

## 1.4. SYMMETRY IN RECIPROCAL SPACE

it was found more convenient to construct a FORTRAN interpreter which would detect in the REDUCE output the basic building blocks of the trigonometric structure factors (see Section 1.4.3.3) and perform the required transformations.

Tables A1.4.3.1–A1.4.3.7 were thus constructed with the aid of a chain composed of (i) a space-group generating routine, (ii) a FORTRAN interface, which processes the space-group input and ‘writes’ a complete REDUCE program, (iii) execution of the REDUCE program and (iv) a FORTRAN interpreter of the REDUCE output in terms of the abbreviated symbols to be used in the tables. The computation was at a ‘one-group-at-a-time’ basis and the automation of its repetition was performed by means of procedural constructs at the operating-system level. The construction of Table A1.4.4.1 involved only the preliminary stage of the processing of the space-group information by the FORTRAN interface. All the computations were carried out on a Cyber 170-855 at the Tel Aviv University Computation Center.

It is of some importance to comment on the recommended usage of the tables included in this chapter in automatic computations. If, for example, we wish to compute the expression:  $A = -8(\text{Escs} + \text{Ossc})$ , use can be made of the facility provided by most versions of FORTRAN of transferring subprogram names as parameters of a FUNCTION. We thus need only two FUNCTIONS for any calculation of  $A$  and  $B$  for a cubic space group, one FUNCTION for the block of even permutations of  $x$ ,  $y$  and  $z$ :

```

FUNCTION E(P, Q, R)
EXTERNAL SIN, COS
COMMON/TSF/TPH, TPK, TPL, X, Y, Z
E = P(TPH * X) * Q(TPK * Y) * R(TPL * Z)
1 + P(TPH * Z) * Q(TPK * X) * R(TPL * Y)
2 + P(TPH * Y) * Q(TPK * Z) * R(TPL * X)
RETURN
END

```

where TPH, TPK and TPL denote  $2\pi h$ ,  $2\pi k$  and  $2\pi l$ , respectively, and a similar FUNCTION, say O(P,Q,R), for the block of odd permutations of  $x$ ,  $y$  and  $z$ . The calling statement in the calling (sub)program can thus be:

$$A = -8 * (E(\text{SIN}, \text{COS}, \text{SIN}) + O(\text{SIN}, \text{SIN}, \text{COS})).$$

A small number of such FUNCTIONS suffices for all the space-group-specific computations that involve trigonometric structure factors.

### Appendix 1.4.2.

#### Space-group symbols for numeric and symbolic computations

##### A1.4.2.1. Introduction (U. SHMUELI, S. R. HALL AND R. W. GROSSE-KUNSTLEVE)

This appendix lists two sets of computer-adapted space-group symbols which are implemented in existing crystallographic software and can be employed in the automated generation of space-group representations. The computer generation of space-group symmetry information is of well known importance in many

crystallographic calculations, numeric as well as symbolic. A prerequisite for a computer program that generates this information is a set of computer-adapted space-group symbols which are based on the generating elements of the space group to be derived. The sets of symbols to be presented are:

(i) *Explicit symbols*. These symbols are based on the classification of crystallographic point groups and space groups by Zachariasen (1945). These symbols are termed *explicit* because they contain in an explicit manner the rotation and translation parts of the space-group generators of the space group to be derived and used. These computer-adapted explicit symbols were proposed by Shmueli (1984), who also describes in detail their implementation in the program *SPGRGEN*. This program was used for the automatic preparation of the structure-factor tables for the 17 plane groups and 230 space groups, listed in Appendix 1.4.3, and the 230 space groups in reciprocal space, listed in Appendix 1.4.4. The explicit symbols presented in this appendix are adapted to the 306 representations of the 230 space groups as presented in *IT A* (1983) with regard to the standard settings and choice of space-group origins.

The symmetry-generating algorithm underlying the explicit symbols, and their definition, are given in Section A1.4.2.2 of this appendix and the explicit symbols are listed in Table A1.4.2.1.

(ii) *Hall symbols*. These symbols are based on the implied-origin notation of Hall (1981*a,b*), who also describes in detail the algorithm implemented in the program *SGNAME* (Hall, 1981*a*). In the first edition of *IT B* (1993), the term ‘concise space-group symbols’ was used for this notation. In recent years, however, the term ‘Hall symbols’ has come into use in symmetry papers (Altermatt & Brown, 1987; Grosse-Kunstleve, 1999), software applications (Hovmöller, 1992; Grosse-Kunstleve, 1995; Larine *et al.*, 1995; Dowty, 1997) and data-handling approaches (Bourne *et al.*, 1998). This term has therefore been adopted for the second edition.

The main difference in the definition of the Hall symbols between this edition and the first edition of *IT B* is the generalization of the origin-shift vector to a full change-of-basis matrix. The examples have been expanded to show how this matrix is applied. The notation has also been made more consistent, and a typographical error in a default axis direction has been corrected.\* The lattice centring symbol ‘H’ has been added to Table A1.4.2.2. In addition, Hall symbols are now provided for 530 settings to include all settings from Table 4.3.1 of *IT A* (1983). Namely, all non-standard symbols for the monoclinic and orthorhombic space groups are included.

Some of the space-group symbols listed in Table A1.4.2.7 differ from those listed in Table B.6 (p. 119) of the first edition of *IT B*. This is because the symmetry of many space groups can be represented by more than one subset of ‘generator’ elements and these lead to different Hall symbols. The symbols listed in this edition have been selected after first sorting the symmetry elements into a strictly prescribed order based on the shape of their Seitz matrices, whereas those in Table B.6 were selected from symmetry elements in the order of *ITI* (1965). Software for selecting the Hall symbols listed in Table A1.4.2.7 is freely available (Hall, 1997). These symbols and their equivalents in the first edition of *IT B* will generate identical symmetry elements, but the former may be used as a reference table in a strict mapping procedure between different symmetry representations (Hall *et al.*, 2000).

The Hall symbols are defined in Section A1.4.2.3 of this appendix and are listed in Table A1.4.2.7.

\* The correct default axis direction  $\mathbf{a} - \mathbf{b}$  of an N preceded by 3 or 6 replaces  $\mathbf{a} + \mathbf{b}$  on p. 117, right-hand column, line 4, in the first edition of *IT B*.

## 1. GENERAL RELATIONSHIPS AND TECHNIQUES

### A1.4.2.2. Explicit symbols (U. SHMUELI)

As shown elsewhere (Shmueli, 1984), the set of representative operators of a crystallographic space group [*i.e.* the set that is listed for each space group in the symmetry tables of *IT A* (1983) and automatically regenerated for the purpose of compiling the symmetry tables in the present chapter] may have one of the following forms:

$$\begin{aligned} & \{(\mathbf{Q}, \mathbf{u})\}, \\ & \{(\mathbf{Q}, \mathbf{u})\} \times \{(\mathbf{R}, \mathbf{v})\}, \quad \text{or} \\ & \{(\mathbf{P}, \mathbf{t})\} \times \{(\mathbf{Q}, \mathbf{u})\} \times \{(\mathbf{R}, \mathbf{v})\}, \end{aligned} \quad (\text{A1.4.2.1})$$

where  $\mathbf{P}$ ,  $\mathbf{Q}$  and  $\mathbf{R}$  are point-group operators, and  $\mathbf{t}$ ,  $\mathbf{u}$  and  $\mathbf{v}$  are zero vectors or translations not belonging to the lattice-translations subgroup. Each of the forms in (A1.4.2.1), enclosed in braces, is evaluated as, *e.g.*,

$$\{(\mathbf{P}, \mathbf{t})\} = \{(\mathbf{I}, \mathbf{0}), (\mathbf{P}, \mathbf{t}), (\mathbf{P}, \mathbf{t})^2, \dots, (\mathbf{P}, \mathbf{t})^{g-1}\}, \quad (\text{A1.4.2.2})$$

where  $\mathbf{I}$  is a unit operator and  $g$  is the order of the rotation operator  $\mathbf{P}$  (*i.e.*  $\mathbf{P}^g = \mathbf{I}$ ). The representative operations of the space group are evaluated by expanding the generators into cyclic groups, as in (A1.4.2.2), and forming, as needed, ordered products of the expanded groups as indicated in (A1.4.2.1) and explained in detail in the original article (Shmueli, 1984). The rotation and translation parts of the generators  $(\mathbf{P}, \mathbf{t})$ ,  $(\mathbf{Q}, \mathbf{u})$  and  $(\mathbf{R}, \mathbf{v})$  presented here were adapted to the settings and choices of origin used in the main symmetry tables of *IT A* (1983).

The general structure of a three-generator symbol, corresponding to the last line of (A1.4.2.1), as represented in Table A1.4.2.1, is

$$\text{LSC}\$r_1\text{P}t_1t_2t_3\$r_2\text{Q}u_1u_2u_3\$r_3\text{R}v_1v_2v_3, \quad (\text{A1.4.2.3})$$

where

L – lattice type; can be P, A, B, C, I, F, or R. The symbol R is used only for the seven rhombohedral space groups in their representations in rhombohedral and hexagonal axes [obverse setting (*IT I*, 1952)].

S – crystal system; can be A (triclinic), M (monoclinic), O (orthorhombic), T (tetragonal), R (trigonal), H (hexagonal) or C (cubic).

C – status of centrosymmetry; can be C or N according as the space group is centrosymmetric or noncentrosymmetric, respectively.

\$ – this character is followed by six characters that define a generator of the space group.

$r_i$  – indicator of the type of rotation that follows:  $r_i$  is P or I according as the rotation part of the  $i$ th generator is proper or improper, respectively.

P, Q, R – two-character symbols of matrix representations of the point-group rotation operators  $\mathbf{P}$ ,  $\mathbf{Q}$  and  $\mathbf{R}$ , respectively (see below).

$t_1t_2t_3, u_1u_2u_3, v_1v_2v_3$  – components of the translation parts of the generators, given in units of  $\frac{1}{12}$ ; *e.g.* the translation part  $(0 \frac{1}{2} \frac{3}{4})$  is given in Table A1.4.2.1 as 069. An *exception*:  $(0 \ 0 \ \frac{5}{6})$  is denoted by 005 and not by 0010.

The two-character symbols for the matrices of rotation, which appear in the explicit space-group symbols in Table A1.4.2.1, are defined as follows:

$$\begin{aligned} 1\text{A} &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} & 2\text{A} &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & \bar{1} & 0 \\ 0 & 0 & \bar{1} \end{pmatrix} & 2\text{B} &= \begin{pmatrix} \bar{1} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \bar{1} \end{pmatrix} \\ 2\text{C} &= \begin{pmatrix} \bar{1} & 0 & 0 \\ 0 & \bar{1} & 0 \\ 0 & 0 & 1 \end{pmatrix} & 2\text{D} &= \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & \bar{1} \end{pmatrix} & 2\text{E} &= \begin{pmatrix} 0 & \bar{1} & 0 \\ \bar{1} & 0 & 0 \\ 0 & 0 & \bar{1} \end{pmatrix} \\ 2\text{F} &= \begin{pmatrix} 1 & \bar{1} & 0 \\ 0 & \bar{1} & 0 \\ 0 & 0 & \bar{1} \end{pmatrix} & 2\text{G} &= \begin{pmatrix} 1 & 0 & 0 \\ 1 & \bar{1} & 0 \\ 0 & 0 & \bar{1} \end{pmatrix} & 3\text{Q} &= \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \\ 3\text{C} &= \begin{pmatrix} 0 & \bar{1} & 0 \\ 1 & \bar{1} & 0 \\ 0 & 0 & 1 \end{pmatrix} & 4\text{C} &= \begin{pmatrix} 0 & \bar{1} & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} & 6\text{C} &= \begin{pmatrix} 1 & \bar{1} & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \end{aligned}$$

where only matrices of proper rotation are given (and required), since the corresponding matrices of improper rotation are created by the program for appropriate value of the  $r_i$  indicator. The first character of a symbol is the order of the axis of rotation and the second character specifies its orientation: in terms of direct-space lattice vectors, we have

$$\begin{aligned} \text{A} &= [100], \text{B} = [010], \text{C} = [001], \text{D} = [110], \\ \text{E} &= [1\bar{1}0], \text{F} = [100], \text{G} = [210] \text{ and } \text{Q} = [111] \end{aligned}$$

for the standard orientations of the axes of rotation. Note that the axes 2F, 2G, 3C and 6C appear in trigonal and hexagonal space groups.

In the above scheme a space group is determined by one, two or at most three generators [see (A1.4.2.1)]. It should be pointed out that a convenient way of achieving a representation of the space group in any setting and relative to any origin is to start from the standard generators in Table A1.4.2.1 and let the computer program perform the appropriate transformation of the generators only, as in equations (1.4.4.4) and (1.4.4.5). The subsequent expansion of the transformed generators and the formation of the required products [see (A1.4.2.1) and (A1.4.2.2)] leads to the new representation of the space group.

In order to illustrate an explicit space-group symbol consider, for example, the symbol for the space group  $Ia\bar{3}d$ , as given in Table A1.4.2.1:

$$\text{ICC}\$I3\text{Q}000\$P4\text{C}393\$P2\text{D}933.$$

The first three characters tell us that the Bravais lattice of this space group is of type I, that the space group is centrosymmetric and that it belongs to the cubic system. We then see that the generators are (i) an improper threefold axis along  $[111]$  ( $I3Q$ ) with a zero translation part, (ii) a proper fourfold axis along  $[001]$  ( $P4C$ ) with translation part  $(1/4, 3/4, 1/4)$  and (iii) a proper twofold axis along  $[110]$  ( $P2D$ ) with translation part  $(3/4, 1/4, 1/4)$ .

