\ 0,0,\frac{1}{2}$			$4\cdot1\cdot1$
134	$P4_2/nnm$ ($\bar{4}2m$)	$P4/mmm$	$\frac{1}{2}(\mathbf{a}-\mathbf{b}), \frac{1}{2}(\mathbf{a}+\mathbf{b}), \frac{1}{2}\mathbf{c}$	$\frac{1}{2},\frac{1}{2},0;\ 0,0,\frac{1}{2}$			$4\cdot1\cdot1$
134	$P4_2/nnm$ ($2/m$)	$P4/mmm$ (mmm)	$\frac{1}{2}(\mathbf{a}-\mathbf{b}), \frac{1}{2}(\mathbf{a}+\mathbf{b}), \frac{1}{2}\mathbf{c}$	$\frac{1}{2},\frac{1}{2},0;\ 0,0,\frac{1}{2}$			$4\cdot1\cdot1$
135	$P4_2/mbc$	$P4/mmm$	$\frac{1}{2}(\mathbf{a}-\mathbf{b}), \frac{1}{2}(\mathbf{a}+\mathbf{b}), \frac{1}{2}\mathbf{c}$	$\frac{1}{2},\frac{1}{2},0;\ 0,0,\frac{1}{2}$			$4\cdot1\cdot1$
136	$P4_2/mnm$	$P4/mmm$	$\frac{1}{2}(\mathbf{a}-\mathbf{b}), \frac{1}{2}(\mathbf{a}+\mathbf{b}), \frac{1}{2}\mathbf{c}$	$\frac{1}{2},\frac{1}{2},0;\ 0,0,\frac{1}{2}$			$4\cdot1\cdot1$
137	$P4_2/nmc$ ($\bar{4}m2$)	$P4/mmm$	$\frac{1}{2}(\mathbf{a}-\mathbf{b}), \frac{1}{2}(\mathbf{a}+\mathbf{b}), \frac{1}{2}\mathbf{c}$	$\frac{1}{2},\frac{1}{2},0;\ 0,0,\frac{1}{2}$			$4\cdot1\cdot1$
137	$P4_2/nmc$ ($\bar{1}$)	$P4/mmm$ (mmm)	$\frac{1}{2}(\mathbf{a}-\mathbf{b}), \frac{1}{2}(\mathbf{a}+\mathbf{b}), \frac{1}{2}\mathbf{c}$	$\frac{1}{2},\frac{1}{2},0;\ 0,0,\frac{1}{2}$			$4\cdot1\cdot1$
138	$P4_2/ncm$ ($\bar{4}$)	$P4/mmm$	$\frac{1}{2}(\mathbf{a}-\mathbf{b}), \frac{1}{2}(\mathbf{a}+\mathbf{b}), \frac{1}{2}\mathbf{c}$	$\frac{1}{2},\frac{1}{2},0;\ 0,0,\frac{1}{2}$			$4\cdot1\cdot1$
138	$P4_2/ncm$ ($2/m$)	$P4/mmm$ (mmm)	$\frac{1}{2}(\mathbf{a}-\mathbf{b}), \frac{1}{2}(\mathbf{a}+\mathbf{b}), \frac{1}{2}\mathbf{c}$	$\frac{1}{2},\frac{1}{2},0;\ 0,0,\frac{1}{2}$			$4\cdot1\cdot1$
139	$I4/mmm$	$P4/mmm$	$\frac{1}{2}(\mathbf{a}-\mathbf{b}), \frac{1}{2}(\mathbf{a}+\mathbf{b}), \frac{1}{2}\mathbf{c}$	$0,0,\frac{1}{2}$			$2\cdot1\cdot1$
140	$I4/mcm$	$P4/mmm$	$\frac{1}{2}(\mathbf{a}-\mathbf{b}), \frac{1}{2}(\mathbf{a}+\mathbf{b}), \frac{1}{2}\mathbf{c}$	$0,0,\frac{1}{2}$			$2\cdot1\cdot1$
141	$I4_1/amd$ ($\bar{4}m2$)	$P4_2/nnm$ ($\bar{4}2m$)	$\frac{1}{2}(\mathbf{a}-\mathbf{b}), \frac{1}{2}(\mathbf{a}+\mathbf{b}), \frac{1}{2}\mathbf{c}$	$0,0,\frac{1}{2}$			$2\cdot1\cdot1$
141	$I4_1/amd$ ($2/m$)	$P4_2/nnm$ ($2/m$)	$\frac{1}{2}(\mathbf{a}-\mathbf{b}), \frac{1}{2}(\mathbf{a}+\mathbf{b}), \frac{1}{2}\mathbf{c}$	$0,0,\frac{1}{2}$			$2\cdot1\cdot1$
142	$I4_1/acd$ ($\bar{4}$)	$P4_2/nnm$ ($\bar{4}2m$)	$\frac{1}{2}(\mathbf{a}-\mathbf{b}), \frac{1}{2}(\mathbf{a}+\mathbf{b}), \frac{1}{2}\mathbf{c}$	$0,0,\frac{1}{2}$			$2\cdot1\cdot1$
142	$I4_1/acd$ ($\bar{1}$)	$P4_2/nnm$ ($2/m$)	$\frac{1}{2}(\mathbf{a}-\mathbf{b}), \frac{1}{2}(\mathbf{a}+\mathbf{b}), \frac{1}{2}\mathbf{c}$	$0,0,\frac{1}{2}$			$2\cdot1\cdot1$
143	$P3$	P^16/mmm	$\frac{2}{3}\mathbf{a}+\frac{1}{3}\mathbf{b},\ -\frac{1}{3}\mathbf{a}+\frac{1}{3}\mathbf{b},\ \varepsilon\mathbf{c}$	$\frac{2}{3},\frac{1}{3},0;\ 0,0,t$	0, 0, 0	$\bar{x},\bar{y},z;\ y,x,z$	$(3\cdot\infty)\cdot2\cdot4$
		$\mathcal{N}_{\mathcal{E}^+}(\mathcal{G})$: P^1622	$\frac{2}{3}\mathbf{a}+\frac{1}{3}\mathbf{b},\ -\frac{1}{3}\mathbf{a}+\frac{1}{3}\mathbf{b},\ \varepsilon\mathbf{c}$	$\frac{2}{3},\frac{1}{3},0;\ 0,0,t$	/	$\bar{x},\bar{y},z;\ y,x,\bar{z}$	$(3\cdot\infty)\cdot4$
144	$P3_1$	P^1622 [$\equiv\mathcal{N}_{\mathcal{E}^+}(\mathcal{G})$]	$\frac{2}{3}\mathbf{a}+\frac{1}{3}\mathbf{b},\ -\frac{1}{3}\mathbf{a}+\frac{1}{3}\mathbf{b},\ \varepsilon\mathbf{c}$	$\frac{2}{3},\frac{1}{3},0;\ 0,0,t$	/	$\bar{x},\bar{y},z;\ y,x,\bar{z}$	$(3\cdot\infty)\cdot4$
145	$P3_2$	P^1622 [$\equiv\mathcal{N}_{\mathcal{E}^+}(\mathcal{G})$]	$\frac{2}{3}\mathbf{a}+\frac{1}{3}\mathbf{b},\ -\frac{1}{3}\mathbf{a}+\frac{1}{3}\mathbf{b},\ \varepsilon\mathbf{c}$	$\frac{2}{3},\frac{1}{3},0;\ 0,0,t$	/	$\bar{x},\bar{y},z;\ y,x,\bar{z}$	$(3\cdot\infty)\cdot4$
146	$R3$ (hexag.)	$P^1\bar{3}1m$	$\frac{2}{3}\mathbf{a}+\frac{1}{3}\mathbf{b},\ -\frac{1}{3}\mathbf{a}+\frac{1}{3}\mathbf{b},\ \varepsilon\mathbf{c}$	$0,0,t$	0, 0, 0	$\bar{y},\bar{x},z$	$\infty\cdot2\cdot2$
		$\mathcal{N}_{\mathcal{E}^+}(\mathcal{G})$: P^1312	$\frac{2}{3}\mathbf{a}+\frac{1}{3}\mathbf{b},\ -\frac{1}{3}\mathbf{a}+\frac{1}{3}\mathbf{b},\ \varepsilon\mathbf{c}$	$0,0,t$	/	$y,x,\bar{z}$	$\infty\cdot2$
146	$R3$ (rhomboh.)	$P^1\bar{3}1m$	$\frac{2}{3}\mathbf{a}-\frac{1}{3}\mathbf{b}-\frac{1}{3}\mathbf{c},\ -\frac{1}{3}\mathbf{a}+\frac{2}{3}\mathbf{b}-\frac{1}{3}\mathbf{c},\ \varepsilon(\mathbf{a}+\mathbf{b}+\mathbf{c})$	r,r,r	0, 0, 0	y,x,z	$\infty\cdot2\cdot2$
		$\mathcal{N}_{\mathcal{E}^+}(\mathcal{G})$: P^1312	$\frac{2}{3}\mathbf{a}-\frac{1}{3}\mathbf{b}-\frac{1}{3}\mathbf{c},\ -\frac{1}{3}\mathbf{a}+\frac{2}{3}\mathbf{b}-\frac{1}{3}\mathbf{c},\ \varepsilon(\mathbf{a}+\mathbf{b}+\mathbf{c})$	r,r,r	/	$\bar{y},\bar{x},\bar{z}$	$\infty\cdot2$
147	$P\bar{3}$	$P6/mmm$	$\mathbf{a},\ \mathbf{b},\ \frac{1}{2}\mathbf{c}$	$0,0,\frac{1}{2}$		$\bar{x},\bar{y},z;\ y,x,z$	$2\cdot1\cdot4$
148	$R\bar{3}$ (hexag.)	$R\bar{3}m$ (hexag.)	$-\mathbf{a},\ -\mathbf{b},\ \frac{1}{2}\mathbf{c}$	$0,0,\frac{1}{2}$		$\bar{y},\bar{x},z$	$2\cdot1\cdot2$
148	$R\bar{3}$ (rhomboh.)	$R\bar{3}m$ (rhomboh.)	$\frac{1}{2}(-\mathbf{a}+\mathbf{b}+\mathbf{c}),\ \frac{1}{2}(\mathbf{a}-\mathbf{b}+\mathbf{c}),\ \frac{1}{2}(\mathbf{a}+\mathbf{b}-\mathbf{c})$	$\frac{1}{2},\frac{1}{2},\frac{1}{2}$		y,x,z	$2\cdot1\cdot2$

