

## 5. DYNAMICAL THEORY AND ITS APPLICATIONS

approximated by the function

$$I_h \approx 1/(4\eta^2).$$

*Width of the total-reflection domain.* The width of the total-reflection domain is equal to  $\Delta\eta = 2$  and its angular width is therefore equal, using (5.1.3.5), to  $2\delta$ , where  $\delta$  is given by (5.1.3.6). It is proportional to the structure factor, the polarization factor  $C$  and the square root of the asymmetry factor  $|\gamma|$ . Using an asymmetric reflection, it is therefore possible to decrease the width at wish. This is used in monochromators to produce a pseudo plane wave [see, for instance, Kikuta & Kohra (1970)]. It is possible to deduce the value of the form factor from very accurate measurements of the rocking curve; see, for instance, Kikuta (1971).

*Integrated intensity.* The integrated intensity is defined by (5.1.6.8):

$$I_{hi} = 8\delta/3. \quad (5.1.7.3)$$

*Penetration depth.* Within the domain of total reflection, there are two wavefields propagating inside the crystal with imaginary wavevectors, one towards the inside of the crystal and the other one in the opposite direction, so that they cancel out and, globally, no energy penetrates the crystal. The absorption coefficient of the waves penetrating the crystal is

$$\mu = -4\pi K_{oi}\gamma_o = 2\pi\gamma_o(1 - \eta^2)^{1/2}/\Lambda_B, \quad (5.1.7.4)$$

where  $\Lambda_B$  is the value taken by  $\Lambda_o$  [equation (5.1.3.8)] in the Bragg case.

The penetration depth is a minimum at the middle of the reflection domain and at this point it is equal to  $\Lambda_B/2\pi$ . This attenuation effect is called *extinction*, and  $\Lambda_B$  is called the *extinction length*. It is a specific property owing to the existence of wavefields. The resulting propagation direction of energy is parallel to the crystal surface, but with a cross section equal to zero: it is an *evanescent wave* [see, for instance, Cowan *et al.* (1986)].

#### 5.1.7.1.2. Absorbing crystals

*Rocking curve.* Since the sign of  $\gamma$  is negative,  $[\eta^2 + S(\gamma h)]^{1/2}$  in (5.1.3.10) has a very large imaginary part when  $|\eta_r| \leq 1$ . It cannot be calculated using the same approximations as in the Laue case. Let us set

$$Z \exp(i\Psi') = \eta \mp (\eta^2 - 1)^{1/2}. \quad (5.1.7.5)$$

The reflecting power is

$$I_h = (F_h/F_{\bar{h}})^{1/2} Z^2, \quad (5.1.7.6)$$

where  $Z = [L - (L^2 - 1)^{1/2}]^{1/2}$ ,  $L = |\eta|^2 + \rho^2$  and  $\rho = |\eta^2 - 1|$  is the modulus of expression (5.1.7.5) where the sign is chosen in such a way that  $Z$  is smaller than 1.

The expression for the reflected intensity in the absorbing Bragg case was first given by Prins (1930). The way of representing it given here was first used by Hirsch & Ramachandran (1950). The properties of the rocking curve have been described by Fingerland (1971).

There is no longer a total-reflection domain and energy penetrates the crystal at all incidence angles, although with a very high absorption coefficient within the domain  $|\eta_r| \leq 1$ . Fig. 5.1.7.2 gives an example of a rocking curve for a thick absorbing crystal. It was first observed by Renninger (1955). The shape is asymmetric and is due to the anomalous-absorption effect: it is lower than normal on the low-angle side, which is associated with wavefields belonging to branch 1 of the dispersion surface, and larger than normal on the high-angle side, which is associated with branch 2 wavefields. The amount of asymmetry depends on the value of the ratio  $A/B$  of the coefficients in the expression for the imaginary part

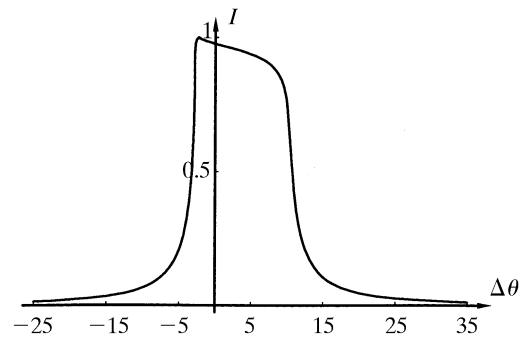


Fig. 5.1.7.2. Theoretical rocking curve in the reflection case for a thick absorbing crystal. The 400 reflection of GaAs using Cu  $K\alpha$  radiation is shown.

of the deviation parameter in (5.1.3.7): the smaller this ratio, the more important the asymmetry.

*Absorption coefficient.* The effective absorption coefficient, taking into account both the Borrmann effect and extinction, is given by (Authier, 1986)

$$\mu = \mu_o + 2(|F_h F_{\bar{h}}|)^{1/2} \frac{R\lambda}{V(|\gamma|)^{1/2}} Z \sin(\beta + \Psi'),$$

where  $\beta$  is defined in equation (5.1.3.7) and  $\Psi'$  in equation (5.1.7.5), and where the sign is chosen in such a way that  $Z$  converges. Fig. 5.1.7.3 shows the variation of the penetration depth  $z_o = \gamma_o/\mu$  with the deviation parameter.

#### 5.1.7.2. Thin crystals

##### 5.1.7.2.1. Non-absorbing crystals

*Boundary conditions.* If the crystal is thin, the wavefield created at the reflecting surface at  $A$  and penetrating inside can reach the back surface at  $B$  (Fig. 5.1.7.4a). The *incident* direction there points towards the *outside* of the crystal, while the *reflected* direction

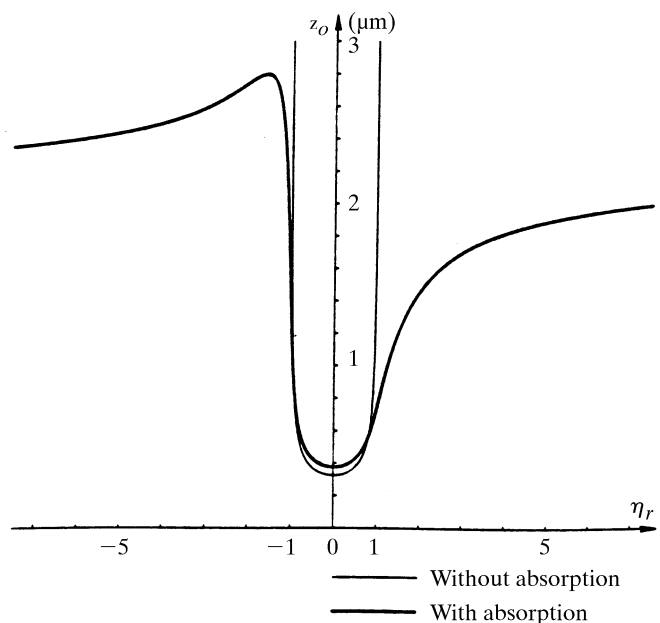


Fig. 5.1.7.3. Bragg case: thick crystals. Variation of the penetration depth with incidence angle (represented here by the dimensionless deviation parameter  $\eta$ ). Thin curve: without absorption; thick curve: with absorption for the 400 reflection of GaAs using Cu  $K\alpha$  radiation.