If we make use of the above-outlined interpretation of the explicit symbol (A1.4.2.3), the space-group symmetry transformations in direct space, corresponding to these three generators of the space group  $Ia\bar{3}d$ , become

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Table A1.4.2.1. *Explicit symbols*

No.	Short Hermann-Mauguin symbol	Comments	Explicit symbols	No.	Short Hermann-Mauguin symbol	Comments	Explicit symbols
1	$P1$		PAN\$P1A000	15	$C2/c$	$C12/c1$	CMC\$I1A000\$P2B006
2	$P\bar{1}$		PAC\$I1A000	15	$C2/c$	$A12/n1$	AMC\$I1A000\$P2B606
3	$P2$	$P121$	PMN\$P2B000	15	$C2/c$	$I12/a1$	IMC\$I1A000\$P2B600
3	$P2$	$P112$	PMN\$P2C000	15	$C2/c$	$A112/a$	AMC\$I1A000\$P2C600
4	$P2_1$	$P12_11$	PMN\$P2B060	15	$C2/c$	$B112/n$	BMC\$I1A000\$P2C660
4	$P2_1$	$P112_1$	PMN\$P2C006	15	$C2/c$	$I112/b$	IMC\$I1A000\$P2C060
5	$C2$	$C121$	CMN\$P2B000	16	$P222$		PON\$P2C000\$P2A000
5	$C2$	$A121$	AMN\$P2B000	17	$P222_1$		PON\$P2C006\$P2A000
5	$C2$	$I121$	IMN\$P2B000	18	$P2_12_12$		PON\$P2C000\$P2A660
5	$C2$	$A112$	AMN\$P2C000	19	$P2_12_12_1$		PON\$P2C606\$P2A660
5	$C2$	$B112$	BMN\$P2C000	20	$C222_1$		CON\$P2C006\$P2A000
5	$C2$	$I112$	IMN\$P2C000	21	$C222$		CON\$P2C000\$P2A000
6	$Pm$	$P1m1$	PMN\$I2B000	22	$F222$		FON\$P2C000\$P2A000
6	$Pm$	$P11m$	PMN\$I2C000	23	$I222$		ION\$P2C000\$P2A000
7	$Pc$	$P1c1$	PMN\$I2B006	24	$I2_12_12_1$		ION\$P2C606\$P2A660
7	$Pc$	$P1n1$	PMN\$I2B606	25	$Pmm2$		PON\$P2C000\$I2A000
7	$Pc$	$P1a1$	PMN\$I2B600	26	$Pmc2_1$		PON\$P2C006\$I2A000
7	$Pc$	$P11a$	PMN\$I2C600	27	$Pcc2$		PON\$P2C000\$I2A006
7	$Pc$	$P11n$	PMN\$I2C660	28	$Pma2$		PON\$P2C000\$I2A600
7	$Pc$	$P11b$	PMN\$I2C060	29	$Pca2_1$		PON\$P2C006\$I2A606
8	$Cm$	$C1m1$	CMN\$I2B000	30	$Pnc2$		PON\$P2C000\$I2A066
8	$Cm$	$A1m1$	AMN\$I2B000	31	$Pmn2_1$		PON\$P2C606\$I2A000
8	$Cm$	$I1m1$	IMN\$I2B000	32	$Pba2$		PON\$P2C000\$I2A660
8	$Cm$	$A11m$	AMN\$I2C000	33	$Pna2_1$		PON\$P2C006\$I2A666
8	$Cm$	$B11m$	BMN\$I2C000	34	$Pnn2$		PON\$P2C000\$I2A666
8	$Cm$	$I11m$	IMN\$I2C000	35	$Cmm2$		CON\$P2C000\$I2A000
9	$Cc$	$C1c1$	CMN\$I2B006	36	$Cmc2_1$		CON\$P2C006\$I2A000
9	$Cc$	$A1n1$	AMN\$I2B606	37	$Ccc2$		CON\$P2C000\$I2A006
9	$Cc$	$I1a1$	IMN\$I2B600	38	$Amm2$		AON\$P2C000\$I2A000
9	$Cc$	$A11a$	AMN\$I2C600	39	$Abm2$		AON\$P2C000\$I2A060
9	$Cc$	$B11n$	BMN\$I2C660	40	$Ama2$		AON\$P2C000\$I2A600
9	$Cc$	$I11b$	IMN\$I2C060	41	$Aba2$		AON\$P2C000\$I2A660
10	$P2/m$	$P12/m1$	PMC\$I1A000\$P2B000	42	$Fmm2$		FON\$P2C000\$I2A000
10	$P2/m$	$P112/m$	PMC\$I1A000\$P2C000	43	$Fdd2$		FON\$P2C000\$I2A333
11	$P2_1/m$	$P12_1/m1$	PMC\$I1A000\$P2B060	44	$Imm2$		ION\$P2C000\$I2A000
11	$P2_1/m$	$P112_1/m$	PMC\$I1A000\$P2C006	45	$Iba2$		ION\$P2C000\$I2A660
12	$C2/m$	$C12/m1$	CMC\$I1A000\$P2B000	46	$Ima2$		ION\$P2C000\$I2A600
12	$C2/m$	$A12/m1$	AMC\$I1A000\$P2B000	47	$Pmmm$		POC\$I1A000\$P2C000\$P2A000
12	$C2/m$	$I12/m1$	IMC\$I1A000\$P2B000	48	$Pnnn$	Origin 1	POC\$I1A666\$P2C000\$P2A000
12	$C2/m$	$A112/m$	AMC\$I1A000\$P2C000	48	$Pnnn$	Origin 2	POC\$I1A000\$P2C660\$P2A066
12	$C2/m$	$B112/m$	BMC\$I1A000\$P2C000	49	$Pccm$		POC\$I1A000\$P2C000\$P2A006
12	$C2/m$	$I112/m$	IMC\$I1A000\$P2C000	50	$Pban$	Origin 1	POC\$I1A660\$P2C000\$P2A000
13	$P2/c$	$P12/c1$	PMC\$I1A000\$P2B006	50	$Pban$	Origin 2	POC\$I1A000\$P2C660\$P2A060
13	$P2/c$	$P12/n1$	PMC\$I1A000\$P2B606	51	$Pmna$		POC\$I1A000\$P2C600\$P2A600
13	$P2/c$	$P12/a1$	PMC\$I1A000\$P2B600	52	$Pnaa$		POC\$I1A000\$P2C600\$P2A066
13	$P2/c$	$P112/a$	PMC\$I1A000\$P2C600	53	$Pmna$		POC\$I1A000\$P2C606\$P2A000
13	$P2/c$	$P112/n$	PMC\$I1A000\$P2C660	54	$Pcca$		POC\$I1A000\$P2C600\$P2A606
13	$P2/c$	$P112/b$	PMC\$I1A000\$P2C060	55	$Pbam$		POC\$I1A000\$P2C000\$P2A660
14	$P2_1/c$	$P12_1/c1$	PMC\$I1A000\$P2B066	56	$Pccn$		POC\$I1A000\$P2C660\$P2A606
14	$P2_1/c$	$P12_1/n1$	PMC\$I1A000\$P2B666	57	$Pbcm$		POC\$I1A000\$P2C006\$P2A060
14	$P2_1/c$	$P12_1/a1$	PMC\$I1A000\$P2B660	58	$Pnmm$		POC\$I1A000\$P2C000\$P2A666
14	$P2_1/c$	$P112_1/a$	PMC\$I1A000\$P2C606	59	$Pmnn$	Origin 1	POC\$I1A660\$P2C000\$P2A660
14	$P2_1/c$	$P112_1/n$	PMC\$I1A000\$P2C666	59	$Pmnn$	Origin 2	POC\$I1A000\$P2C660\$P2A600
14	$P2_1/c$	$P112_1/b$	PMC\$I1A000\$P2C066	60	$Pbcn$		POC\$I1A000\$P2C666\$P2A660

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Table A1.4.2.1. *Explicit symbols (cont.)*