Table 3.5.2.5 (continued)

Space group $\mathcal{G}$		Euclidean normalizer $\mathcal{N}_{\mathcal{E}}(\mathcal{G})$ and chirality-preserving normalizer $\mathcal{N}_{\mathcal{E}^+}(\mathcal{G})$		Additional generators of $\mathcal{N}_{\mathcal{E}}(\mathcal{G})$ and $\mathcal{N}_{\mathcal{E}^+}(\mathcal{G})$			Index of $\mathcal{G}$ in $\mathcal{N}_{\mathcal{E}}(\mathcal{G})$ or $\mathcal{N}_{\mathcal{E}^+}(\mathcal{G})$
No.	Hermann–Mauguin symbol	Symbol	Basis vectors	Translations	Inversion through a centre at	Further generators	
149	$P312$	$P6/mmm$	$\frac{2}{3}\mathbf{a}+\frac{1}{3}\mathbf{b},\ -\frac{1}{3}\mathbf{a}+\frac{1}{3}\mathbf{b},\ \frac{1}{2}\mathbf{c}$	$\frac{2}{3},\frac{1}{3},0;\ 0,0,\frac{1}{2}$	$0,0,0$	$\bar{x},\bar{y},z$	$6\cdot2\cdot2$
		$\mathcal{N}_{\mathcal{E}^+}(\mathcal{G})$: $P622$	$\frac{2}{3}\mathbf{a}+\frac{1}{3}\mathbf{b},\ -\frac{1}{3}\mathbf{a}+\frac{1}{3}\mathbf{b},\ \frac{1}{2}\mathbf{c}$	$\frac{2}{3},\frac{1}{3},0;\ 0,0,\frac{1}{2}$	/	$\bar{x},\bar{y},z$	$6\cdot2$
150	$P321$	$P6/mmm$	$\mathbf{a},\ \mathbf{b},\ \frac{1}{2}\mathbf{c}$	$0,0,\frac{1}{2}$	$0,0,0$	$\bar{x},\bar{y},z$	$2\cdot2\cdot2$
		$\mathcal{N}_{\mathcal{E}^+}(\mathcal{G})$: $P622$	$\mathbf{a},\ \mathbf{b},\ \frac{1}{2}\mathbf{c}$	$0,0,\frac{1}{2}$	/	$\bar{x},\bar{y},z$	$2\cdot2$
151	$P3_112$	$P6_222\ [\equiv\mathcal{N}_{\mathcal{E}^+}(\mathcal{G})]$	$\frac{2}{3}\mathbf{a}+\frac{1}{3}\mathbf{b},\ -\frac{1}{3}\mathbf{a}+\frac{1}{3}\mathbf{b},\ \frac{1}{2}\mathbf{c}$	$\frac{2}{3},\frac{1}{3},0;\ 0,0,\frac{1}{2}$	/	$\bar{x},\bar{y},z$	$6\cdot2$
152	$P3_121$	$P6_222\ [\equiv\mathcal{N}_{\mathcal{E}^+}(\mathcal{G})]$	$\mathbf{a}+\mathbf{b},\ -\mathbf{a},\ \frac{1}{2}\mathbf{c}$	$0,0,\frac{1}{2}$	/	$\bar{x},\bar{y},z$	$2\cdot2$
153	$P3_212$	$P6_422\ [\equiv\mathcal{N}_{\mathcal{E}^+}(\mathcal{G})]$	$\frac{2}{3}\mathbf{a}+\frac{1}{3}\mathbf{b},\ -\frac{1}{3}\mathbf{a}+\frac{1}{3}\mathbf{b},\ \frac{1}{2}\mathbf{c}$	$\frac{2}{3},\frac{1}{3},0;\ 0,0,\frac{1}{2}$	/	$\bar{x},\bar{y},z$	$6\cdot2$
154	$P3_221$	$P6_422\ [\equiv\mathcal{N}_{\mathcal{E}^+}(\mathcal{G})]$	$\mathbf{a}+\mathbf{b},\ -\mathbf{a},\ \frac{1}{2}\mathbf{c}$	$0,0,\frac{1}{2}$	/	$\bar{x},\bar{y},z$	$2\cdot2$
155	$R32$ (hexag.)	$R\bar{3}m$ (hexag.)	$-\mathbf{a},\ -\mathbf{b},\ \frac{1}{2}\mathbf{c}$	$0,0,\frac{1}{2}$	$0,0,0$		$2\cdot2\cdot1$
		$\mathcal{N}_{\mathcal{E}^+}(\mathcal{G})$: $R32$ (hexag.)	$-\mathbf{a},\ -\mathbf{b},\ \frac{1}{2}\mathbf{c}$	$0,0,\frac{1}{2}$	/		$2\cdot1$
155	$R32$ (rhomboh.)	$R\bar{3}m$ (rhomboh.)	$\frac{1}{2}(-\mathbf{a}+\mathbf{b}+\mathbf{c}),\ \frac{1}{2}(\mathbf{a}-\mathbf{b}+\mathbf{c}),\ \frac{1}{2}(\mathbf{a}+\mathbf{b}-\mathbf{c})$	$\frac{1}{2},\frac{1}{2},\frac{1}{2}$	$0,0,0$		$2\cdot2\cdot1$
		$\mathcal{N}_{\mathcal{E}^+}(\mathcal{G})$: $R32$ (rhomboh.)	$\frac{1}{2}(-\mathbf{a}+\mathbf{b}+\mathbf{c}),\ \frac{1}{2}(\mathbf{a}-\mathbf{b}+\mathbf{c}),\ \frac{1}{2}(\mathbf{a}+\mathbf{b}-\mathbf{c})$	$\frac{1}{2},\frac{1}{2},\frac{1}{2}$	/		$2\cdot1$
