

## 5.1. DYNAMICAL THEORY OF X-RAY DIFFRACTION

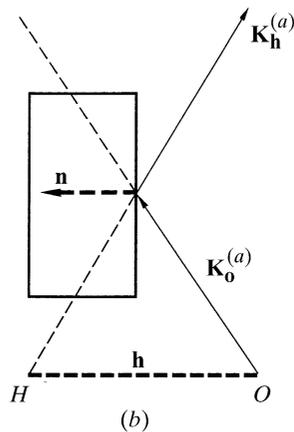
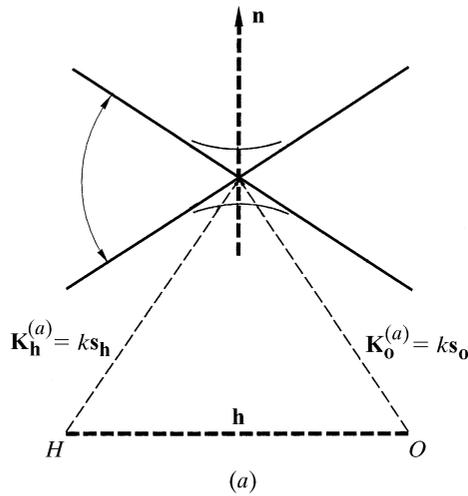


Fig. 5.1.3.3. Reflection, or Bragg, geometry. (a) Reciprocal space; (b) direct space.

$$\gamma_o = \cos(\mathbf{n}, \mathbf{s}_o); \quad \gamma_h = \cos(\mathbf{n}, \mathbf{s}_h). \quad (5.1.3.2)$$

It will be noted that they are both positive, as is their ratio,

$$\gamma = \gamma_h / \gamma_o. \quad (5.1.3.3)$$

This is the *asymmetry ratio*, which is very important since the width of the rocking curve is proportional to its square root [equation (5.1.3.6)].

(b) *Reflection, or Bragg case* (Fig. 5.1.3.3). In this case there are three possible situations: the normal to the crystal surface drawn from  $M$  intersects either branch 1 or branch 2 of the dispersion surface, or the intersection points are imaginary (Fig. 5.1.3.3a). The reflected wave is directed towards the *outside* of the crystal (Fig. 5.1.3.3b). The cosines defined by (5.1.3.2) are now positive for  $\gamma_o$  and negative for  $\gamma_h$ . The asymmetry factor is therefore also negative.

### 5.1.3.3. Middle of the reflection domain

It will be apparent from the equations given later that the incident wavevector corresponding to the middle of the reflection domain is, in both cases,  $\mathbf{OI}$ , where  $I$  is the intersection of the normal to the crystal surface drawn from the Lorentz point,  $L_o$ , with  $T'_o$  (Figs. 5.1.3.4 and 5.1.3.5), while, according to Bragg's law, it should be  $\mathbf{OL}_a$ . The angle  $\Delta\theta$  between the incident wavevectors  $\mathbf{OL}_a$  and  $\mathbf{OI}$ , corresponding to the middle of the reflecting domain according to the geometrical and dynamical theories, respectively, is

$$\Delta\theta_o = \overline{L_a I} / k = R\lambda^2 F_o (1 - \gamma) / (2\pi V \sin 2\theta). \quad (5.1.3.4)$$

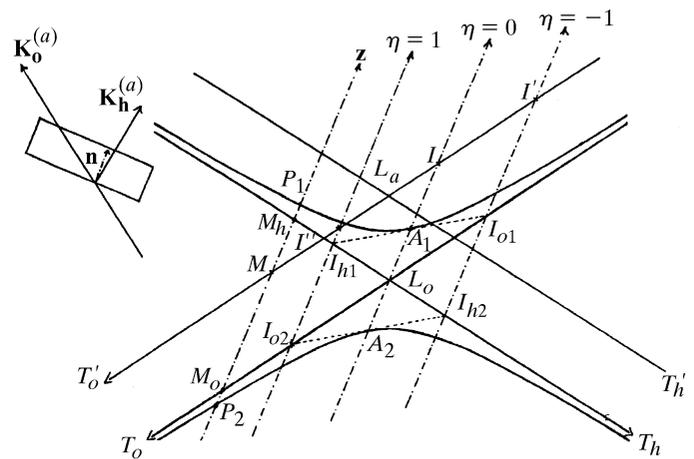


Fig. 5.1.3.4. Boundary conditions at the entrance surface for transmission geometry.

In the Bragg case, the asymmetry ratio  $\gamma$  is negative and  $\Delta\theta_o$  is never equal to zero. This difference in Bragg angle between the two theories is due to the refraction effect, which is neglected in geometrical theory. In the Laue case,  $\Delta\theta_o$  is equal to zero for symmetric reflections ( $\gamma = 1$ ).

### 5.1.3.4. Deviation parameter

The solutions of dynamical theory are best described by introducing a reduced parameter called the *deviation parameter*,

$$\eta = (\Delta\theta - \Delta\theta_o) / \delta, \quad (5.1.3.5)$$

where

$$\delta = R\lambda^2 |C| (|\gamma| F_h F_{\bar{h}})^{1/2} / (\pi V \sin 2\theta), \quad (5.1.3.6)$$

whose real part is equal to the half width of the rocking curve (Sections 5.1.6 and 5.1.7). The width  $2\delta$  of the rocking curve is sometimes called the *Darwin width*.

The definition (5.1.3.5) of the deviation parameter is independent of the geometrical situation (reflection or transmission case); this is not followed by some authors. The present convention has the advantage of being quite general.

In an absorbing crystal,  $\eta$ ,  $\Delta\theta_o$  and  $\delta$  are complex, and it is the real part,  $\Delta\theta_{or}$ , of  $\Delta\theta_o$  which has the geometrical interpretation given in Section 5.1.3.3. One obtains

$$\begin{aligned} \eta &= \eta_r + i\eta_i \\ \eta_r &= (\Delta\theta - \Delta\theta_{or}) / \delta_r; \quad \eta_i = A\eta_r + B \end{aligned} \quad (5.1.3.7)$$

$$\begin{aligned} A &= -\tan \beta \\ B &= \left\{ \chi_{io} / \left[ |C| (|\chi_h \chi_{\bar{h}}|)^{1/2} \cos \beta \right] \right\} (1 - \gamma) / 2 (|\gamma|)^{1/2}, \end{aligned}$$

where  $\beta$  is the phase angle of  $(\chi_h \chi_{\bar{h}})^{1/2}$  [or that of  $(F_h F_{\bar{h}})^{1/2}$ ].

### 5.1.3.5. Pendellösung and extinction distances

Let

$$\Lambda_o = \pi V (\gamma_o |\gamma_h|)^{1/2} / [R\lambda |C| (F_h F_{\bar{h}})^{1/2}]. \quad (5.1.3.8)$$

This length plays a very important role in the dynamical theory of diffraction by both perfect and deformed crystals. For example, it is  $15.3 \mu\text{m}$  for the 220 reflection of silicon, with Mo  $K\alpha$  radiation and a symmetric reflection.

In *transmission* geometry, it gives the period of the interference between the two excited wavefields which constitutes the *Pendellösung* effect first described by Ewald (1917) (see Section