No.	Short Hermann–Mauguin symbol	Comments	Explicit symbols	No.	Short Hermann–Mauguin symbol	Comments	Explicit symbols
61	<i>Pbca</i>		POC\$I1A000\$P2C606\$P2A660	110	<i>I4<sub>1</sub>cd</i>		ITN\$P4C063\$I2A660
62	<i>Pnma</i>		POC\$I1A000\$P2C606\$P2A666	111	<i>P4<sub>2</sub>m</i>		PTN\$I4C000\$P2A000
63	<i>Cmcm</i>		COC\$I1A000\$P2C006\$P2A000	112	<i>P4<sub>2</sub>c</i>		PTN\$I4C000\$P2A006
64	<i>Cmca</i>		COC\$I1A000\$P2C066\$P2A000	113	<i>P4<sub>2</sub>1m</i>		PTN\$I4C000\$P2A660
65	<i>Cmmm</i>		COC\$I1A000\$P2C000\$P2A000	114	<i>P4<sub>2</sub>1c</i>		PTN\$I4C000\$P2A666
66	<i>Cccm</i>		COC\$I1A000\$P2C000\$P2A006	115	<i>P4<sub>2</sub>m2</i>		PTN\$I4C000\$P2D000
67	<i>Cmma</i>		COC\$I1A000\$P2C060\$P2A000	116	<i>P4<sub>2</sub>c2</i>		PTN\$I4C000\$P2D006
68	<i>Ccca</i>	Origin 1	COC\$I1A066\$P2C660\$P2A660	117	<i>P4<sub>2</sub>b2</i>		PTN\$I4C000\$P2D660
68	<i>Ccca</i>	Origin 2	COC\$I1A000\$P2C600\$P2A606	118	<i>P4<sub>2</sub>n2</i>		PTN\$I4C000\$P2D666
69	<i>Fmmm</i>		FOC\$I1A000\$P2C000\$P2A000	119	<i>I4<sub>2</sub>m2</i>		ITN\$I4C000\$P2D000
70	<i>Fddd</i>	Origin 1	FOC\$I1A333\$P2C000\$P2A000	120	<i>I4<sub>2</sub>c2</i>		ITN\$I4C000\$P2D006
70	<i>Fddd</i>	Origin 2	FOC\$I1A000\$P2C990\$P2A099	121	<i>I4<sub>2</sub>m</i>		ITN\$I4C000\$P2A000
71	<i>Immm</i>		IOC\$I1A000\$P2C000\$P2A000	122	<i>I4<sub>2</sub>d</i>		ITN\$I4C000\$P2A609
72	<i>Ibam</i>		IOC\$I1A000\$P2C000\$P2A660	123	<i>P4/mmm</i>		PTC\$I1A000\$P4C000\$P2A000
73	<i>Ibca</i>		IOC\$I1A000\$P2C066\$P2A660	124	<i>P4/mcc</i>		PTC\$I1A000\$P4C000\$P2A006
74	<i>Imma</i>		IOC\$I1A000\$P2C060\$P2A000	125	<i>P4/nbm</i>	Origin 1	PTC\$I1A660\$P4C000\$P2A000
75	<i>P4</i>		PTN\$P4C000	125	<i>P4/nbm</i>	Origin 2	PTC\$I1A000\$P4C600\$P2A060
76	<i>P4<sub>1</sub></i>		PTN\$P4C003	126	<i>P4/nnc</i>	Origin 1	PTC\$I1A666\$P4C000\$P2A000
77	<i>P4<sub>2</sub></i>		PTN\$P4C006	126	<i>P4/nnc</i>	Origin 2	PTC\$I1A000\$P4C600\$P2A066
78	<i>P4<sub>3</sub></i>		PTN\$P4C009	127	<i>P4/mbm</i>		PTC\$I1A000\$P4C000\$P2A660
79	<i>I4</i>		ITN\$P4C000	128	<i>P4/mnc</i>		PTC\$I1A000\$P4C000\$P2A666
80	<i>I4<sub>1</sub></i>		ITN\$P4C063	129	<i>P4/nmm</i>	Origin 1	PTC\$I1A660\$P4C660\$P2A660
81	<i>P4<sub>1</sub></i>		PTN\$I4C000	129	<i>P4/nmm</i>	Origin 2	PTC\$I1A000\$P4C600\$P2A600
82	<i>I4<sub>1</sub></i>		ITN\$I4C000	130	<i>P4/ncc</i>	Origin 1	PTC\$I1A660\$P4C660\$P2A666
83	<i>P4/m</i>		PTC\$I1A000\$P4C000	130	<i>P4/ncc</i>	Origin 2	PTC\$I1A000\$P4C600\$P2A606
84	<i>P4<sub>2</sub>/m</i>		PTC\$I1A000\$P4C006	131	<i>P4<sub>2</sub>/mnc</i>		PTC\$I1A000\$P4C006\$P2A000
85	<i>P4/n</i>	Origin 1	PTC\$I1A660\$P4C660	132	<i>P4<sub>2</sub>/mcm</i>		PTC\$I1A000\$P4C006\$P2A006
85	<i>P4/n</i>	Origin 2	PTC\$I1A000\$P4C600	133	<i>P4<sub>2</sub>/nbc</i>	Origin 1	PTC\$I1A666\$P4C666\$P2A006
86	<i>P4<sub>2</sub>/n</i>	Origin 1	PTC\$I1A666\$P4C666	133	<i>P4<sub>2</sub>/nbc</i>	Origin 2	PTC\$I1A000\$P4C606\$P2A060
86	<i>P4<sub>2</sub>/n</i>	Origin 2	PTC\$I1A000\$P4C066	134	<i>P4<sub>2</sub>/nmm</i>	Origin 1	PTC\$I1A666\$P4C666\$P2A000
87	<i>I4/m</i>		ITC\$I1A000\$P4C000	134	<i>P4<sub>2</sub>/nmm</i>	Origin 2	PTC\$I1A000\$P4C606\$P2A066
88	<i>I4<sub>1</sub>/a</i>	Origin 1	ITC\$I1A063\$P4C063	135	<i>P4<sub>2</sub>/mbc</i>		PTC\$I1A000\$P4C006\$P2A660
88	<i>I4<sub>1</sub>/a</i>	Origin 2	ITC\$I1A000\$P4C933	136	<i>P4<sub>2</sub>/mnm</i>		PTC\$I1A000\$P4C666\$P2A666
89	<i>P422</i>		PTN\$P4C000\$P2A000	137	<i>P4<sub>2</sub>/nmc</i>	Origin 1	PTC\$I1A666\$P4C666\$P2A666
90	<i>P4<sub>2</sub>2</i>		PTN\$P4C660\$P2A660	137	<i>P4<sub>2</sub>/nmc</i>	Origin 2	PTC\$I1A000\$P4C606\$P2A600
91	<i>P4<sub>1</sub>22</i>		PTN\$P4C003\$P2A006	138	<i>P4<sub>2</sub>/ncm</i>	Origin 1	PTC\$I1A666\$P4C666\$P2A660
92	<i>P4<sub>1</sub>2<sub>1</sub>2</i>		PTN\$P4C663\$P2A669	138	<i>P4<sub>2</sub>/ncm</i>	Origin 2	PTC\$I1A000\$P4C606\$P2A606
93	<i>P4<sub>2</sub>22</i>		PTN\$P4C006\$P2A000	139	<i>I4/mmm</i>		ITC\$I1A000\$P4C000\$P2A000
94	<i>P4<sub>2</sub>2<sub>1</sub>2</i>		PTN\$P4C666\$P2A666	140	<i>I4/mcm</i>		ITC\$I1A000\$P4C000\$P2A006
95	<i>P4<sub>3</sub>22</i>		PTN\$P4C009\$P2A006	141	<i>I4<sub>1</sub>/amd</i>	Origin 1	ITC\$I1A063\$P4C063\$P2A063
96	<i>P4<sub>3</sub>2<sub>1</sub>2</i>		PTN\$P4C669\$P2A663	141	<i>I4<sub>1</sub>/amd</i>	Origin 2	ITC\$I1A000\$P4C393\$P2A000
97	<i>I422</i>		ITN\$P4C000\$P2A000	142	<i>I4<sub>1</sub>/acd</i>	Origin 1	ITC\$I1A063\$P4C063\$P2A069
98	<i>I4<sub>1</sub>22</i>		ITN\$P4C063\$P2A063	142	<i>I4<sub>1</sub>/acd</i>	Origin 2	ITC\$I1A000\$P4C393\$P2A006
99	<i>P4mm</i>		PTN\$P4C000\$I2A000	143	<i>P3</i>		PRN\$P3C000
100	<i>P4bm</i>		PTN\$P4C000\$I2A660	144	<i>P3<sub>1</sub></i>		PRN\$P3C004
101	<i>P4<sub>2</sub>cm</i>		PTN\$P4C006\$I2A006	145	<i>P3<sub>2</sub></i>		PRN\$P3C008
102	<i>P4<sub>2</sub>nm</i>		PTN\$P4C666\$I2A666	146	<i>R3</i>	Hexagonal axes	RRN\$P3C000
103	<i>P4cc</i>		PTN\$P4C000\$I2A006	146	<i>R3</i>	Rhombohedral axes	PRN\$P3Q000
104	<i>P4nc</i>		PTN\$P4C000\$I2A666	147	<i>P3<sub>1</sub></i>		PRC\$I3C000
105	<i>P4<sub>2</sub>mc</i>		PTN\$P4C006\$I2A000	148	<i>R3<sub>1</sub></i>	Hexagonal axes	RRC\$I3C000
106	<i>P4<sub>2</sub>bc</i>		PTN\$P4C006\$I2A660	148	<i>R3<sub>2</sub></i>	Rhombohedral axes	PRC\$I3Q000
107	<i>I4mm</i>		ITN\$P4C000\$I2A000	149	<i>P312</i>		PRN\$P3C000\$P2G000
108	<i>I4cm</i>		ITN\$P4C000\$I2A006	150	<i>P321</i>		PRN\$P3C000\$P2F000
109	<i>I4<sub>1</sub>md</i>		ITN\$P4C063\$I2A666	151	<i>P3<sub>1</sub>12</i>		PRN\$P3C004\$P2G000

1.4. SYMMETRY IN RECIPROCAL SPACE

Table A1.4.2.1. *Explicit symbols (cont.)*

No.	Short Hermann–Mauguin symbol	Comments	Explicit symbols	No.	Short Hermann–Mauguin symbol	Comments	Explicit symbols
152	$P3_121$		PRN\$P3C004\$P2F008	192	$P6/mcc$		PHC\$I1A000\$P6C000\$P2F006
153	$P3_212$		PRN\$P3C008\$P2G000	193	$P6_3/mcm$		PHC\$I1A000\$P6C006\$P2F006
154	$P3_221$		PRN\$P3C008\$P2F004	194	$P6_3/mmc$		PHC\$I1A000\$P6C006\$P2F000
155	$R32$	Hexagonal axes	RRN\$P3C000\$P2F000	195	$P23$		PCN\$P3Q000\$P2C000\$P2A000
155	$R32$	Rhombohedral axes	PRN\$P3Q000\$P2E000	196	$F23$		FCN\$P3Q000\$P2C000\$P2A000
156	$P3m1$		PRN\$P3C000\$I2F000	197	$I23$		ICN\$P3Q000\$P2C000\$P2A000
157	$P31m$		PRN\$P3C000\$I2G000	198	$P2_13$		PCN\$P3Q000\$P2C606\$P2A660
158	$P3c1$		PRN\$P3C000\$I2F006	199	$I2_13$		ICN\$P3Q000\$P2C606\$P2A660
159	$P31c$		PRN\$P3C000\$I2G006	200	$Pm\bar{3}$		PCC\$I3Q000\$P2C000\$P2A000
160	$R3m$	Hexagonal axes	RRN\$P3C000\$I2F000	201	$Pn\bar{3}$	Origin 1	PCC\$I3Q666\$P2C000\$P2A000
160	$R3m$	Rhombohedral axes	PRN\$P3Q000\$I2E000	201	$Pn\bar{3}$	Origin 2	PCC\$I3Q000\$P2C660\$P2A066
161	$R3c$	Hexagonal axes	RRN\$P3C000\$I2F006	202	$Fm\bar{3}$		FCC\$I3Q000\$P2C000\$P2A000
161	$R3c$	Rhombohedral axes	PRN\$P3Q000\$I2E666	203	$Fd\bar{3}$	Origin 1	FCC\$I3Q333\$P2C000\$P2A000
162	$P\bar{3}1m$		PRC\$I3C000\$P2G000	203	$Fd\bar{3}$	Origin 2	FCC\$I3Q000\$P2C330\$P2A033
163	$P\bar{3}1c$		PRC\$I3C000\$P2G006	204	$Im\bar{3}$		ICC\$I3Q000\$P2C000\$P2A000
164	$P\bar{3}m1$		PRC\$I3C000\$P2F000	205	$Pa\bar{3}$		PCC\$I3Q000\$P2C606\$P2A660
165	$P\bar{3}c1$		PRC\$I3C000\$P2F006	206	$Ia\bar{3}$		ICC\$I3Q000\$P2C606\$P2A660
166	$R\bar{3}m$	Hexagonal axes	RRC\$I3C000\$P2F000	207	$P432$		PCN\$P3Q000\$P4C000\$P2D000
166	$R\bar{3}m$	Rhombohedral axes	PRC\$I3Q000\$P2E000	208	$P4_232$		PCN\$P3Q000\$P4C666\$P2D666
167	$R\bar{3}c$	Hexagonal axes	RRC\$I3C000\$P2F006	209	$F432$		FCN\$P3Q000\$P4C000\$P2D000
167	$R\bar{3}c$	Rhombohedral axes	PRC\$I3Q000\$P2E666	210	$F4_132$		FCN\$P3Q000\$P4C993\$P2D939
168	$P6$		PHN\$P6C000	211	$I432$		ICN\$P3Q000\$P4C000\$P2D000
169	$P6_1$		PHN\$P6C002	212	$P4_332$		PCN\$P3Q000\$P4C939\$P2D399
170	$P6_5$		PHN\$P6C005	213	$P4_132$		PCN\$P3Q000\$P4C393\$P2D933
171	$P6_2$		PHN\$P6C004	214	$I4_132$		ICN\$P3Q000\$P4C393\$P2D933
172	$P6_4$		PHN\$P6C008	215	$P\bar{4}3m$		PCN\$P3Q000\$I4C000\$I2D000
173	$P6_3$		PHN\$P6C006	216	$F\bar{4}3m$		FCN\$P3Q000\$I4C000\$I2D000
174	$P\bar{6}$		PHN\$I6C000	217	$I\bar{4}3m$		ICN\$P3Q000\$I4C000\$I2D000
175	$P6/m$		PHC\$I1A000\$P6C000	218	$P\bar{4}3n$		PCN\$P3Q000\$I4C666\$I2D666
176	$P6_3/m$		PHC\$I1A000\$P6C006	219	$F\bar{4}3c$		FCN\$P3Q000\$I4C666\$I2D666
177	$P622$		PHN\$P6C000\$P2F000	220	$I\bar{4}3d$		ICN\$P3Q000\$I4C939\$I2D399
178	$P6_122$		PHN\$P6C002\$P2F000	221	$Pm\bar{3}m$		PCC\$I3Q000\$P4C000\$P2D000
179	$P6_522$		PHN\$P6C005\$P2F000	222	$Pn\bar{3}n$	Origin 1	PCC\$I3Q666\$P4C000\$P2D000
180	$P6_222$		PHN\$P6C004\$P2F000	222	$Pn\bar{3}n$	Origin 2	PCC\$I3Q000\$P4C600\$P2D006
181	$P6_422$		PHN\$P6C008\$P2F000	223	$Pm\bar{3}n$		PCC\$I3Q000\$P4C666\$P2D666
182	$P6_322$		PHN\$P6C006\$P2F000	224	$Pn\bar{3}m$	Origin 1	PCC\$I3Q666\$P4C666\$P2D666
183	$P6mm$		PHN\$P6C000\$I2F000	224	$Pn\bar{3}m$	Origin 2	PCC\$I3Q000\$P4C066\$P2D660
184	$P6cc$		PHN\$P6C000\$I2F006	225	$Fm\bar{3}m$		FCC\$I3Q000\$P4C000\$P2D000
185	$P6_3cm$		PHN\$P6C006\$I2F006	226	$Fm\bar{3}c$		FCC\$I3Q000\$P4C666\$P2D666
186	$P6_3mc$		PHN\$P6C006\$I2F000	227	$Fd\bar{3}m$	Origin 1	FCC\$I3Q333\$P4C993\$P2D939
187	$P\bar{6}m2$		PHN\$I6C000\$P2G000	227	$Fd\bar{3}m$	Origin 2	FCC\$I3Q000\$P4C693\$P2D936
188	$P\bar{6}c2$		PHN\$I6C006\$P2G000	228	$Fd\bar{3}c$	Origin 1	FCC\$I3Q999\$P4C993\$P2D939
189	$P\bar{6}2m$		PHN\$I6C000\$P2F000	228	$Fd\bar{3}c$	Origin 2	FCC\$I3Q000\$P4C093\$P2D930
190	$P\bar{6}2c$		PHN\$I6C006\$P2F000	229	$Im\bar{3}m$		ICC\$I3Q000\$P4C000\$P2D000
191	$P6/mmm$		PHC\$I1A000\$P6C000\$P2F000	230	$Ia\bar{3}d$		ICC\$I3Q000\$P4C393\$P2D933

# 1. GENERAL RELATIONSHIPS AND TECHNIQUES

Table A1.4.2.2. *Lattice symbol L*

The lattice symbol L implies Seitz matrices for the lattice translations. For noncentrosymmetric lattices the rotation parts of the Seitz matrices are for  $I$  (see Table A1.4.2.4). For centrosymmetric lattices the rotation parts are  $I$  and  $-I$ . The translation parts in the fourth columns of the Seitz matrices are listed in the last column of the table. The total number of matrices implied by each symbol is given by **nS**.