156	$P3m1$	P^16/mmm	$\frac{2}{3}\mathbf{a}+\frac{1}{3}\mathbf{b},\ -\frac{1}{3}\mathbf{a}+\frac{1}{3}\mathbf{b},\ \varepsilon\mathbf{c}$	$\frac{2}{3},\frac{1}{3},0;\ 0,0,t$	$0,0,0$	$\bar{x},\bar{y},z$	$(3\cdot\infty)\cdot2\cdot2$
157	$P31m$	P^16/mmm	$\mathbf{a},\ \mathbf{b},\ \varepsilon\mathbf{c}$	$0,0,t$	$0,0,0$	$\bar{x},\bar{y},z$	$\infty\cdot2\cdot2$
158	$P3c1$	P^16/mmm	$\frac{2}{3}\mathbf{a}+\frac{1}{3}\mathbf{b},\ -\frac{1}{3}\mathbf{a}+\frac{1}{3}\mathbf{b},\ \varepsilon\mathbf{c}$	$\frac{2}{3},\frac{1}{3},0;\ 0,0,t$	$0,0,0$	$\bar{x},\bar{y},z$	$(3\cdot\infty)\cdot2\cdot2$
159	$P31c$	P^16/mmm	$\mathbf{a},\ \mathbf{b},\ \varepsilon\mathbf{c}$	$0,0,t$	$0,0,0$	$\bar{x},\bar{y},z$	$\infty\cdot2\cdot2$
160	$R3m$ (hexag.)	$P^1\bar{3}1m$	$\frac{2}{3}\mathbf{a}+\frac{1}{3}\mathbf{b},\ -\frac{1}{3}\mathbf{a}+\frac{1}{3}\mathbf{b},\ \varepsilon\mathbf{c}$	$0,0,t$	$0,0,0$		$\infty\cdot2\cdot1$
160	$R3m$ (rhomboh.)	$P^1\bar{3}1m$	$\frac{2}{3}\mathbf{a}-\frac{1}{3}\mathbf{b}-\frac{1}{3}\mathbf{c},\ -\frac{1}{3}\mathbf{a}+\frac{2}{3}\mathbf{b}-\frac{1}{3}\mathbf{c},\ \varepsilon(\mathbf{a}+\mathbf{b}+\mathbf{c})$	r,r,r	$0,0,0$		$\infty\cdot2\cdot1$
161	$R3c$ (hexag.)	$P^1\bar{3}1m$	$\frac{2}{3}\mathbf{a}+\frac{1}{3}\mathbf{b},\ -\frac{1}{3}\mathbf{a}+\frac{1}{3}\mathbf{b},\ \varepsilon\mathbf{c}$	$0,0,t$	$0,0,0$		$\infty\cdot2\cdot1$
161	$R3c$ (rhomboh.)	$P^1\bar{3}1m$	$\frac{2}{3}\mathbf{a}-\frac{1}{3}\mathbf{b}-\frac{1}{3}\mathbf{c},\ -\frac{1}{3}\mathbf{a}+\frac{2}{3}\mathbf{b}-\frac{1}{3}\mathbf{c},\ \varepsilon(\mathbf{a}+\mathbf{b}+\mathbf{c})$	r,r,r	$0,0,0$		$\infty\cdot2\cdot1$
162	$P\bar{3}1m$	$P6/mmm$	$\mathbf{a},\ \mathbf{b},\ \frac{1}{2}\mathbf{c}$	$0,0,\frac{1}{2}$		$\bar{x},\bar{y},z$	$2\cdot1\cdot2$
163	$P\bar{3}1c$	$P6/mmm$	$\mathbf{a},\ \mathbf{b},\ \frac{1}{2}\mathbf{c}$	$0,0,\frac{1}{2}$		$\bar{x},\bar{y},z$	$2\cdot1\cdot2$
164	$P\bar{3}m1$	$P6/mmm$	$\mathbf{a},\ \mathbf{b},\ \frac{1}{2}\mathbf{c}$	$0,0,\frac{1}{2}$		$\bar{x},\bar{y},z$	$2\cdot1\cdot2$
165	$P\bar{3}c1$	$P6/mmm$	$\mathbf{a},\ \mathbf{b},\ \frac{1}{2}\mathbf{c}$	$0,0,\frac{1}{2}$		$\bar{x},\bar{y},z$	$2\cdot1\cdot2$
166	$R\bar{3}m$ (hexag.)	$R\bar{3}m$ (hexag.)	$-\mathbf{a},\ -\mathbf{b},\ \frac{1}{2}\mathbf{c}$	$0,0,\frac{1}{2}$			$2\cdot1\cdot1$
166	$R\bar{3}m$ (rhomboh.)	$R\bar{3}m$ (rhomboh.)	$\frac{1}{2}(-\mathbf{a}+\mathbf{b}+\mathbf{c}),\ \frac{1}{2}(\mathbf{a}-\mathbf{b}+\mathbf{c}),\ \frac{1}{2}(\mathbf{a}+\mathbf{b}-\mathbf{c})$	$\frac{1}{2},\frac{1}{2},\frac{1}{2}$			$2\cdot1\cdot1$
167	$R\bar{3}c$ (hexag.)	$R\bar{3}m$ (hexag.)	$-\mathbf{a},\ -\mathbf{b},\ \frac{1}{2}\mathbf{c}$	$0,0,\frac{1}{2}$			$2\cdot1\cdot1$
167	$R\bar{3}c$ (rhomboh.)	$R\bar{3}m$ (rhomboh.)	$\frac{1}{2}(-\mathbf{a}+\mathbf{b}+\mathbf{c}),\ \frac{1}{2}(\mathbf{a}-\mathbf{b}+\mathbf{c}),\ \frac{1}{2}(\mathbf{a}+\mathbf{b}-\mathbf{c})$	$\frac{1}{2},\frac{1}{2},\frac{1}{2}$			$2\cdot1\cdot1$
