3.5.4. Normalizers of point groups

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Normalizers with respect to the Euclidean or affine group may be defined for any group of isometries (*cf.* Gubler, 1982*a*,*b*). For a point group, however, it seems inadequate to use a supergroup that contains transformations that do not map a fixed point of that point group onto itself. Appropriate supergroups for the definition of normalizers of point groups are the full isometry groups of the sphere, $m\overline{\infty}$, and of the circle, ∞m, in three-dimensional and two-dimensional space (*cf.* Galiulin, 1978).

These normalizers are listed in Tables 3.5.4.1 and 3.5.4.2. It has to be noticed that the normalizer of a crystallographic point group may contain continuous rotations, *i.e.* rotations with infinitesimal rotation angle, or noncrystallographic rotations (∞m; $m\overline{\infty}$, ∞/mm, $8mm$, $12mm$; $8/mmm$, $12/mmm$). In analogy to space groups, these normalizers define equivalence relationships on the 'Wyckoff positions' of the point groups (*cf.* Sections 3.2.3 and 3.2.4). They give also the relation between the different but equivalent morphological descriptions of a crystal.

Table 3.5.4.1
Normalizers of the two-dimensional point groups with respect to the full isometry group of the circle

The upper part refers to the crystallographic, the lower part to the noncrystallographic point groups as listed in Table 3.2.1.5. The letter n represents an arbitrary integer; $(2n)$ represents an even number.

Normalizer	Point groups
∞m	1, 2, 4, 3, 6
$12mm$	$6mm$
$8mm$	$4mm$
$6mm$	$3m$
$4mm$	$2mm$
$2mm$	m
∞m	n, ∞, ∞m
$(2n)mm$	nmm, nm

Table 3.5.4.2
Normalizers of the three-dimensional point groups with respect to the full isometry group of the sphere

The upper part refers to the crystallographic, the lower part to the noncrystallographic point groups as listed in Table 3.2.1.6. The letter n represents an arbitrary integer; $(2n)$ represents an even number.

Normalizer	Point groups
$m\overline{\infty}$	1, $\overline{1}$
$m\overline{3}m$	222, mmm, 23, $m\overline{3}$, 432, $\overline{4}3m$, $m\overline{3}m$
∞/mm	2, m, $2/m$, 4, $\overline{4}$, $4/m$, 3, $\overline{3}$, 6, $\overline{6}$, $6/m$
$12/mmm$	622, $6mm$, $6/mmm$
$8/mmm$	422, $4mm$, $4/mmm$
$6/mmm$	32, $3m$, $\overline{3}m$, $\overline{6}2m$
$4/mmm$	$mm2$, $\overline{4}2m$
$m\overline{\infty}$	2∞, $m\overline{\infty}$
$m\overline{35}$	235, $m\overline{35}$
∞/mm	n, $\overline{n}$, n/m, ∞, ∞/m, $\infty 2$, ∞m, ∞/mm
$(2n)/mmm$	$n22$, nmm, n/mmm, $n2$, nm, $\overline{n}m$
n/mmm	$\overline{n}2m$

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