Noncentrosymmetric		Centrosymmetric		Implied lattice translation(s)
Symbol	nS	Symbol	nS	
P	1	-P	2	0, 0, 0
A	2	-A	4	0, 0, 0    0, $\frac{1}{2}$ , $\frac{1}{2}$
B	2	-B	4	0, 0, 0 $\frac{1}{2}$ , 0, $\frac{1}{2}$
C	2	-C	4	0, 0, 0 $\frac{1}{2}$ , $\frac{1}{2}$ , 0
I	2	-I	4	0, 0, 0 $\frac{1}{2}$ , $\frac{1}{2}$ , $\frac{1}{2}$
R	3	-R	6	0, 0, 0 $\frac{2}{3}$ , $\frac{1}{3}$ , $\frac{1}{3}$ $\frac{1}{3}$ , $\frac{2}{3}$ , $\frac{2}{3}$
H	3	-H	6	0, 0, 0 $\frac{2}{3}$ , $\frac{1}{3}$ , 0 $\frac{1}{3}$ , $\frac{2}{3}$ , 0
F	4	-F	8	0, 0, 0    0, $\frac{1}{2}$ , $\frac{1}{2}$ $\frac{1}{2}$ , 0, $\frac{1}{2}$ $\frac{1}{2}$ , $\frac{1}{2}$ , 0

$$\begin{aligned} \left[ \begin{pmatrix} 0 & 0 & \bar{1} \\ \bar{1} & 0 & 0 \\ 0 & \bar{1} & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \right] &= \begin{pmatrix} \bar{z} \\ \bar{x} \\ \bar{y} \end{pmatrix}, \\ \left[ \begin{pmatrix} 0 & \bar{1} & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} + \begin{pmatrix} \frac{1}{4} \\ \frac{3}{4} \\ \frac{1}{4} \end{pmatrix} \right] &= \begin{pmatrix} \frac{1}{4} - y \\ \frac{3}{4} + x \\ \frac{1}{4} + z \end{pmatrix}, \\ \left[ \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & \bar{1} \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} + \begin{pmatrix} \frac{3}{4} \\ \frac{1}{4} \\ \frac{1}{4} \end{pmatrix} \right] &= \begin{pmatrix} \frac{3}{4} + y \\ \frac{1}{4} + x \\ \frac{1}{4} - z \end{pmatrix}. \end{aligned}$$

The corresponding symmetry transformations in reciprocal space, in the notation of Section 1.4.4, are

$$\left[ (hkl) \begin{pmatrix} 0 & 0 & \bar{1} \\ \bar{1} & 0 & 0 \\ 0 & \bar{1} & 0 \end{pmatrix} : -(hkl) \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \right] = [\bar{k}\bar{l}h : 0];$$

similarly,  $[\bar{k}h\bar{l} : -131/4]$  and  $[k\bar{h}\bar{l} : -311/4]$  are obtained from the second and third generator of  $Ia\bar{3}d$ , respectively.

The first column of Table A1.4.2.1 lists the conventional space-group number. The second column shows the conventional short Hermann–Mauguin or international space-group symbol, and the third column, *Comments*, shows the full international space-group symbol *only* for the different settings of the monoclinic space groups that are given in the main space-group tables of *IT A* (1983). Other comments pertain to the choice of the space-group origin – where there are alternatives – and to axial systems. The fourth column shows the explicit space-group symbols described above for each of the settings considered in *IT A* (1983).

### A1.4.2.3. *Hall symbols* (S. R. HALL AND R. W. GROSSE-KUNSTLEVE)

The explicit-origin space-group notation proposed by Hall (1981a) is based on a subset of the symmetry operations, in the form of Seitz matrices, sufficient to uniquely define a space group. The concise unambiguous nature of this notation makes it well suited to handling symmetry in computing and database applications.

Table A1.4.2.7 lists space-group notation in several formats. The first column of Table A1.4.2.7 lists the space-group numbers with axis codes appended to identify the non-standard settings. The second column lists the Hermann–Mauguin symbols in computer-entry format with appended codes to identify the origin and cell choice when there are alternatives. The general forms of the Hall notation are listed in the fourth column and the computer-entry representations of these symbols are listed in the third column. The computer-entry format is the general notation expressed as case-insensitive ASCII characters with the overline (bar) symbol replaced by a minus sign.

The Hall notation has the general form:

$$\mathbf{L}[\mathbf{N}_T^A]_1 \dots [\mathbf{N}_T^A]_p \mathbf{V}. \quad (\text{A1.4.2.4})$$

**L** is the symbol specifying the lattice translational symmetry (see Table A1.4.2.2). The integral translations are implicitly included in the set of generators. If **L** has a leading minus sign, it also specifies an inversion centre at the origin.  $[\mathbf{N}_T^A]_n$  specifies the  $4 \times 4$  Seitz matrix  $\mathbf{S}_n$  of a symmetry element in the minimum set which defines the space-group symmetry (see Tables A1.4.2.3 to A1.4.2.6), and **p** is the number of elements in the set. **V** is a change-of-basis operator needed for less common descriptions of the space-group symmetry.

Table A1.4.2.3. *Translation symbol T*

The symbol T specifies the translation elements of a Seitz matrix. Alphabetical symbols (given in the first column) specify translations along a fixed direction. Numerical symbols (given in the third column) specify translations as a fraction of the rotation order  $|N|$  and in the direction of the implied or explicitly defined axis.

Translation symbol	Translation vector	Subscript symbol	Fractional translation
<i>a</i>	$\frac{1}{2}, 0, 0$	<i>l</i> in $3_1$	$\frac{1}{3}$
<i>b</i>	$0, \frac{1}{2}, 0$	<i>2</i> in $3_2$	$\frac{2}{3}$
<i>c</i>	$0, 0, \frac{1}{2}$	<i>l</i> in $4_1$	$\frac{1}{4}$
<i>n</i>	$\frac{1}{2}, \frac{1}{2}, \frac{1}{2}$	<i>3</i> in $4_3$	$\frac{3}{4}$
<i>u</i>	$\frac{1}{4}, 0, 0$	<i>l</i> in $6_1$	$\frac{1}{6}$
<i>v</i>	$0, \frac{1}{4}, 0$	<i>2</i> in $6_2$	$\frac{1}{3}$
<i>w</i>	$0, 0, \frac{1}{4}$	<i>4</i> in $6_4$	$\frac{2}{3}$
<i>d</i>	$\frac{1}{4}, \frac{1}{4}, \frac{1}{4}$	<i>5</i> in $6_5$	$\frac{5}{6}$

## 1.4. SYMMETRY IN RECIPROCAL SPACE

Table A1.4.2.4. *Rotation matrices for principal axes*

The  $3 \times 3$  matrices for *proper* rotations along the three principal unit-cell directions are given below. The matrices for *improper* rotations ( $-1$ ,  $-2$ ,  $-3$ ,  $-4$  and  $-6$ ) are identical except that the signs of the elements are reversed.

Axis	Symbol A	Rotation order						
		1	2	3	4	6		
<b>a</b>	<i>x</i>	$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \bar{1} & 0 \\ 0 & 0 & \bar{1} \end{pmatrix}$	$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & \bar{1} \\ 0 & 1 & \bar{1} \end{pmatrix}$	$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & \bar{1} \\ 0 & 1 & 0 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & \bar{1} \\ 0 & 1 & 0 \end{pmatrix}$		
		<b>b</b>	<i>y</i>	$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} \bar{1} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \bar{1} \end{pmatrix}$	$\begin{pmatrix} \bar{1} & 0 & 1 \\ 0 & 1 & 0 \\ \bar{1} & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ \bar{1} & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ \bar{1} & 0 & 1 \end{pmatrix}$
				<b>c</b>	<i>z</i>	$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} \bar{1} & 0 & 0 \\ 0 & \bar{1} & 0 \\ 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 & \bar{1} & 0 \\ 1 & \bar{1} & 0 \\ 0 & 0 & 1 \end{pmatrix}$

The matrix symbol  $\mathbf{N}_T^A$  is composed of three parts: **N** is the symbol denoting the  $|\mathbf{N}|$ -fold order of the rotation matrix (see Tables A1.4.2.4, A1.4.2.5 and A1.4.2.6), **T** is a subscript symbol denoting the *translation* vector (see Table A1.4.2.3) and **A** is a superscript symbol denoting the *axis* of rotation.

The computer-entry format of the Hall notation contains the rotation-order symbol **N** as positive integers 1, 2, 3, 4, or 6 for proper rotations and as negative integers  $-1$ ,  $-2$ ,  $-3$ ,  $-4$  or  $-6$  for improper rotations. The **T** translation symbols 1, 2, 3, 4, 5, 6, *a*, *b*, *c*, *n*, *u*, *v*, *w*, *d* are described in Table A1.4.2.3. These translations apply additively [*e.g.* *ad* signifies a  $(\frac{3}{4}, \frac{1}{4}, \frac{1}{4})$  translation]. The **A** axis symbols *x*, *y*, *z* denote rotations about the axes **a**, **b** and **c**, respectively (see Table A1.4.2.4). The axis symbols '' and ' signal rotations about the body-diagonal vectors **a + b** (or alternatively **b + c** or **c + a**) and **a - b** (or alternatively **b - c** or **c - a**) (see Table

A1.4.2.5). The axis symbol \* always refers to a threefold rotation along **a + b + c** (see Table A1.4.2.6).

The change-of-basis operator **V** has the general form  $(v_x, v_y, v_z)$ . The vectors  $v_x$ ,  $v_y$ , and  $v_z$  are specified by

$$\begin{aligned} v_x &= r_{1,1}X + r_{1,2}Y + r_{1,3}Z + \mathbf{t}_1 \\ v_y &= r_{2,1}X + r_{2,2}Y + r_{2,3}Z + \mathbf{t}_2, \\ v_z &= r_{3,1}X + r_{3,2}Y + r_{3,3}Z + \mathbf{t}_3 \end{aligned}$$

where  $r_{i,j}$  and  $\mathbf{t}_i$  are fractions or real numbers. Terms in which  $r_{i,j}$  or  $\mathbf{t}_i$  are zero need not be specified. The  $4 \times 4$  change-of-basis matrix operator **V** is defined as

$$\mathbf{V} = \begin{pmatrix} r_{1,1} & r_{1,2} & r_{1,3} & \mathbf{t}_1 \\ r_{2,1} & r_{2,2} & r_{2,3} & \mathbf{t}_2 \\ r_{3,1} & r_{3,2} & r_{3,3} & \mathbf{t}_3 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

Table A1.4.2.5. *Rotation matrices for face-diagonal axes*

The symbols for face-diagonal twofold rotations are  $2'$  and  $2''$ . The face-diagonal axis direction is determined by the axis of the preceding rotation  $\mathbf{N}^x$ ,  $\mathbf{N}^y$  or  $\mathbf{N}^z$ . Note that the single prime ' is the default and may be omitted.

Preceding rotation	Rotation	Axis	Matrix
$\mathbf{N}^x$	$2'$	<b>b - c</b>	$\begin{pmatrix} \bar{1} & 0 & 0 \\ 0 & 0 & \bar{1} \\ 0 & \bar{1} & 0 \end{pmatrix}$
	$2''$	<b>b + c</b>	$\begin{pmatrix} \bar{1} & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$
$\mathbf{N}^y$	$2'$	<b>a - c</b>	$\begin{pmatrix} 0 & 0 & \bar{1} \\ 0 & \bar{1} & 0 \\ \bar{1} & 0 & 0 \end{pmatrix}$
	$2''$	<b>a + c</b>	$\begin{pmatrix} 0 & 0 & 1 \\ 0 & \bar{1} & 0 \\ 1 & 0 & 0 \end{pmatrix}$
$\mathbf{N}^z$	$2'$	<b>a - b</b>	$\begin{pmatrix} 0 & \bar{1} & 0 \\ \bar{1} & 0 & 0 \\ 0 & 0 & \bar{1} \end{pmatrix}$
	$2''$	<b>a + b</b>	$\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & \bar{1} \end{pmatrix}$

The transformed symmetry operations are derived from the specified Seitz matrices  $\mathbf{S}_n$  as

$$\mathbf{S}'_n = \mathbf{V} \cdot \mathbf{S}_n \cdot \mathbf{V}^{-1}$$

and from the integral translations  $\mathbf{t}(1, 0, 0)$ ,  $\mathbf{t}(0, 1, 0)$  and  $\mathbf{t}(0, 0, 1)$  as

$$(\mathbf{t}'_n, \mathbf{1})^T = \mathbf{V} \cdot (\mathbf{t}_n, \mathbf{1})^T.$$

A shorthand form of **V** may be used when the change-of-basis operator only translates the origin of the basis system. In this form  $v_x$ ,  $v_y$  and  $v_z$  are specified simply as shifts in twelfths, implying the matrix operator

Table A1.4.2.6. *Rotation matrix for the body-diagonal axis*

The symbol for the threefold rotation in the **a + b + c** direction is  $3^*$ . Note that for cubic space groups the body-diagonal axis is implied and the asterisk \* may be omitted.

Axis	Rotation	Matrix
<b>a + b + c</b>	$3^*$	$\begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$

## 1. GENERAL RELATIONSHIPS AND TECHNIQUES

$$\mathbf{V} = \begin{pmatrix} 1 & 0 & 0 & v_x/12 \\ 0 & 1 & 0 & v_y/12 \\ 0 & 0 & 1 & v_z/12 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

In the shorthand form of  $\mathbf{V}$ , the commas separating the vectors may be omitted.