168	$P6$	P^16/mmm	$\mathbf{a},\ \mathbf{b},\ \varepsilon\mathbf{c}$	$0,0,t$	$0,0,0$	y,x,z	$\infty\cdot2\cdot2$
		$\mathcal{N}_{\mathcal{E}^+}(\mathcal{G})$: P^1622	$\mathbf{a},\ \mathbf{b},\ \varepsilon\mathbf{c}$	$0,0,t$	/	$y,x,\bar{z}$	$\infty\cdot2$
169	$P6_1$	$P^1622\ [\equiv\mathcal{N}_{\mathcal{E}^+}(\mathcal{G})]$	$\mathbf{a},\ \mathbf{b},\ \varepsilon\mathbf{c}$	$0,0,t$	/	$y,x,\bar{z}$	$\infty\cdot2$
170	$P6_5$	$P^1622\ [\equiv\mathcal{N}_{\mathcal{E}^+}(\mathcal{G})]$	$\mathbf{a},\ \mathbf{b},\ \varepsilon\mathbf{c}$	$0,0,t$	/	$y,x,\bar{z}$	$\infty\cdot2$
171	$P6_2$	$P^1622\ [\equiv\mathcal{N}_{\mathcal{E}^+}(\mathcal{G})]$	$\mathbf{a},\ \mathbf{b},\ \varepsilon\mathbf{c}$	$0,0,t$	/	$y,x,\bar{z}$	$\infty\cdot2$
172	$P6_4$	$P^1622\ [\equiv\mathcal{N}_{\mathcal{E}^+}(\mathcal{G})]$	$\mathbf{a},\ \mathbf{b},\ \varepsilon\mathbf{c}$	$0,0,t$	/	$y,x,\bar{z}$	$\infty\cdot2$
173	$P6_3$	P^16/mmm	$\mathbf{a},\ \mathbf{b},\ \varepsilon\mathbf{c}$	$0,0,t$	$0,0,0$	y,x,z	$\infty\cdot2\cdot2$
		$\mathcal{N}_{\mathcal{E}^+}(\mathcal{G})$: P^1622	$\mathbf{a},\ \mathbf{b},\ \varepsilon\mathbf{c}$	$0,0,t$	/	$y,x,\bar{z}$	$\infty\cdot2$
174	$P\bar{6}$	$P6/mmm$	$\frac{2}{3}\mathbf{a}+\frac{1}{3}\mathbf{b},\ -\frac{1}{3}\mathbf{a}+\frac{1}{3}\mathbf{b},\ \frac{1}{2}\mathbf{c}$	$\frac{2}{3},\frac{1}{3},0;\ 0,0,\frac{1}{2}$	$0,0,0$	y,x,z	$6\cdot2\cdot2$
175	$P6/m$	$P6/mmm$	$\mathbf{a},\ \mathbf{b},\ \frac{1}{2}\mathbf{c}$	$0,0,\frac{1}{2}$		y,x,z	$2\cdot1\cdot2$
176	$P6_3/m$	$P6/mmm$	$\mathbf{a},\ \mathbf{b},\ \frac{1}{2}\mathbf{c}$	$0,0,\frac{1}{2}$		y,x,z	$2\cdot1\cdot2$
177	$P622$	$P6/mmm$	$\mathbf{a},\ \mathbf{b},\ \frac{1}{2}\mathbf{c}$	$0,0,\frac{1}{2}$	$0,0,0$		$2\cdot2\cdot1$
		$\mathcal{N}_{\mathcal{E}^+}(\mathcal{G})$: $P622$	$\mathbf{a},\ \mathbf{b},\ \frac{1}{2}\mathbf{c}$	$0,0,\frac{1}{2}$	/		$2\cdot1$
178	$P6_122$	$P6_222\ [\equiv\mathcal{N}_{\mathcal{E}^+}(\mathcal{G})]$	$\mathbf{a},\ \mathbf{b},\ \frac{1}{2}\mathbf{c}$	$0,0,\frac{1}{2}$	/		$2\cdot1$
179	$P6_522$	$P6_422\ [\equiv\mathcal{N}_{\mathcal{E}^+}(\mathcal{G})]$	$\mathbf{a},\ \mathbf{b},\ \frac{1}{2}\mathbf{c}$	$0,0,\frac{1}{2}$	/		$2\cdot1$
180	$P6_222$	$P6_422\ [\equiv\mathcal{N}_{\mathcal{E}^+}(\mathcal{G})]$	$\mathbf{a},\ \mathbf{b},\ \frac{1}{2}\mathbf{c}$	$0,0,\frac{1}{2}$	/		$2\cdot1$
181	$P6_422$	$P6_222\ [\equiv\mathcal{N}_{\mathcal{E}^+}(\mathcal{G})]$	$\mathbf{a},\ \mathbf{b},\ \frac{1}{2}\mathbf{c}$	$0,0,\frac{1}{2}$	/		$2\cdot1$
182	$P6_322$	$P6/mmm$	$\mathbf{a},\ \mathbf{b},\ \frac{1}{2}\mathbf{c}$	$0,0,\frac{1}{2}$	$0,0,0$		$2\cdot2\cdot1$
		$\mathcal{N}_{\mathcal{E}^+}(\mathcal{G})$: $P622$	$\mathbf{a},\ \mathbf{b},\ \frac{1}{2}\mathbf{c}$	$0,0,\frac{1}{2}$	/		$2\cdot1$
183	$P6mm$	P^16/mmm	$\mathbf{a},\ \mathbf{b},\ \varepsilon\mathbf{c}$	$0,0,t$	$0,0,0$		$\infty\cdot2\cdot1$
184	$P6cc$	P^16/mmm	$\mathbf{a},\ \mathbf{b},\ \varepsilon\mathbf{c}$	$0,0,t$	$0,0,0$		$\infty\cdot2\cdot1$

