2.3. Patterson and molecular-replacement techniques

2.3.1. Introduction

2.3.1.1. Background

Historically, the Patterson has been used in a variety of ways to effect the solutions of crystal structures. While some simple structures (Ketelaar & de Vries, 1939; Hughes, 1940; Speakman, 1949; Shoemaker et al., 1950) were solved by direct analysis of Patterson syntheses, alternative methods have largely superseded this procedure. An early innovation was the heavy-atom method which depends on the location of a small number of relatively strong scatterers (Harker, 1936). Image-seeking methods and Patterson superposition techniques were first contemplated in the late 1930s (Winch, 1939) and applied sometime later (Beevers & Robertson, 1950; Clastre & Gay, 1950; Garrido, 1950; Nordman & Nakatsu, 1963; Kraut, 1961; Hamilton, 1965; Simpson et al., 1965) or to position known molecular fragments in unknown crystal structures (Nordman & Nakatsu, 1963; Huber, 1965). The Patterson function has been used extensively in conjunction with the isomorphous replacement method (Rossmann, 1960; Blow, 1958) or anomalous dispersion (Rossmann, 1961a) to determine the position of heavy-atom substitution. Pattersons have been used to detect the presence and relative orientation of multiple copies of a given chemical motif in the crystallographic asymmetric unit in the same or different crystals (Rossmann & Blow, 1962). Finally, the orientation and placement of known molecular structures (‘molecular replacement’) into unknown crystal structures can be accomplished via Patterson techniques.

The function, introduced by Patterson in 1934 (Patterson, 1934a,b), is a convolution of electron density with itself and may be defined as

$$P(\mathbf{u}) = \int_V \rho(\mathbf{x}) \cdot \rho(\mathbf{x} + \mathbf{u}) \, d\mathbf{x}, \quad (2.3.1.1)$$

where $P(\mathbf{u})$ is the ‘Patterson’ function at $\mathbf{u}$, $\rho(\mathbf{x})$ is the crystal’s periodic electron density and $V$ is the volume of the unit cell. The Patterson function, or $F^2$ series, can be calculated directly from the experimentally derived X-ray intensities as

$$P(\mathbf{u}) = \frac{2}{V^2} \sum_{\mathbf{h}} |F_\mathbf{h}|^2 \cos 2\pi\mathbf{h} \cdot \mathbf{u}. \quad (2.3.1.2)$$

The derivation of (2.3.1.2) from (2.3.1.1) can be found in this volume (see Section 1.3.4.2.1.6) along with a discussion of the physical significance and symmetry of the Patterson function, although the principal properties will be restated here.

The Patterson can be considered to be a vector map of all the pairwise interactions between the atoms in a unit cell. The vectors in a Patterson correspond to vectors in the real (direct) crystal cell but translated to the Patterson origin. Their weights are proportional to the product of densities at the tips of the vectors in the real cell. The Patterson unit cell has the same size as the real crystal cell. The symmetry of the Patterson comprises the Laue point group of the crystal cell plus any additional lattice symmetry due to Bravais centring. The reduction of the real space group to the Laue symmetry is produced by the translation of all vectors to the Patterson origin and the introduction of a centre of symmetry. The latter is a consequence of the relationship between the vectors AB and BA. The Patterson symmetries for all 230 space groups are tabulated in IT A (1983).

An analysis of Patterson peaks can be obtained by considering $N$ atoms with form factors $f_i$ in the unit cell. Then

$$F_\mathbf{h} = \sum_{i=1}^N f_i \exp(2\pi i \mathbf{h} \cdot \mathbf{x}_i).$$

Using Friedel’s law,

$$|F_\mathbf{h}|^2 = F_\mathbf{h} \cdot F^*_\mathbf{h} = \left[ \sum_{i=1}^N f_i \exp(2\pi i \mathbf{h} \cdot \mathbf{x}_i) \right] \left[ \sum_{i=1}^N f_i \exp(-2\pi i \mathbf{h} \cdot \mathbf{x}_i) \right],$$

which can be decomposed to

$$|F_\mathbf{h}|^2 = \sum_{i=1}^N |f_i|^2 + \sum_{i \neq j} |f_i f_j| \exp[2\pi i \mathbf{h} \cdot (\mathbf{x}_i - \mathbf{x}_j)]. \quad (2.3.1.3)$$

On substituting (2.3.1.3) in (2.3.1.2), we see that the Patterson consists of the sum of $N^2$ total interactions of which $N$ are of weight $|f_i|^2$ at the origin and $N(N-1)$ are of weight $|f_i f_j|$ at $\mathbf{x}_i - \mathbf{x}_j$.

The weight of a peak in a real cell is given by

$$w_i = \int_U \rho_i(x) \, dx = Z_i \, \text{(atomic number)},$$

where $U$ is the volume of the atom $i$. By analogy, the weight of a peak in a Patterson (form factor $f_i f_j$) will be given by

$$w_{ij} = \int_U P_{ij}(u) \, du = ZZ_i.$$