### A1.4.2.3.1. Default axes

For most symbols the rotation axes applicable to each  $\mathbf{N}$  are implied and an explicit axis symbol  $\mathbf{A}$  is not needed. The rules for *default* axis directions are:

- (i) the *first* rotation or roto-inversion has an axis direction of  $\mathbf{c}$ ;
- (ii) the *second* rotation (if  $|\mathbf{N}|$  is 2) has an axis direction of  $\mathbf{a}$  if preceded by an  $|\mathbf{N}|$  of 2 or 4,  $\mathbf{a}-\mathbf{b}$  if preceded by an  $|\mathbf{N}|$  of 3 or 6;
- (iii) the *third* rotation (if  $|\mathbf{N}|$  is 3) has an axis direction of  $\mathbf{a} + \mathbf{b} + \mathbf{c}$ .

### A1.4.2.3.2. Example matrices

The following examples show how the notation expands to Seitz matrices.

The notation  $\bar{2}_c^x$  represents an improper twofold rotation along  $\mathbf{a}$  and a  $\mathbf{c}/2$  translation:

$$-2xc = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \frac{1}{2} \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

The notation  $3^*$  represents a threefold rotation along  $\mathbf{a} + \mathbf{b} + \mathbf{c}$ :

$$3^* = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

The notation  $4_{vw}$  represents a fourfold rotation along  $\mathbf{c}$  (implied) and translation of  $\mathbf{b}/4$  and  $\mathbf{c}/4$ :

$$4vw = \begin{pmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & \frac{1}{4} \\ 0 & 0 & 1 & \frac{1}{4} \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

The notation  $6_1 2 (0 0 -1)$  represents a  $6_1$  screw along  $\mathbf{c}$ , a twofold rotation along  $\mathbf{a} - \mathbf{b}$  and an origin shift of  $-\mathbf{c}/12$ . Note that the  $6_1$  matrix is unchanged by the shifted origin whereas the 2 matrix is changed by  $-\mathbf{c}/6$ .

$6_1 2 (0 0 -1)$

$$= \begin{pmatrix} 1 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & \frac{1}{6} \\ 0 & 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & \frac{5}{6} \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

The change-of-basis vector  $(0 0 -1)$  could also be entered as  $(x, y, z - 1/12)$ .

The *reverse setting of the R-centred lattice* (hexagonal axes) is specified using a change-of-basis transformation applied to the standard *obverse setting* (see Table A1.4.2.2). The obverse Seitz matrices are

$$R 3 = \begin{pmatrix} 1 & 0 & 0 & \frac{1}{3} \\ 0 & 1 & 0 & \frac{2}{3} \\ 0 & 0 & 1 & \frac{2}{3} \\ 0 & 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 0 & \frac{2}{3} \\ 0 & 1 & 0 & \frac{1}{3} \\ 0 & 0 & 1 & \frac{1}{3} \\ 0 & 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & -1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

The reverse-setting Seitz matrices are

$$R 3 (-x, -y, z) = \begin{pmatrix} 1 & 0 & 0 & \frac{1}{3} \\ 0 & 1 & 0 & \frac{2}{3} \\ 0 & 0 & 1 & \frac{1}{3} \\ 0 & 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 0 & \frac{2}{3} \\ 0 & 1 & 0 & \frac{1}{3} \\ 0 & 0 & 1 & \frac{2}{3} \\ 0 & 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & -1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

The conventional primitive hexagonal lattice may be transformed to a *C-centred orthohexagonal setting* using the change-of-basis operator

$$P 6 (x - 1/2y, 1/2y, z) = \begin{pmatrix} \frac{1}{2} & -\frac{3}{2} & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

In this case the lattice translation for the *C* centring is obtained by transforming the integral translation  $t(0, 1, 0)$ :

$$\mathbf{V} \cdot (0 \ 1 \ 0 \ 1)^T = \begin{pmatrix} 1 & -\frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -\frac{1}{2} & \frac{1}{2} & 0 & 1 \end{pmatrix}^T.$$

The standard setting of an *I*-centred tetragonal space group may be transformed to a primitive setting using the change-of-basis operator

$$I 4 (y + z, x + z, x + y) = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

Note that in the primitive setting, the fourfold axis is along  $\mathbf{a} + \mathbf{b}$ .



### 1.4. SYMMETRY IN RECIPROCAL SPACE

Table A1.4.2.7. *Hall symbols*

The first column, n:c, lists the space-group numbers and axis codes separated by a colon. The second column lists the Hermann–Mauguin symbols in computer-entry format. The third column lists the Hall symbols in computer-entry format and the fourth column lists the Hall symbols as described in Tables A1.4.2.2–A1.4.2.6.

n:c	H–M entry	Hall entry	Hall symbol	n:c	H–M entry	Hall entry	Hall symbol
1	P 1	p 1	P 1	9:-a2	B n 1 1	b -2xab	$B \bar{2}^x_{ab}$
2	P -1	-p 1	$\bar{P} 1$	9:-a3	I b 1 1	i -2xb	$I \bar{2}^x_b$
3:b	P 1 2 1	p 2y	P 2 <sup>y</sup>	10:b	P 1 2/m 1	-p 2y	$\bar{P} 2^y$
3:c	P 1 1 2	p 2	P 2	10:c	P 1 1 2/m	-p 2	$\bar{P} 2$
3:a	P 2 1 1	p 2x	P 2 <sup>x</sup>	10:a	P 2/m 1 1	-p 2x	$\bar{P} 2^x$
4:b	P 1 21 1	p 2yb	P 2 <sup>y</sup> <sub>b</sub>	11:b	P 1 21/m 1	-p 2yb	$\bar{P} 2^y_b$
4:c	P 1 1 21	p 2c	P 2 <sub>c</sub>	11:c	P 1 1 21/m	-p 2c	$\bar{P} 2_c$
4:a	P 21 1 1	p 2xa	P 2 <sup>x</sup> <sub>a</sub>	11:a	P 21/m 1 1	-p 2xa	$\bar{P} 2^x_a$
5:b1	C 1 2 1	c 2y	C 2 <sup>y</sup>	12:b1	C 1 2/m 1	-c 2y	$\bar{C} 2^y$
5:b2	A 1 2 1	a 2y	A 2 <sup>y</sup>	12:b2	A 1 2/m 1	-a 2y	$\bar{A} 2^y$
5:b3	I 1 2 1	i 2y	I 2 <sup>y</sup>	12:b3	I 1 2/m 1	-i 2y	$\bar{I} 2^y$
5:c1	A 1 1 2	a 2	A 2	12:c1	A 1 1 2/m	-a 2	$\bar{A} 2$
5:c2	B 1 1 2	b 2	B 2	12:c2	B 1 1 2/m	-b 2	$\bar{B} 2$
5:c3	I 1 1 2	i 2	I 2	12:c3	I 1 1 2/m	-i 2	$\bar{I} 2$
5:a1	B 2 1 1	b 2x	B 2 <sup>x</sup>	12:a1	B 2/m 1 1	-b 2x	$\bar{B} 2^x$
5:a2	C 2 1 1	c 2x	C 2 <sup>x</sup>	12:a2	C 2/m 1 1	-c 2x	$\bar{C} 2^x$
5:a3	I 2 1 1	i 2x	I 2 <sup>x</sup>	12:a3	I 2/m 1 1	-i 2x	$\bar{I} 2^x$
6:b	P 1 m 1	p -2y	P $\bar{2}^y$	13:b1	P 1 2/c 1	-p 2yc	$\bar{P} 2^y_c$
6:c	P 1 1 m	p -2	P $\bar{2}$	13:b2	P 1 2/n 1	-p 2yac	$\bar{P} 2^y_{ac}$
6:a	P m 1 1	p -2x	P $\bar{2}^x$	13:b3	P 1 2/a 1	-p 2ya	$\bar{P} 2^y_a$
7:b1	P 1 c 1	p -2yc	P $\bar{2}^y_c$	13:c1	P 1 1 2/a	-p 2a	$\bar{P} 2_a$
7:b2	P 1 n 1	p -2yac	P $\bar{2}^y_{ac}$	13:c2	P 1 1 2/n	-p 2ab	$\bar{P} 2_{ab}$
7:b3	P 1 a 1	p -2ya	P $\bar{2}^y_a$	13:c3	P 1 1 2/b	-p 2b	$\bar{P} 2_b$
7:c1	P 1 1 a	p -2a	P $\bar{2}_a$	13:a1	P 2/b 1 1	-p 2xb	$\bar{P} 2^x_b$
7:c2	P 1 1 n	p -2ab	P $\bar{2}_{ab}$	13:a2	P 2/n 1 1	-p 2xbc	$\bar{P} 2^x_{bc}$
7:c3	P 1 1 b	p -2b	P $\bar{2}_b$	13:a3	P 2/c 1 1	-p 2xc	$\bar{P} 2^x_c$
7:a1	P b 1 1	p -2xb	P $\bar{2}^x_b$	14:b1	P 1 21/c 1	-p 2ybc	$\bar{P} 2^y_{bc}$
7:a2	P n 1 1	p -2xbc	P $\bar{2}^x_{bc}$	14:b2	P 1 21/n 1	-p 2yn	$\bar{P} 2^y_n$
7:a3	P c 1 1	p -2xc	P $\bar{2}^x_c$	14:b3	P 1 21/a 1	-p 2yab	$\bar{P} 2^y_{ab}$
8:b1	C 1 m 1	c -2y	C $\bar{2}^y$	14:c1	P 1 1 21/a	-p 2ac	$\bar{P} 2_{ac}$
8:b2	A 1 m 1	a -2y	A $\bar{2}^y$	14:c2	P 1 1 21/n	-p 2n	$\bar{P} 2_n$
8:b3	I 1 m 1	i -2y	I $\bar{2}^y$	14:c3	P 1 1 21/b	-p 2bc	$\bar{P} 2_{bc}$
8:c1	A 1 1 m	a -2	A $\bar{2}$	14:a1	P 21/b 1 1	-p 2xab	$\bar{P} 2^x_{ab}$
8:c2	B 1 1 m	b -2	B $\bar{2}$	14:a2	P 21/n 1 1	-p 2xn	$\bar{P} 2^x_n$
8:c3	I 1 1 m	i -2	I $\bar{2}$	14:a3	P 21/c 1 1	-p 2xac	$\bar{P} 2^x_{ac}$
8:a1	B m 1 1	b -2x	B $\bar{2}^x$	15:b1	C 1 2/c 1	-c 2yc	$\bar{C} 2^y_c$
8:a2	C m 1 1	c -2x	C $\bar{2}^x$	15:b2	A 1 2/n 1	-a 2yab	$\bar{A} 2^y_{ab}$
8:a3	I m 1 1	i -2x	I $\bar{2}^x$	15:b3	I 1 2/a 1	-i 2ya	$\bar{I} 2^y_a$
9:b1	C 1 c 1	c -2yc	C $\bar{2}^y_c$	15:-b1	A 1 2/a 1	-a 2ya	$\bar{A} 2^y_a$
9:b2	A 1 n 1	a -2yab	A $\bar{2}^y_{ab}$	15:-b2	C 1 2/n 1	-c 2yac	$\bar{C} 2^y_{ac}$
9:b3	I 1 a 1	i -2ya	I $\bar{2}^y_a$	15:-b3	I 1 2/c 1	-i 2yc	$\bar{I} 2^y_c$
9:-b1	A 1 a 1	a -2ya	A $\bar{2}^y_a$	15:c1	A 1 1 2/a	-a 2a	$\bar{A} 2_a$
9:-b2	C 1 n 1	c -2yac	C $\bar{2}^y_{ac}$	15:c2	B 1 1 2/n	-b 2ab	$\bar{B} 2_{ab}$
9:-b3	I 1 c 1	i -2yc	I $\bar{2}^y_c$	15:c3	I 1 1 2/b	-i 2b	$\bar{I} 2_b$
9:c1	A 1 1 a	a -2a	A $\bar{2}_a$	15:-c1	B 1 1 2/b	-b 2b	$\bar{B} 2_b$
9:c2	B 1 1 n	b -2ab	B $\bar{2}_{ab}$	15:-c2	A 1 1 2/n	-a 2ab	$\bar{A} 2_{ab}$
9:c3	I 1 1 b	i -2b	I $\bar{2}_b$	15:-c3	I 1 1 2/a	-i 2a	$\bar{I} 2_a$
9:-c1	B 1 1 b	b -2b	B $\bar{2}_b$	15:a1	B 2/b 1 1	-b 2xb	$\bar{B} 2^x_b$
9:-c2	A 1 1 n	a -2ab	A $\bar{2}_{ab}$	15:a2	C 2/n 1 1	-c 2xac	$\bar{C} 2^x_{ac}$
9:-c3	I 1 1 a	i -2a	I $\bar{2}_a$	15:a3	I 2/c 1 1	-i 2xc	$\bar{I} 2^x_c$
9:a1	B b 1 1	b -2xb	B $\bar{2}^x_b$	15:-a1	C 2/c 1 1	-c 2xc	$\bar{C} 2^x_c$
9:a2	C n 1 1	c -2xac	C $\bar{2}^x_{ac}$	15:-a2	B 2/n 1 1	-b 2xab	$\bar{B} 2^x_{ab}$
9:a3	I c 1 1	i -2xc	I $\bar{2}^x_c$	15:-a3	I 2/b 1 1	-i 2xb	$\bar{I} 2^x_b$
9:-a1	C c 1 1	c -2xc	C $\bar{2}^x_c$	16	P 2 2 2	p 2 2	P 2 2