Table 3.5.2.5 (continued)

Space group $\mathcal{G}$		Euclidean normalizer $\mathcal{N}_{\mathcal{E}}(\mathcal{G})$ and chirality-preserving normalizer $\mathcal{N}_{\mathcal{E}^+}(\mathcal{G})$		Additional generators of $\mathcal{N}_{\mathcal{E}}(\mathcal{G})$ and $\mathcal{N}_{\mathcal{E}^+}(\mathcal{G})$			Index of $\mathcal{G}$ in $\mathcal{N}_{\mathcal{E}}(\mathcal{G})$ or $\mathcal{N}_{\mathcal{E}^+}(\mathcal{G})$
No.	Hermann–Mauguin symbol	Symbol	Basis vectors	Translations	Inversion through a centre at	Further generators	
185	$P6_3cm$	P^16/mmm	$\mathbf{a}, \mathbf{b}, \varepsilon\mathbf{c}$	$0, 0, t$	$0, 0, 0$		$\infty \cdot 2 \cdot 1$
186	$P6_3mc$	P^16/mmm	$\mathbf{a}, \mathbf{b}, \varepsilon\mathbf{c}$	$0, 0, t$	$0, 0, 0$		$\infty \cdot 2 \cdot 1$
187	$P\bar{6}m2$	$P6/mmm$	$\frac{2}{3}\mathbf{a}+\frac{1}{3}\mathbf{b}, -\frac{1}{3}\mathbf{a}+\frac{1}{3}\mathbf{b}, \frac{1}{2}\mathbf{c}$	$\frac{2}{3}, \frac{1}{3}, 0;\ 0, 0, \frac{1}{2}$	$0, 0, 0$		$6 \cdot 2 \cdot 1$
188	$P\bar{6}c2$	$P6/mmm$	$\frac{2}{3}\mathbf{a}+\frac{1}{3}\mathbf{b}, -\frac{1}{3}\mathbf{a}+\frac{1}{3}\mathbf{b}, \frac{1}{2}\mathbf{c}$	$\frac{2}{3}, \frac{1}{3}, 0;\ 0, 0, \frac{1}{2}$	$0, 0, 0$		$6 \cdot 2 \cdot 1$
189	$P\bar{6}2m$	$P6/mmm$	$\mathbf{a}, \mathbf{b}, \frac{1}{2}\mathbf{c}$	$0, 0, \frac{1}{2}$	$0, 0, 0$		$2 \cdot 2 \cdot 1$
190	$P\bar{6}2c$	$P6/mmm$	$\mathbf{a}, \mathbf{b}, \frac{1}{2}\mathbf{c}$	$0, 0, \frac{1}{2}$	$0, 0, 0$		$2 \cdot 2 \cdot 1$
191	$P6/mmm$	$P6/mmm$	$\mathbf{a}, \mathbf{b}, \frac{1}{2}\mathbf{c}$	$0, 0, \frac{1}{2}$			$2 \cdot 1 \cdot 1$
192	$P6/mcc$	$P6/mmm$	$\mathbf{a}, \mathbf{b}, \frac{1}{2}\mathbf{c}$	$0, 0, \frac{1}{2}$			$2 \cdot 1 \cdot 1$
193	$P6_3/mcm$	$P6/mmm$	$\mathbf{a}, \mathbf{b}, \frac{1}{2}\mathbf{c}$	$0, 0, \frac{1}{2}$			$2 \cdot 1 \cdot 1$
194	$P6_3/mmc$	$P6/mmm$	$\mathbf{a}, \mathbf{b}, \frac{1}{2}\mathbf{c}$	$0, 0, \frac{1}{2}$			$2 \cdot 1 \cdot 1$
195	$P23$	$Im\bar{3}m$	$\mathbf{a}, \mathbf{b}, \mathbf{c}$	$\frac{1}{2}, \frac{1}{2}, \frac{1}{2}$	$0, 0, 0$	y, x, z	$2 \cdot 2 \cdot 2$
		$\mathcal{N}_{\mathcal{E}^+}(\mathcal{G})$: $I432$	$\mathbf{a}, \mathbf{b}, \mathbf{c}$	$\frac{1}{2}, \frac{1}{2}, \frac{1}{2}$	/	$y, x, \bar{z}$	$2 \cdot 2$
196	$F23$	$Im\bar{3}m$	$\frac{1}{2}\mathbf{a}, \frac{1}{2}\mathbf{b}, \frac{1}{2}\mathbf{c}$	$\frac{1}{4}, \frac{1}{4}, \frac{1}{4}$	$0, 0, 0$	y, x, z	$4 \cdot 2 \cdot 2$
		$\mathcal{N}_{\mathcal{E}^+}(\mathcal{G})$: $I432$	$\frac{1}{2}\mathbf{a}, \frac{1}{2}\mathbf{b}, \frac{1}{2}\mathbf{c}$	$\frac{1}{4}, \frac{1}{4}, \frac{1}{4}$	/	$y, x, \bar{z}$	$4 \cdot 2$
197	$I23$	$Im\bar{3}m$	$\mathbf{a}, \mathbf{b}, \mathbf{c}$		$0, 0, 0$	y, x, z	$1 \cdot 2 \cdot 2$
		$\mathcal{N}_{\mathcal{E}^+}(\mathcal{G})$: $I432$	$\mathbf{a}, \mathbf{b}, \mathbf{c}$		/	$y, x, \bar{z}$	$1 \cdot 2$
198	$P2_13$	$Ia\bar{3}d$	$\mathbf{a}, \mathbf{b}, \mathbf{c}$	$\frac{1}{2}, \frac{1}{2}, \frac{1}{2}$	$0, 0, 0$	$y+\frac{1}{4}, x+\frac{1}{4}, z+\frac{1}{4}$	$2 \cdot 2 \cdot 2$
		$\mathcal{N}_{\mathcal{E}^+}(\mathcal{G})$: $I4_132$	$\mathbf{a}, \mathbf{b}, \mathbf{c}$	$\frac{1}{2}, \frac{1}{2}, \frac{1}{2}$	/	$\bar{y}+\frac{1}{4}, \bar{x}+\frac{1}{4}, \bar{z}+\frac{1}{4}$	$2 \cdot 2$
199	$I2_13$	$Ia\bar{3}d$	$\mathbf{a}, \mathbf{b}, \mathbf{c}$		$0, 0, 0$	$y+\frac{1}{4}, x+\frac{1}{4}, z+\frac{1}{4}$	$1 \cdot 2 \cdot 2$
		$\mathcal{N}_{\mathcal{E}^+}(\mathcal{G})$: $I4_132$	$\mathbf{a}, \mathbf{b}, \mathbf{c}$		/	$\bar{y}+\frac{1}{4}, \bar{x}+\frac{1}{4}, \bar{z}+\frac{1}{4}$	$1 \cdot 2$
200	$Pm\bar{3}$	$Im\bar{3}m$	$\mathbf{a}, \mathbf{b}, \mathbf{c}$	$\frac{1}{2}, \frac{1}{2}, \frac{1}{2}$		y, x, z	$2 \cdot 1 \cdot 2$
201	$Pn\bar{3}$ (23)	$Im\bar{3}m$	$\mathbf{a}, \mathbf{b}, \mathbf{c}$	$\frac{1}{2}, \frac{1}{2}, \frac{1}{2}$		y, x, z	$2 \cdot 1 \cdot 2$
201	$Pn\bar{3}$ ($\bar{3}$)	$Im\bar{3}m$ ($\bar{3}m$)	$\mathbf{a}, \mathbf{b}, \mathbf{c}$	$\frac{1}{2}, \frac{1}{2}, \frac{1}{2}$		y, x, z	$2 \cdot 1 \cdot 2$
202	$Fm\bar{3}$	$Pm\bar{3}m$	$\frac{1}{2}\mathbf{a}, \frac{1}{2}\mathbf{b}, \frac{1}{2}\mathbf{c}$	$\frac{1}{2}, \frac{1}{2}, \frac{1}{2}$		y, x, z	$2 \cdot 1 \cdot 2$
203	$Fd\bar{3}$ (23)	$Pn\bar{3}m$ ($\bar{4}3m$)	$\frac{1}{2}\mathbf{a}, \frac{1}{2}\mathbf{b}, \frac{1}{2}\mathbf{c}$	$\frac{1}{2}, \frac{1}{2}, \frac{1}{2}$		y, x, z	$2 \cdot 1 \cdot 2$
203	$Fd\bar{3}$ ($\bar{3}$)	$Pn\bar{3}m$ ($\bar{3}m$)	$\frac{1}{2}\mathbf{a}, \frac{1}{2}\mathbf{b}, \frac{1}{2}\mathbf{c}$	$\frac{1}{2}, \frac{1}{2}, \frac{1}{2}$		y, x, z	$2 \cdot 1 \cdot 2$
204	$Im\bar{3}$	$Im\bar{3}m$	$\mathbf{a}, \mathbf{b}, \mathbf{c}$			y, x, z	$1 \cdot 1 \cdot 2$
205	$Pa\bar{3}$	$Ia\bar{3}$	$\mathbf{a}, \mathbf{b}, \mathbf{c}$	$\frac{1}{2}, \frac{1}{2}, \frac{1}{2}$			$2 \cdot 1 \cdot 1$
206	$Ia\bar{3}$	$Ia\bar{3}d$	$\mathbf{a}, \mathbf{b}, \mathbf{c}$			$y+\frac{1}{4}, x+\frac{1}{4}, z+\frac{1}{4}$	$1 \cdot 1 \cdot 2$
207	$P432$	$Im\bar{3}m$	$\mathbf{a}, \mathbf{b}, \mathbf{c}$	$\frac{1}{2}, \frac{1}{2}, \frac{1}{2}$	$0, 0, 0$		$2 \cdot 2 \cdot 1$
		$\mathcal{N}_{\mathcal{E}^+}(\mathcal{G})$: $I432$	$\mathbf{a}, \mathbf{b}, \mathbf{c}$	$\frac{1}{2}, \frac{1}{2}, \frac{1}{2}$	/		$2 \cdot 1$
208	$P4_232$	$Im\bar{3}m$	$\mathbf{a}, \mathbf{b}, \mathbf{c}$	$\frac{1}{2}, \frac{1}{2}, \frac{1}{2}$	$0, 0, 0$		$2 \cdot 2 \cdot 1$
		$\mathcal{N}_{\mathcal{E}^+}(\mathcal{G})$: $I432$	$\mathbf{a}, \mathbf{b}, \mathbf{c}$	$\frac{1}{2}, \frac{1}{2}, \frac{1}{2}$	/		$2 \cdot 1$
209	$F432$	$Pm\bar{3}m$	$\frac{1}{2}\mathbf{a}, \frac{1}{2}\mathbf{b}, \frac{1}{2}\mathbf{c}$	$\frac{1}{2}, \frac{1}{2}, \frac{1}{2}$	$0, 0, 0$		$2 \cdot 2 \cdot 1$
		$\mathcal{N}_{\mathcal{E}^+}(\mathcal{G})$: $P432$	$\frac{1}{2}\mathbf{a}, \frac{1}{2}\mathbf{b}, \frac{1}{2}\mathbf{c}$	$\frac{1}{2}, \frac{1}{2}, \frac{1}{2}$	/		$2 \cdot 1$
210	$F4_132$	$Pn\bar{3}m$ ($\bar{4}3m$)	$\frac{1}{2}\mathbf{a}, \frac{1}{2}\mathbf{b}, \frac{1}{2}\mathbf{c}$	$\frac{1}{2}, \frac{1}{2}, \frac{1}{2}$	$\frac{1}{8}, \frac{1}{8}, \frac{1}{8}$		$2 \cdot 2 \cdot 1$
		$\mathcal{N}_{\mathcal{E}^+}(\mathcal{G})$: $P4_232$	$\frac{1}{2}\mathbf{a}, \frac{1}{2}\mathbf{b}, \frac{1}{2}\mathbf{c}$	$\frac{1}{2}, \frac{1}{2}, \frac{1}{2}$	/		$2 \cdot 1$
211	$I432$	$Im\bar{3}m$	$\mathbf{a}, \mathbf{b}, \mathbf{c}$		$0, 0, 0$		$1 \cdot 2 \cdot 1$
		$\mathcal{N}_{\mathcal{E}^+}(\mathcal{G})$: $I432$	$\mathbf{a}, \mathbf{b}, \mathbf{c}$		/		$1 \cdot 1$
212	$P4_332$	$I4_132$ [$\equiv \mathcal{N}_{\mathcal{E}^+}(\mathcal{G})$]	$\mathbf{a}, \mathbf{b}, \mathbf{c}$	$\frac{1}{2}, \frac{1}{2}, \frac{1}{2}$	/		$2 \cdot 1$
213	$P4_132$	$I4_132$ [$\equiv \mathcal{N}_{\mathcal{E}^+}(\mathcal{G})$]	$\mathbf{a}, \mathbf{b}, \mathbf{c}$	$\frac{1}{2}, \frac{1}{2}, \frac{1}{2}$	/		$2 \cdot 1$
214	$I4_132$	$Ia\bar{3}d$	$\mathbf{a}, \mathbf{b}, \mathbf{c}$		$0, 0, 0$		$1 \cdot 2 \cdot 1$
		$\mathcal{N}_{\mathcal{E}^+}(\mathcal{G})$: $I4_132$	$\mathbf{a}, \mathbf{b}, \mathbf{c}$		/		$1 \cdot 1$
215	$P\bar{4}3m$	$Im\bar{3}m$	$\mathbf{a}, \mathbf{b}, \mathbf{c}$	$\frac{1}{2}, \frac{1}{2}, \frac{1}{2}$	$0, 0, 0$		$2 \cdot 2 \cdot 1$
216	$F\bar{4}3m$	$Im\bar{3}m$	$\frac{1}{2}\mathbf{a}, \frac{1}{2}\mathbf{b}, \frac{1}{2}\mathbf{c}$	$\frac{1}{4}, \frac{1}{4}, \frac{1}{4}$	$0, 0, 0$		$4 \cdot 2 \cdot 1$
217	$I\bar{4}3m$	$Im\bar{3}m$	$\mathbf{a}, \mathbf{b}, \mathbf{c}$		$0, 0, 0$		$1 \cdot 2 \cdot 1$
218	$P\bar{4}3n$	$Im\bar{3}m$	$\mathbf{a}, \mathbf{b}, \mathbf{c}$	$\frac{1}{2}, \frac{1}{2}, \frac{1}{2}$	$0, 0, 0$		$2 \cdot 2 \cdot 1$
219	$F\bar{4}3c$	$Im\bar{3}m$	$\frac{1}{2}\mathbf{a}, \frac{1}{2}\mathbf{b}, \frac{1}{2}\mathbf{c}$	$\frac{1}{4}, \frac{1}{4}, \frac{1}{4}$	$0, 0, 0$		$4 \cdot 2 \cdot 1$
220	$I\bar{4}3d$	$Ia\bar{3}d$	$\mathbf{a}, \mathbf{b}, \mathbf{c}$		$0, 0, 0$		$1 \cdot 2 \cdot 1$
221	$Pm\bar{3}m$	$Im\bar{3}m$	$\mathbf{a}, \mathbf{b}, \mathbf{c}$	$\frac{1}{2}, \frac{1}{2}, \frac{1}{2}$			$2 \cdot 1 \cdot 1$
222	$Pn\bar{3}n$ (432)	$Im\bar{3}m$	$\mathbf{a}, \mathbf{b}, \mathbf{c}$	$\frac{1}{2}, \frac{1}{2}, \frac{1}{2}$			$2 \cdot 1 \cdot 1$
222	$Pn\bar{3}n$ ($\bar{3}$)	$Im\bar{3}m$ ($\bar{3}m$)	$\mathbf{a}, \mathbf{b}, \mathbf{c}$	$\frac{1}{2}, \frac{1}{2}, \frac{1}{2}$			$2 \cdot 1 \cdot 1$
223	$Pm\bar{3}n$	$Im\bar{3}m$	$\mathbf{a}, \mathbf{b}, \mathbf{c}$	$\frac{1}{2}, \frac{1}{2}, \frac{1}{2}$			$2 \cdot 1 \cdot 1$