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Table A1.4.2.7. Hall symbols (cont.)

n:c	H-M entry	Hall entry	Hall symbol	n:c	H-M entry	Hall entry	Hall symbol
17	P 2 2 2 1	p 2c 2	P 2 <sub>c</sub> 2	33:ba-c	P b n 2 1	p 2c -2ab	P 2 <sub>c</sub> 2 <sub>ab</sub>
17:cab	P 2 1 2 2	p 2a 2a	P 2 <sub>a</sub> 2 <sub>a</sub>	33:cab	P 2 1 n b	p -2bc 2a	P 2 <sub>bc</sub> 2 <sub>a</sub>
17:bca	P 2 2 1 2	p 2 2b	P 2 2 <sub>b</sub>	33:-cba	P 2 1 c n	p -2n 2a	P 2 <sub>n</sub> 2 <sub>a</sub>
18	P 2 1 2 1 2	p 2 2ab	P 2 2 <sub>ab</sub>	33:bca	P c 2 1 n	p -2n -2ac	P 2 <sub>n</sub> 2 <sub>ac</sub>
18:cab	P 2 2 1 2 1	p 2bc 2	P 2 <sub>bc</sub> 2	33:a-cb	P n 2 1 a	p -2ac -2n	P 2 <sub>ac</sub> 2 <sub>n</sub>
18:bca	P 2 1 2 2 1	p 2ac 2ac	P 2 <sub>ac</sub> 2 <sub>ac</sub>	34	P n n 2	p 2 -2n	P 2 2 <sub>n</sub>
19	P 2 1 2 1 2 1	p 2ac 2ab	P 2 <sub>ac</sub> 2 <sub>ab</sub>	34:cab	P 2 n n	p -2n 2	P 2 <sub>n</sub> 2
20	C 2 2 2 1	c 2c 2	C 2 <sub>c</sub> 2	34:bca	P n 2 n	p -2n -2n	P 2 <sub>n</sub> 2 <sub>n</sub>
20:cab	A 2 1 2 2	a 2a 2a	A 2 <sub>a</sub> 2 <sub>a</sub>	35	C m m 2	c 2 -2	C 2 2
20:bca	B 2 2 1 2	b 2 2b	B 2 2 <sub>b</sub>	35:cab	A 2 m m	a -2 2	A 2 2
21	C 2 2 2 2	c 2 2	C 2 2	35:bca	B m 2 m	b -2 -2	B 2 2
21:cab	A 2 2 2 2	a 2 2	A 2 2	36	C m c 2 1	c 2c -2	C 2 <sub>c</sub> 2
21:bca	B 2 2 2 2	b 2 2	B 2 2	36:ba-c	C c m 2 1	c 2c -2c	C 2 <sub>c</sub> 2 <sub>c</sub>
22	F 2 2 2 2	f 2 2	F 2 2	36:cab	A 2 1 m a	a -2a 2a	A 2 <sub>a</sub> 2 <sub>a</sub>
23	I 2 2 2 2	i 2 2	I 2 2	36:-cba	A 2 1 a m	a -2 2a	A 2 2 <sub>a</sub>
24	I 2 1 2 1 2 1	i 2b 2c	I 2 <sub>b</sub> 2 <sub>c</sub>	36:bca	B b 2 1 m	b -2 -2b	B 2 2 <sub>b</sub>
25	P m m 2	p 2 -2	P 2 2	36:a-cb	B m 2 1 b	b -2b -2	B 2 <sub>b</sub> 2
25:cab	P 2 m m	p -2 2	P 2 2	37	C c c 2	c 2 -2c	C 2 2 <sub>c</sub>
25:bca	P m 2 m	p -2 -2	P 2 2	37:cab	A 2 a a	a -2a 2	A 2 <sub>a</sub> 2
26	P m c 2 1	p 2c -2	P 2 <sub>c</sub> 2	37:bca	B b 2 b	b -2b -2b	B 2 <sub>b</sub> 2 <sub>b</sub>
26:ba-c	P c m 2 1	p 2c -2c	P 2 <sub>c</sub> 2 <sub>c</sub>	38	A m m 2	a 2 -2	A 2 2
26:cab	P 2 1 m a	p -2a 2a	P 2 <sub>a</sub> 2 <sub>a</sub>	38:ba-c	B m m 2	b 2 -2	B 2 2
26:-cba	P 2 1 a m	p -2 2a	P 2 2 <sub>a</sub>	38:cab	B 2 m m	b -2 2	B 2 2
26:bca	P b 2 1 m	p -2 -2b	P 2 2 <sub>b</sub>	38:-cba	C 2 m m	c -2 2	C 2 2
26:a-cb	P m 2 1 b	p -2b -2	P 2 <sub>b</sub> 2	38:bca	C m 2 m	c -2 -2	C 2 2
27	P c c 2	p 2 -2c	P 2 2 <sub>c</sub>	38:a-cb	A m 2 m	a -2 -2	A 2 2
27:cab	P 2 a a	p -2a 2	P 2 <sub>a</sub> 2	39	A b m 2	a 2 -2b	A 2 2 <sub>b</sub>
27:bca	P b 2 b	p -2b -2b	P 2 <sub>b</sub> 2 <sub>b</sub>	39:ba-c	B m a 2	b 2 -2a	B 2 2 <sub>a</sub>
28	P m a 2	p 2 -2a	P 2 2 <sub>a</sub>	39:cab	B 2 c m	b -2a 2	B 2 <sub>a</sub> 2
28:ba-c	P b m 2	p 2 -2b	P 2 2 <sub>b</sub>	39:-cba	C 2 m b	c -2a 2	C 2 <sub>a</sub> 2
28:cab	P 2 m b	p -2b 2	P 2 <sub>b</sub> 2	39:bca	C m 2 a	c -2a -2a	C 2 <sub>a</sub> 2 <sub>a</sub>
28:-cba	P 2 c m	p -2c 2	P 2 <sub>c</sub> 2	39:a-cb	A c 2 m	a -2b -2b	A 2 <sub>b</sub> 2 <sub>b</sub>
28:bca	P c 2 m	p -2c -2c	P 2 <sub>c</sub> 2 <sub>c</sub>	40	A m a 2	a 2 -2a	A 2 2 <sub>a</sub>
28:a-cb	P m 2 a	p -2a -2a	P 2 <sub>a</sub> 2 <sub>a</sub>	40:ba-c	B b m 2	b 2 -2b	B 2 2 <sub>b</sub>
29	P c a 2 1	p 2c -2ac	P 2 <sub>c</sub> 2 <sub>ac</sub>	40:cab	B 2 m b	b -2b 2	B 2 <sub>b</sub> 2
29:ba-c	P b c 2 1	p 2c -2b	P 2 <sub>c</sub> 2 <sub>b</sub>	40:-cba	C 2 c m	c -2c 2	C 2 <sub>c</sub> 2
29:cab	P 2 1 a b	p -2b 2a	P 2 <sub>b</sub> 2 <sub>a</sub>	40:bca	C c 2 m	c -2c -2c	C 2 <sub>c</sub> 2 <sub>c</sub>
29:-cba	P 2 1 c a	p -2ac 2a	P 2 <sub>ac</sub> 2 <sub>a</sub>	40:a-cb	A m 2 a	a -2a -2a	A 2 <sub>a</sub> 2 <sub>a</sub>
29:bca	P c 2 1 b	p -2bc -2c	P 2 <sub>bc</sub> 2 <sub>c</sub>	41	A b a 2	a 2 -2ab	A 2 2 <sub>ab</sub>
29:a-cb	P b 2 1 a	p -2a -2ab	P 2 <sub>a</sub> 2 <sub>ab</sub>	41:ba-c	B b a 2	b 2 -2ab	B 2 2 <sub>ab</sub>
30	P n c 2	p 2 -2bc	P 2 2 <sub>bc</sub>	41:cab	B 2 c b	b -2ab 2	B 2 <sub>ab</sub> 2
30:ba-c	P c n 2	p 2 -2ac	P 2 2 <sub>ac</sub>	41:-cba	C 2 c b	c -2ac 2	C 2 <sub>ac</sub> 2
30:cab	P 2 n a	p -2ac 2	P 2 <sub>ac</sub> 2	41:bca	C c 2 a	c -2ac -2ac	C 2 <sub>ac</sub> 2 <sub>ac</sub>
30:-cba	P 2 a n	p -2ab 2	P 2 <sub>ab</sub> 2	41:a-cb	A c 2 a	a -2ab -2ab	A 2 <sub>ab</sub> 2 <sub>ab</sub>
30:bca	P b 2 n	p -2ab -2ab	P 2 <sub>ab</sub> 2 <sub>ab</sub>	42	F m m 2	f 2 -2	F 2 2
30:a-cb	P n 2 b	p -2bc -2bc	P 2 <sub>bc</sub> 2 <sub>bc</sub>	42:cab	F 2 m m	f -2 2	F 2 2
31	P m n 2 1	p 2ac -2	P 2 <sub>ac</sub> 2	42:bca	F m 2 m	f -2 -2	F 2 2
31:ba-c	P n m 2 1	p 2bc -2bc	P 2 <sub>bc</sub> 2 <sub>bc</sub>	43	F d d 2	f 2 -2d	F 2 2 <sub>d</sub>
31:cab	P 2 1 m n	p -2ab 2ab	P 2 <sub>ab</sub> 2 <sub>ab</sub>	43:cab	F 2 d d	f -2d 2	F 2 <sub>d</sub> 2
31:-cba	P 2 1 n m	p -2 2ac	P 2 2 <sub>ac</sub>	43:bca	F d 2 d	f -2d -2d	F 2 <sub>d</sub> 2 <sub>d</sub>
31:bca	P n 2 1 m	p -2 -2bc	P 2 2 <sub>bc</sub>	44	I m m 2	i 2 -2	I 2 2
31:a-cb	P m 2 1 n	p -2ab -2	P 2 <sub>ab</sub> 2	44:cab	I 2 m m	i -2 2	I 2 2
32	P b a 2	p 2 -2ab	P 2 2 <sub>ab</sub>	44:bca	I m 2 m	i -2 -2	I 2 2
32:cab	P 2 c b	p -2bc 2	P 2 <sub>bc</sub> 2	45	I b a 2	i 2 -2c	I 2 2 <sub>c</sub>
32:bca	P c 2 a	p -2ac -2ac	P 2 <sub>ac</sub> 2 <sub>ac</sub>	45:cab	I 2 c b	i -2a 2	I 2 <sub>a</sub> 2
33	P n a 2 1	p 2c -2n	P 2 <sub>c</sub> 2 <sub>n</sub>	45:bca	I c 2 a	i -2b -2b	I 2 <sub>b</sub> 2 <sub>b</sub>

1.4. SYMMETRY IN RECIPROCAL SPACE

Table A1.4.2.7. Hall symbols (cont.)

n:c	H-M entry	Hall entry	Hall symbol	n:c	H-M entry	Hall entry	Hall symbol
46	I m a 2	i 2 -2a	I 2 $\bar{2}_a$	58:bca	P n m n	-p 2n 2n	$\bar{P} 2_n 2_n$
46:ba-c	I b m 2	i 2 -2b	I 2 $\bar{2}_b$	59:1	P m m n:1	p 2 2ab -1ab	P 2 $2_{ab} \bar{1}_{ab}$
46:cab	I 2 m b	i -2b 2	I $\bar{2}_b 2$	59:2	P m m n:2	-p 2ab 2a	$\bar{P} 2_{ab} 2_a$
46:-cba	I 2 c m	i -2c 2	I $\bar{2}_c 2$	59:1cab	P n m m:1	p 2bc 2 -1bc	P $2_{bc} 2 \bar{1}_{bc}$
46:bca	I c 2 m	i -2c -2c	I $\bar{2}_c \bar{2}_c$	59:2cab	P n m m:2	-p 2c 2bc	$\bar{P} 2_c 2_{bc}$
46:a-cb	I m 2 a	i -2a -2a	I $\bar{2}_a \bar{2}_a$	59:1bca	P m n m:1	p 2ac 2ac -1ac	P $2_{ac} 2_{ac} \bar{1}_{ac}$
47	P m m m	-p 2 2	$\bar{P} 2 2$	59:2bca	P m n m:2	-p 2c 2a	$\bar{P} 2_c 2_a$
48:1	P n n n:1	p 2 2 -1n	P 2 2 $\bar{1}_n$	60	P b c n	-p 2n 2ab	$\bar{P} 2_n 2_{ab}$
48:2	P n n n:2	-p 2ab 2bc	$\bar{P} 2_{ab} 2_{bc}$	60:ba-c	P c a n	-p 2n 2c	$\bar{P} 2_n 2_c$
49	P c c m	-p 2 2c	$\bar{P} 2 2_c$	60:cab	P n c a	-p 2a 2n	$\bar{P} 2_a 2_n$
49:cab	P m a a	-p 2a 2	$\bar{P} 2_a 2$	60:-cba	P n a b	-p 2bc 2n	$\bar{P} 2_{bc} 2_n$
49:bca	P b m b	-p 2b 2b	$\bar{P} 2_b 2_b$	60:bca	P b n a	-p 2ac 2b	$\bar{P} 2_{ac} 2_b$
50:1	P b a n:1	p 2 2 -1ab	P 2 2 $\bar{1}_{ab}$	60:a-cb	P c n b	-p 2b 2ac	$\bar{P} 2_b 2_{ac}$
50:2	P b a n:2	-p 2ab 2b	$\bar{P} 2_{ab} 2_b$	61	P b c a	-p 2ac 2ab	$\bar{P} 2_{ac} 2_{ab}$
50:1cab	P n c b:1	p 2 2 -1bc	P 2 2 $\bar{1}_{bc}$	61:ba-c	P c a b	-p 2bc 2ac	$\bar{P} 2_{bc} 2_{ac}$
50:2cab	P n c b:2	-p 2b 2bc	$\bar{P} 2_b 2_{bc}$	62	P n m a	-p 2ac 2n	$\bar{P} 2_{ac} 2_n$
50:1bca	P c n a:1	p 2 2 -1ac	P 2 2 $\bar{1}_{ac}$	62:ba-c	P m n b	-p 2bc 2a	$\bar{P} 2_{bc} 2_a$
50:2bca	P c n a:2	-p 2a 2c	$\bar{P} 2_a 2_c$	62:cab	P b n m	-p 2c 2ab	$\bar{P} 2_c 2_{ab}$
51	P m m a	-p 2a 2a	$\bar{P} 2_a 2_a$	62:-cba	P c m n	-p 2n 2ac	$\bar{P} 2_n 2_{ac}$
51:ba-c	P m m b	-p 2b 2	$\bar{P} 2_b 2$	62:bca	P m c n	-p 2n 2a	$\bar{P} 2_n 2_a$
51:cab	P b m m	-p 2 2b	$\bar{P} 2 2_b$	62:a-cb	P n a m	-p 2c 2n	$\bar{P} 2_c 2_n$
51:-cba	P c m m	-p 2c 2c	$\bar{P} 2_c 2_c$	63	C m c m	-c 2c 2	$\bar{C} 2_c 2$
51:bca	P m c m	-p 2c 2	$\bar{P} 2_c 2$	63:ba-c	C c m m	-c 2c 2c	$\bar{C} 2_c 2_c$
51:a-cb	P m a m	-p 2 2a	$\bar{P} 2 2_a$	63:cab	A m m a	-a 2a 2a	$\bar{A} 2_a 2_a$
52	P n n a	-p 2a 2bc	$\bar{P} 2_a 2_{bc}$	63:-cba	A m a m	-a 2 2a	$\bar{A} 2 2_a$
52:ba-c	P n n b	-p 2b 2n	$\bar{P} 2_b 2_n$	63:bca	B b m m	-b 2 2b	$\bar{B} 2 2_b$
52:cab	P b n n	-p 2n 2b	$\bar{P} 2_n 2_b$	63:a-cb	B m m b	-b 2b 2	$\bar{B} 2_b 2$
52:-cba	P c n n	-p 2ab 2c	$\bar{P} 2_{ab} 2_c$	64	C m c a	-c 2ac 2	$\bar{C} 2_{ac} 2$
52:bca	P n c n	-p 2ab 2n	$\bar{P} 2_{ab} 2_n$	64:ba-c	C c m b	-c 2ac 2ac	$\bar{C} 2_{ac} 2_{ac}$
52:a-cb	P n a n	-p 2n 2bc	$\bar{P} 2_n 2_{bc}$	64:cab	A b m a	-a 2ab 2ab	$\bar{A} 2_{ab} 2_{ab}$
53	P m n a	-p 2ac 2	$\bar{P} 2_{ac} 2$	64:-cba	A c a m	-a 2 2ab	$\bar{A} 2 2_{ab}$
53:ba-c	P n m b	-p 2bc 2bc	$\bar{P} 2_{bc} 2_{bc}$	64:bca	B b c m	-b 2 2ab	$\bar{B} 2 2_{ab}$
53:cab	P b m n	-p 2ab 2ab	$\bar{P} 2_{ab} 2_{ab}$	64:a-cb	B m a b	-b 2ab 2	$\bar{B} 2_{ab} 2$
53:-cba	P c n m	-p 2 2ac	$\bar{P} 2 2_{ac}$	65	C m m m	-c 2 2	$\bar{C} 2 2$
53:bca	P n c m	-p 2 2bc	$\bar{P} 2 2_{bc}$	65:cab	A m m m	-a 2 2	$\bar{A} 2 2$
53:a-cb	P m a n	-p 2ab 2	$\bar{P} 2_{ab} 2$	65:bca	B m m m	-b 2 2	$\bar{B} 2 2$
54	P c c a	-p 2a 2ac	$\bar{P} 2_a 2_{ac}$	66	C c c m	-c 2 2c	$\bar{C} 2 2_c$
54:ba-c	P c c b	-p 2b 2c	$\bar{P} 2_b 2_c$	66:cab	A m a a	-a 2a 2	$\bar{A} 2_a 2$
54:cab	P b a a	-p 2a 2b	$\bar{P} 2_a 2_b$	66:bca	B b m b	-b 2b 2b	$\bar{B} 2_b 2_b$
54:-cba	P c a a	-p 2ac 2c	$\bar{P} 2_{ac} 2_c$	67	C m m a	-c 2a 2	$\bar{C} 2_a 2$
54:bca	P b c b	-p 2bc 2b	$\bar{P} 2_{bc} 2_b$	67:ba-c	C m m b	-c 2a 2a	$\bar{C} 2_a 2_a$
54:a-cb	P b a b	-p 2b 2ab	$\bar{P} 2_b 2_{ab}$	67:cab	A b m m	-a 2b 2b	$\bar{A} 2_b 2_b$
55	P b a m	-p 2 2ab	$\bar{P} 2 2_{ab}$	67:-cba	A c m m	-a 2 2b	$\bar{A} 2 2_b$
55:cab	P m c b	-p 2bc 2	$\bar{P} 2_{bc} 2$	67:bca	B m c m	-b 2 2a	$\bar{B} 2 2_a$
55:bca	P c m a	-p 2ac 2ac	$\bar{P} 2_{ac} 2_{ac}$	67:a-cb	B m a m	-b 2a 2	$\bar{B} 2_a 2$
56	P c c n	-p 2ab 2ac	$\bar{P} 2_{ab} 2_{ac}$	68:1	C c c a:1	c 2 2 -1ac	C 2 2 $\bar{1}_{ac}$
56:cab	P n a a	-p 2ac 2bc	$\bar{P} 2_{ac} 2_{bc}$	68:2	C c c a:2	-c 2a 2ac	$\bar{C} 2_a 2_{ac}$
56:bca	P b n b	-p 2bc 2ab	$\bar{P} 2_{bc} 2_{ab}$	68:1ba-c	C c c b:1	c 2 2 -1ac	C 2 2 $\bar{1}_{ac}$
57	P b c m	-p 2c 2b	$\bar{P} 2_c 2_b$	68:2ba-c	C c c b:2	-c 2a 2c	$\bar{C} 2_a 2_c$
57:ba-c	P c a m	-p 2c 2ac	$\bar{P} 2_c 2_{ac}$	68:1cab	A b a a:1	a 2 2 -1ab	A 2 2 $\bar{1}_{ab}$
57:cab	P m c a	-p 2ac 2a	$\bar{P} 2_{ac} 2_a$	68:2cab	A b a a:2	-a 2a 2b	$\bar{A} 2_a 2_b$
57:-cba	P m a b	-p 2b 2a	$\bar{P} 2_b 2_a$	68:1-cba	A c a a:1	a 2 2 -1ab	A 2 2 $\bar{1}_{ab}$
57:bca	P b m a	-p 2a 2ab	$\bar{P} 2_a 2_{ab}$	68:2-cba	A c a a:2	-a 2ab 2b	$\bar{A} 2_{ab} 2_b$
57:a-cb	P c m b	-p 2bc 2c	$\bar{P} 2_{bc} 2_c$	68:1bca	B b c b:1	b 2 2 -1ab	B 2 2 $\bar{1}_{ab}$
58	P n n m	-p 2 2n	$\bar{P} 2 2_n$	68:2bca	B b c b:2	-b 2ab 2b	$\bar{B} 2_{ab} 2_b$
58:cab	P m n n	-p 2n 2	$\bar{P} 2_n 2$	68:1a-cb	B b a b:1	b 2 2 -1ab	B 2 2 $\bar{1}_{ab}$

1. GENERAL RELATIONSHIPS AND TECHNIQUES

Table A1.4.2.7. Hall symbols (cont.)

n:c	H-M entry	Hall entry	Hall symbol	n:c	H-M entry	Hall entry	Hall symbol
68:2a-cb	B b a b:2	-b 2b 2ab	$\bar{B} 2_b 2_{ab}$	112	P -4 2 c	p -4 2c	$P \bar{4} 2_c$
69	F m m m	-f 2 2	$\bar{F} 2 2$	113	P -4 21 m	p -4 2ab	$P \bar{4} 2_{ab}$
70:1	F d d d:1	f 2 2 -1d	$F 2 2 \bar{1}_d$	114	P -4 21 c	p -4 2n	$P \bar{4} 2_n$
70:2	F d d d:2	-f 2uv 2vw	$\bar{F} 2_{uv} 2_{vw}$	115	P -4 m 2	p -4 -2	$P \bar{4} \bar{2}$
71	I m m m	-i 2 2	$\bar{I} 2 2$	116	P -4 c 2	p -4 -2c	$P \bar{4} \bar{2}_c$
72	I b a m	-i 2 2c	$\bar{I} 2 2_c$	117	P -4 b 2	p -4 -2ab	$P \bar{4} \bar{2}_{ab}$
72:cab	I m c b	-i 2a 2	$\bar{I} 2_a 2$	118	P -4 n 2	p -4 -2n	$P \bar{4} \bar{2}_n$
72:bca	I c m a	-i 2b 2b	$\bar{I} 2_b 2_b$	119	I -4 m 2	i -4 -2	$I \bar{4} \bar{2}$
73	I b c a	-i 2b 2c	$\bar{I} 2_b 2_c$	120	I -4 c 2	i -4 -2c	$I \bar{4} \bar{2}_c$
73:ba-c	I c a b	-i 2a 2b	$\bar{I} 2_a 2_b$	121	I -4 2 m	i -4 2	$I \bar{4} 2$
74	I m m a	-i 2b 2	$\bar{I} 2_b 2$	122	I -4 2 d	i -4 2bw	$I \bar{4} 2_{bw}$
74:ba-c	I m m b	-i 2a 2a	$\bar{I} 2_a 2_a$	123	P 4/m m m	-p 4 2	$\bar{P} 4 2$
74:cab	I b m m	-i 2c 2c	$\bar{I} 2_c 2_c$	124	P 4/m c c	-p 4 2c	$\bar{P} 4 2_c$
74:-cba	I c m m	-i 2 2b	$\bar{I} 2 2_b$	125:1	P 4/n b m:1	p 4 2 -1ab	$P 4 2 \bar{1}_{ab}$
74:bca	I m c m	-i 2 2a	$\bar{I} 2 2_a$	125:2	P 4/n b m:2	-p 4a 2b	$\bar{P} 4_a 2_b$
74:a-cb	I m a m	-i 2c 2	$\bar{I} 2_c 2$	126:1	P 4/n n c:1	p 4 2 -1n	$P 4 2 \bar{1}_n$
75	P 4	p 4	P 4	126:2	P 4/n n c:2	-p 4a 2bc	$\bar{P} 4_a 2_{bc}$
76	P 41	p 4w	P 4 <sub>w</sub>	127	P 4/m b m	-p 4 2ab	$\bar{P} 4 2_{ab}$
77	P 42	p 4c	P 4 <sub>c</sub>	128	P 4/m n c	-p 4 2n	$\bar{P} 4 2_n$
78	P 43	p 4cw	P 4 <sub>cw</sub>	129:1	P 4/n m m:1	p 4ab 2ab -1ab	$P 4_{ab} 2_{ab} \bar{1}_{ab}$
79	I 4	i 4	I 4	129:2	P 4/n m m:2	-p 4a 2a	$\bar{P} 4_a 2_a$
80	I 41	i 4bw	I 4 <sub>bw</sub>	130:1	P 4/n c c:1	p 4ab 2n -1ab	$P 4_{ab} 2_n \bar{1}_{ab}$
81	P -4	p -4	$P \bar{4}$	130:2	P 4/n c c:2	-p 4a 2ac	$\bar{P} 4_a 2_{ac}$
82	I -4	i -4	$I \bar{4}$	131	P 42/m m c	-p 4c 2	$\bar{P} 4_c 2$
83	P 4/m	-p 4	$\bar{P} 4$	132	P 42/m c m	-p 4c 2c	$\bar{P} 4_c 2_c$
84	P 42/m	-p 4c	$\bar{P} 4_c$	133:1	P 42/n b c:1	p 4n 2c -1n	$P 4_n 2_c \bar{1}_n$
85:1	P 4/n:1	p 4ab -1ab	$P 4_{ab} \bar{1}_{ab}$	133:2	P 42/n b c:2	-p 4ac 2b	$\bar{P} 4_{ac} 2_b$
85:2	P 4/n:2	-p 4a	$\bar{P} 4_a$	134:1	P 42/n n m:1	p 4n 2 -1n	$P 4_n 2 \bar{1}_n$
86:1	P 42/n:1	p 4n -1n	$P 4_n \bar{1}_n$	134:2	P 42/n n m:2	-p 4ac 2bc	$\bar{P} 4_{ac} 2_{bc}$
86:2	P 42/n:2	-p 4bc	$\bar{P} 4_{bc}$	135	P 42/m b c	-p 4c 2ab	$\bar{P} 4_c 2_{ab}$
87	I 4/m	-i 4	$\bar{I} 4$	136	P 42/m n m	-p 4n 2n	$\bar{P} 4_n 2_n$
88:1	I 41/a:1	i 4bw -1bw	$I 4_{bw} \bar{1}_{bw}$	137:1	P 42/n m c:1	p 4n 2n -1n	$P 4_n 2_n \bar{1}_n$
88:2	I 41/a:2	-i 4ad	$\bar{I} 4_{ad}$	137:2	P 42/n m c:2	-p 4ac 2a	$\bar{P} 4_{ac} 2_a$
89	P 4 2 2	p 4 2	P 4 2	138:1	P 42/n c m:1	p 4n 2ab -1n	$P 4_n 2_{ab} \bar{1}_n$
90	P 4 21 2	p 4ab 2ab	$P 4_{ab} 2_{ab}$	138:2	P 42/n c m:2	-p 4ac 2ac	$\bar{P} 4_{ac} 2_{ac}$
91	P 41 2 2	p 4w 2c	$P 4_w 2_c$	139	I 4/m m m	-i 4 2	$\bar{I} 4 2$
92	P 41 21 2	p 4abw 2nw	$P 4_{abw} 2_{nw}$	140	I 4/m c m	-i 4 2c	$\bar{I} 4 2_c$
93	P 42 2 2	p 4c 2	$P 4_c 2$	141:1	I 41/a m d:1	i 4bw 2bw -1bw	$I 4_{bw} 2_{bw} \bar{1}_{bw}$
94	P 42 21 2	p 4n 2n	$P 4_n 2_n$	141:2	I 41/a m d:2	-i 4bd 2	$\bar{I} 4_{bd} 2$
95	P 43 2 2	p 4cw 2c	$P 4_{cw} 2_c$	142:1	I 41/a c d:1	i 4bw 2aw -1bw	$I 4_{bw} 2_{aw} \bar{1}_{bw}$
96	P 43 21 2	p 4nw 2abw	$P 4_{nw} 2_{abw}$	142:2	I 41/a c d:2	-i 4bd 2c	$\bar{I} 4_{bd} 2_c$
97	I 4 2 2	i 4 2	$I 4 2$	143	P 3	p 3	P 3
98	I 41 2 2	i 4bw 2bw	$I 4_{bw} 2_{bw}$	144	P 31	p 31	P 3 <sub>1</sub>
99	P 4 m m	p 4 -2	$P 4 \bar{2}$	145	P 32	p 32	P 3 <sub>2</sub>
100	P 4 b m	p 4 -2ab	$P 4 \bar{2}_{ab}$	146:h	R 3:h	r 3	R 3
101	P 42 c m	p 4c -2c	$P 4_c \bar{2}_c$	146:r	R 3:r	p 3*	P 3*
102	P 42 n m	p 4n -2n	$P 4_n \bar{2}_n$	147	P -3	-p 3	$\bar{P} 3$
103	P 4 c c	p 4 -2c	$P 4 \bar{2}_c$	148:h	R -3:h	-r 3	$\bar{R} 3$
104	P 4 n c	p 4 -2n	$P 4 \bar{2}_n$	148:r	R -3:r	-p 3*	$\bar{P} 3^*$
105	P 42 m c	p 4c -2	$P 4_c \bar{2}$	149	P 3 1 2	p 3 2	P 3 2
106	P 42 b c	p 4c -2ab	$P 4_c \bar{2}_{ab}$	150	P 3 2 1	p 3 2"	P 3 2"
107	I 4 m m	i 4 -2	$I 4 \bar{2}$	151	P 31 1 2	p 31 2 (0 0 4)	$P 3_1 2 (0 0 4)$
108	I 4 c m	i 4 -2c	$I 4 \bar{2}_c$	152	P 31 2 1	p 31 2"	P 3 <sub>1</sub> 2"
109	I 41 m d	i 4bw -2	$I 4_{bw} \bar{2}$	153	P 32 1 2	p 32 2 (0 0 2)	$P 3_2 2 (0 0 2)$
110	I 41 c d	i 4bw -2c	$I 4_{bw} \bar{2}_c$	154	P 32 2 1	p 32 2"	P 3 <sub>2</sub> 2"
111	P -4 2 m	p -4 2	$P \bar{4} 2$	155:h	R 3 2:h	r 3 2"	$R 3 2"$