Table 3.5.2.5 (continued)

Space group $\mathcal{G}$		Euclidean normalizer $\mathcal{N}_{\mathcal{E}}(\mathcal{G})$ and chirality-preserving normalizer $\mathcal{N}_{\mathcal{E}^+}(\mathcal{G})$		Additional generators of $\mathcal{N}_{\mathcal{E}}(\mathcal{G})$ and $\mathcal{N}_{\mathcal{E}^+}(\mathcal{G})$			Index of $\mathcal{G}$ in $\mathcal{N}_{\mathcal{E}}(\mathcal{G})$ or $\mathcal{N}_{\mathcal{E}^+}(\mathcal{G})$
No.	Hermann–Mauguin symbol	Symbol	Basis vectors	Translations	Inversion through a centre at	Further generators	
224	$Pn\bar{3}m$ ($\bar{4}3m$)	$Im\bar{3}m$	**a**, **b**, **c**	$\frac{1}{2}, \frac{1}{2}, \frac{1}{2}$			$2 \cdot 1 \cdot 1$
224	$Pn\bar{3}m$ ($\bar{3}m$)	$Im\bar{3}m$ ($\bar{3}m$)	**a**, **b**, **c**	$\frac{1}{2}, \frac{1}{2}, \frac{1}{2}$			$2 \cdot 1 \cdot 1$
225	$Fm\bar{3}m$	$Pm\bar{3}m$	$\frac{1}{2}$**a**, $\frac{1}{2}$**b**, $\frac{1}{2}$**c**	$\frac{1}{2}, \frac{1}{2}, \frac{1}{2}$			$2 \cdot 1 \cdot 1$
226	$Fm\bar{3}c$	$Pm\bar{3}m$	$\frac{1}{2}$**a**, $\frac{1}{2}$**b**, $\frac{1}{2}$**c**	$\frac{1}{2}, \frac{1}{2}, \frac{1}{2}$			$2 \cdot 1 \cdot 1$
227	$Fd\bar{3}m$ ($\bar{4}3m$)	$Pn\bar{3}m$ ($\bar{4}3m$)	$\frac{1}{2}$**a**, $\frac{1}{2}$**b**, $\frac{1}{2}$**c**	$\frac{1}{2}, \frac{1}{2}, \frac{1}{2}$			$2 \cdot 1 \cdot 1$
227	$Fd\bar{3}m$ ($\bar{3}m$)	$Pn\bar{3}m$ ($\bar{3}m$)	$\frac{1}{2}$**a**, $\frac{1}{2}$**b**, $\frac{1}{2}$**c**	$\frac{1}{2}, \frac{1}{2}, \frac{1}{2}$			$2 \cdot 1 \cdot 1$
228	$Fd\bar{3}c$ (23)	$Pn\bar{3}m$ ($\bar{4}3m$)	$\frac{1}{2}$**a**, $\frac{1}{2}$**b**, $\frac{1}{2}$**c**	$\frac{1}{2}, \frac{1}{2}, \frac{1}{2}$			$2 \cdot 1 \cdot 1$
228	$Fd\bar{3}c$ ($\bar{3}$)	$Pn\bar{3}m$ ($\bar{3}m$)	$\frac{1}{2}$**a**, $\frac{1}{2}$**b**, $\frac{1}{2}$**c**	$\frac{1}{2}, \frac{1}{2}, \frac{1}{2}$			$2 \cdot 1 \cdot 1$
229	$Im\bar{3}m$	$Im\bar{3}m$	**a**, **b**, **c**				$1 \cdot 1 \cdot 1$
230	$Ia\bar{3}d$	$Ia\bar{3}d$	**a**, **b**, **c**				$1 \cdot 1 \cdot 1$