1.4. SYMMETRY IN RECIPROCAL SPACE

Table A1.4.2.7. Hall symbols (cont.)

n:c	H-M entry	Hall entry	Hall symbol	n:c	H-M entry	Hall entry	Hall symbol
155:r	R 3 2:r	p 3* 2	P 3* 2	194	P 63/m m c	-p 6c 2c	$\bar{P} 6_c 2_c$
156	P 3 m 1	p 3 -2"	P 3 $\bar{2}$ "	195	P 2 3	p 2 2 3	P 2 2 3
157	P 3 1 m	p 3 -2	P 3 $\bar{2}$	196	F 2 3	f 2 2 3	F 2 2 3
158	P 3 c 1	p 3 -2"c	P 3 $\bar{2}_c$	197	I 2 3	i 2 2 3	I 2 2 3
159	P 3 1 c	p 3 -2c	P 3 $\bar{2}_c$	198	P 21 3	p 2ac 2ab 3	P 2 <sub>ac</sub> 2 <sub>ab</sub> 3
160:h	R 3 m:h	r 3 -2"	R 3 $\bar{2}$ "	199	I 21 3	i 2b 2c 3	I 2 <sub>b</sub> 2 <sub>c</sub> 3
160:r	R 3 m:r	p 3* -2	P 3* $\bar{2}$	200	P m -3	-p 2 2 3	$\bar{P} 2 2 3$
161:h	R 3 c:h	r 3 -2"c	R 3 $\bar{2}_c$	201:1	P n -3:1	p 2 2 3 -1n	P 2 2 3 $\bar{1}_n$
161:r	R 3 c:r	p 3* -2n	P 3* $\bar{2}_n$	201:2	P n -3:2	-p 2ab 2bc 3	$\bar{P} 2_{ab} 2_{bc} 3$
162	P -3 1 m	-p 3 2	$\bar{P} 3 2$	202	F m -3	-f 2 2 3	$\bar{F} 2 2 3$
163	P -3 1 c	-p 3 2c	$\bar{P} 3 2_c$	203:1	F d -3:1	f 2 2 3 -1d	F 2 2 3 $\bar{1}_d$
164	P -3 m 1	-p 3 2"	$\bar{P} 3 2$ "	203:2	F d -3:2	-f 2uv 2vw 3	$\bar{F} 2_{uv} 2_{vw} 3$
165	P -3 c 1	-p 3 2"c	$\bar{P} 3 2_c$ "	204	I m -3	-i 2 2 3	$\bar{I} 2 2 3$
166:h	R -3 m:h	-r 3 2"	$\bar{R} 3 2$ "	205	P a -3	-p 2ac 2ab 3	$\bar{P} 2_{ac} 2_{ab} 3$
166:r	R -3 m:r	-p 3* 2	$\bar{P} 3^* 2$	206	I a -3	-i 2b 2c 3	$\bar{I} 2_b 2_c 3$
167:h	R -3 c:h	-r 3 2"c	$\bar{R} 3 2_c$ "	207	P 4 3 2	p 4 2 3	P 4 2 3
167:r	R -3 c:r	-p 3* 2n	$\bar{P} 3^* 2_n$	208	P 42 3 2	p 4n 2 3	P 4 <sub>n</sub> 2 3
168	P 6	p 6	P 6	209	F 4 3 2	f 4 2 3	F 4 2 3
169	P 61	p 61	P 6 <sub>1</sub>	210	F 41 3 2	f 4d 2 3	F 4 <sub>d</sub> 2 3
170	P 65	p 65	P 6 <sub>5</sub>	211	I 4 3 2	i 4 2 3	I 4 2 3
171	P 62	p 62	P 6 <sub>2</sub>	212	P 43 3 2	p 4acd 2ab 3	P 4 <sub>acd</sub> 2 <sub>ab</sub> 3
172	P 64	p 64	P 6 <sub>4</sub>	213	P 41 3 2	p 4bd 2ab 3	P 4 <sub>bd</sub> 2 <sub>ab</sub> 3
173	P 63	p 6c	P 6 <sub>c</sub>	214	I 41 3 2	i 4bd 2c 3	I 4 <sub>bd</sub> 2 <sub>c</sub> 3
174	P -6	p -6	P $\bar{6}$	215	P -4 3 m	p -4 2 3	P $\bar{4} 2 3$
175	P 6/m	-p 6	$\bar{P} 6$	216	F -4 3 m	f -4 2 3	F $\bar{4} 2 3$
176	P 63/m	-p 6c	$\bar{P} 6_c$	217	I -4 3 m	i -4 2 3	I $\bar{4} 2 3$
177	P 6 2 2	p 6 2	P 6 2	218	P -4 3 n	p -4n 2 3	P $\bar{4}_n 2 3$
178	P 61 2 2	p 61 2 (0 0 5)	P 6 <sub>1</sub> 2 (0 0 5)	219	F -4 3 c	f -4a 2 3	F $\bar{4}_a 2 3$
179	P 65 2 2	p 65 2 (0 0 1)	P 6 <sub>5</sub> 2 (0 0 1)	220	I -4 3 d	i -4bd 2c 3	I $\bar{4}_{bd} 2_c 3$
180	P 62 2 2	p 62 2 (0 0 4)	P 6 <sub>2</sub> 2 (0 0 4)	221	P m -3 m	-p 4 2 3	$\bar{P} 4 2 3$
181	P 64 2 2	p 64 2 (0 0 2)	P 6 <sub>4</sub> 2 (0 0 2)	222:1	P n -3 n:1	p 4 2 3 -1n	P 4 2 3 $\bar{1}_n$
182	P 63 2 2	p 6c 2c	P 6 <sub>c</sub> 2 <sub>c</sub>	222:2	P n -3 n:2	-p 4a 2bc 3	$\bar{P} 4_a 2_{bc} 3$
183	P 6 m m	p 6 -2	P 6 $\bar{2}$	223	P m -3 n	-p 4n 2 3	$\bar{P} 4_n 2 3$
184	P 6 c c	p 6 -2c	P 6 $\bar{2}_c$	224:1	P n -3 m:1	p 4n 2 3 -1n	P 4 <sub>n</sub> 2 3 $\bar{1}_n$
185	P 63 c m	p 6c -2	P 6 <sub>c</sub> $\bar{2}$	224:2	P n -3 m:2	-p 4bc 2bc 3	$\bar{P} 4_{bc} 2_{bc} 3$
186	P 63 m c	p 6c -2c	P 6 <sub>c</sub> $\bar{2}_c$	225	F m -3 m	-f 4 2 3	$\bar{F} 4 2 3$
187	P -6 m 2	p -6 2	P $\bar{6} 2$	226	F m -3 c	-f 4a 2 3	$\bar{F} 4_a 2 3$
188	P -6 c 2	p -6c 2	P $\bar{6}_c 2$	227:1	F d -3 m:1	f 4d 2 3 -1d	F 4 <sub>d</sub> 2 3 $\bar{1}_d$
189	P -6 2 m	p -6 -2	P $\bar{6} \bar{2}$	227:2	F d -3 m:2	-f 4vw 2vw 3	$\bar{F} 4_{vw} 2_{vw} 3$
190	P -6 2 c	p -6c -2c	P $\bar{6}_c \bar{2}_c$	228:1	F d -3 c:1	f 4d 2 3 -1ad	F 4 <sub>d</sub> 2 3 $\bar{1}_{ad}$
191	P 6/m m m	-p 6 2	$\bar{P} 6 2$	228:2	F d -3 c:2	-f 4ud 2vw 3	$\bar{F} 4_{ud} 2_{vw} 3$
192	P 6/m c c	-p 6 2c	$\bar{P} 6 2_c$	229	I m -3 m	-i 4 2 3	$\bar{I} 4 2 3$
193	P 63/m c m	-p 6c 2	$\bar{P} 6_c 2$	230	I a -3 d	-i 4bd 2c 3	$\bar{I} 4_{bd} 2_c 3$

The codes appended to the space-group numbers listed in the first column identify the relationship between the symmetry elements and the crystal cell. Where no code is given the first choice listed below applies.

*Monoclinic.* Code = <unique axis><cell choice>: unique axis choices [cf. IT A (1983) Table 4.3.1] b, -b, c, -c, a, -a; cell choices [cf. IT A (1983) Table 4.3.1] 1, 2, 3.

*Orthorhombic.* Code = <origin choice><setting>: origin choices 1, 2; setting choices [cf. IT A (1983) Table 4.3.1] abc, ba-c, cab, -cba, bca, a-cb.

*Tetragonal, cubic.* Code = <origin choice>: origin choices 1, 2.

*Trigonal.* Code = <cell choice>: cell choices h (hexagonal), r (rhombohedral).