The following rule holds without exception: The number of point configurations equivalent to a given one is equal to the quotient i/i_s, with i being the subgroup index of $\mathcal{G}$ in its Euclidean or affine normalizer and i_s the subgroup index between the corresponding two site-symmetry groups of any point in the original point configuration.

When referring to the Euclidean normalizer, i_s may be higher than 1 only if the eigensymmetry of the point configuration under consideration (*i.e.* the group of all motions that maps the point configuration onto itself, *cf.* Sections 1.4.4.4 and 3.4.1.3) is a proper supergroup of $\mathcal{G}$. If $\mathcal{D}$ designates the intersection group of $\mathcal{N}_{\mathcal{E}}(\mathcal{G})$ with the eigensymmetry group of the point configuration, the number of Euclidean-equivalent point configurations equals the index of $\mathcal{D}$ in $\mathcal{N}_{\mathcal{E}}(\mathcal{G})$.

Example

The Euclidean and affine normalizer of $P2_13$ is $Ia\bar{3}d$ with index 8. Point configuration $4a\ x, x, x$ with $x_1 = 0$ forms a face-centred cubic lattice with eigensymmetry $Fm\bar{3}m$. The reference point $0, 0, 0$ has site symmetry .3. in $P2_13$ but $.\bar{3}.$ in $Ia\bar{3}d$. The number of equivalent point configurations, therefore, is $i/i_s = 8/2 = 4$. One additional point configuration is generated by the translation $t(\frac{1}{2}, \frac{1}{2}, \frac{1}{2})$: $4a\ x, x, x$ with $x_2 = \frac{1}{2}$, the two others by applying the d-glide reflection $y + \frac{1}{4}, x + \frac{1}{4}, z + \frac{1}{4}$ to the first two point configurations: $4a\ x, x, x$ with $x_3 = \frac{1}{4}$ and $x_4 = \frac{3}{4}$. The intersection group $\mathcal{D}$ of the eigensymmetry $Fm\bar{3}m$ with the normalizer $Ia\bar{3}d$ is $Pa\bar{3}$. Its index 4 in $Ia\bar{3}d$ gives again the number of equivalent point configurations.

The set of equivalent point configurations is always infinite if the normalizer contains continuous translations, but this set may be described by a finite number of subsets due to non-continuous translations.

Example

The Euclidean and affine normalizer of $P6_1$ is P^1622 (**a**, **b**, ε**c**). With the aid of the 'additional generators' given in Table 3.5.2.5, one can calculate two subsets of point configurations that are equivalent to a given general point configuration $6a\ x, y, z$ with $x = x_0,\ y = y_0,\ z = z_0$: $6a\ x, y, z$ with $x_0, y_0, z_0 + t$ and $y_0, x_0, -z_0 + t$. If, however, the coordinates for the original point configuration are specialized, *e.g.* to $x = y = x_1,\ z = z_1$ or to $x = y = 0,\ z = z_2$, only one subset exists, namely $x_1, x_1, z_1 + t$ or $0, 0, z_2 + t$, respectively. The reduction of the number of subsets is a consequence of the enhancement of the site symmetry in the normalizer (.2. or 622, respectively), but the index i_s, as introduced above, does not necessarily give the reduction factor for the number of subsets.

It has to be noticed that for most space groups with a Euclidean normalizer containing continuous translations the index i_s is larger than 1 for *all* point configurations, *i.e.* the number of subsets of equivalent point configurations is necessarily reduced. The general Wyckoff position of such a space group does not belong to a characteristic type of Wyckoff sets (*cf.* Sections 1.4.4 and 3.4.1.3) and the eigensymmetry of all corresponding point configurations is enhanced.

Example

The Euclidean and affine normalizer of $P6$ is P^16/mmm (**a**, **b**, ε**c**). As a consequence of the continuous translations, the site symmetry of any point is at least $m..$ in P^16/mmm. With the aid of the 'additional generators', one calculates four subsets of point configurations that are equivalent to a given general point configuration $6d\ x, y, z$ with $x = x_0,\ y = y_0,\ z = z_0$: $x_0, y_0, z_0 + t$; $\quad -x_0, -y_0, -z_0 + t$; $\quad y_0, x_0, z_0 + t$; $\quad -y_0, -x_0, -z_0 + t$. The first two and the second two subsets coincide, however.

According to the above examples, Euclidean- (affine-) equivalent point configurations may or may not belong to the same Wyckoff position. Consequently, normalizers also define equivalence relations on Wyckoff positions:

Two *Wyckoff positions* of a space group $\mathcal{G}$ are called *Euclidean-* or $\mathcal{N}_{\mathcal{E}}$-equivalent (*affine-* or $\mathcal{N}_{\mathcal{A}}$*-equivalent*) if their point configurations are mapped onto each other by the Euclidean (affine) normalizer of $\mathcal{G}$.

Euclidean-equivalent Wyckoff positions are important for the description or comparison of crystal structures in terms of atomic coordinates. Affine-equivalent Wyckoff positions result in *Wyckoff sets* (*cf.* Sections 1.4.4 and 3.4.1.2) and form the necessary basis for the *definition of lattice complexes.* All site-symmetry groups corresponding to equivalent Wyckoff positions are conjugate in the respective normalizer.

references