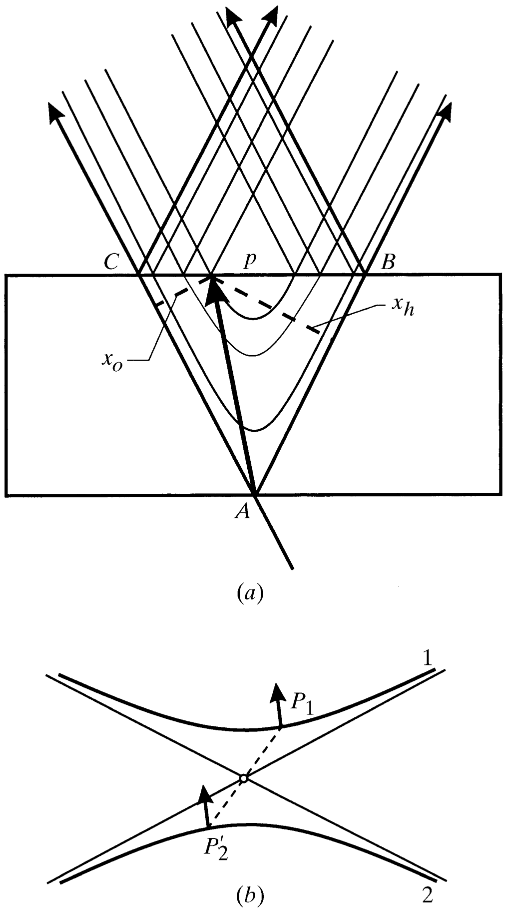

Fig. 5.1.8.4. Interference at the origin of the *Pendellösung* fringes in the case of an incident spherical wave. (*a*) Direct space; (*b*) reciprocal space.

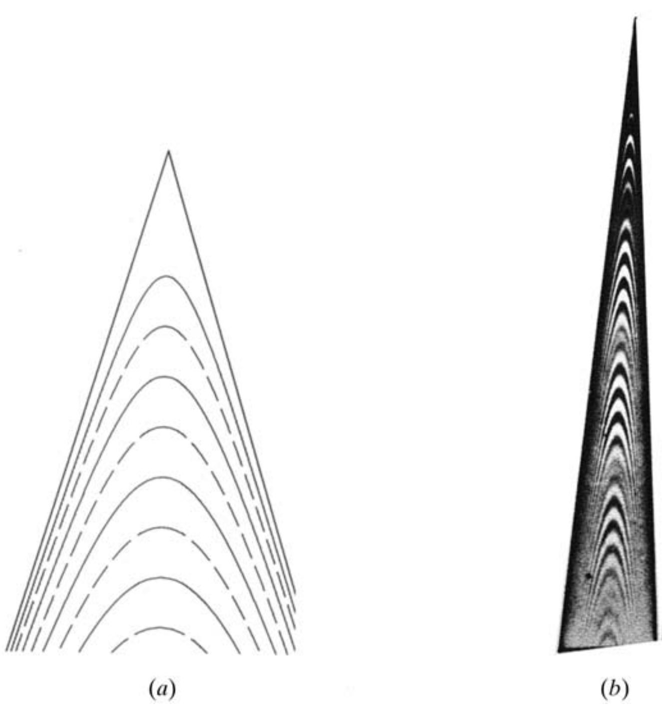

Fig. 5.1.8.5. Spherical-wave *Pendellösung* fringes observed on a wedge-shaped crystal. (*a*) Computer simulation (solid lines: maxima; dashed lines: minima). (*b*) X-ray section topograph of a wedge-shaped silicon crystal (444 reflection, Mo $K\alpha$ radiation).

state [see equation (5.1.3.8)]. If the incident wave is unpolarized, one observes the superposition of the *Pendellösung* fringes corresponding to the two states of polarization, parallel and perpendicular to the plane of incidence. This results in a beat effect, which is clearly visible in Fig. 5.1.8.5.

Appendix 5.1.1.

A5.1.1.1. *Dielectric susceptibility – classical derivation*

Under the influence of the incident electromagnetic radiation, the medium becomes polarized. The dielectric susceptibility, which relates this polarization to the electric field, thus characterizes the interaction of the medium and the electromagnetic wave. The classical derivation of the dielectric susceptibility, χ, which is summarized here is only valid for a very high frequency which is also far from an absorption edge. Let us consider an electromagnetic wave,

$$\mathbf{E} = \mathbf{E_o} \exp 2\pi i(\nu t - \mathbf{k}\cdot\mathbf{r}),$$

incident on a bound electron. The electron behaves as if it were held by a spring with a linear restoring force and is an oscillator with a resonant frequency ν_o. The equation of its motion is written in the following way:

$$m\,\mathrm{d}^2\mathbf{a}/\mathrm{d}t^2 = -4\pi^2\nu_o m\mathbf{a} = \mathbf{F},$$

where the driving force $\mathbf{F}$ is due to the electric field of the wave and is equal to $-e\mathbf{E}$. The magnetic interaction is neglected here.

The solution of the equation of motion is

$$\mathbf{a} = -e\mathbf{E}/[4\pi^2 m(\nu_o - \nu^2)].$$

The resonant frequencies of the electrons in atoms are of the order of the ultraviolet frequencies and are therefore much smaller than X-ray frequencies. They can be neglected and the expression of the amplitude of the electron reduces to

$$\mathbf{a} = e\mathbf{E}/(4\pi^2\nu^2).$$

The dipolar moment is therefore

$$\mathcal{M} = -e\mathbf{a} = -e^2\mathbf{E}/(4\pi^2\nu^2 m).$$

von Laue assumes that the negative charge is distributed continuously all over space and that the charge of a volume element $\mathrm{d}\tau$ is $-e\rho\,\mathrm{d}\tau$, where ρ is the electronic density. The electric moment of the volume element is

$$\mathrm{d}\mathcal{M} = -e^2\rho\mathbf{E}\,\mathrm{d}\tau/(4\pi^2\nu^2 m).$$

The polarization is equal to the moment per unit volume:

$$\mathbf{P} = \mathrm{d}\mathcal{M}/\mathrm{d}\tau = -e^2\rho\mathbf{E}/(4\pi^2\nu^2 m).$$

It is related to the electric field and electric displacement through

$$\mathbf{D} = \varepsilon_o\mathbf{E} + \mathbf{P} = \varepsilon_o(1+\chi)\mathbf{E}. \qquad \text{(A5.1.1.1)}$$

We finally obtain the expression of the dielectric susceptibility,

$$\chi = -e^2\rho/(4\pi^2\varepsilon_o\nu^2 m) = -R\lambda^2\rho/\pi, \qquad \text{(A5.1.1.2)}$$

where $R = e^2/(4\pi\varepsilon_o mc^2)$ ($= 2.81794 \times 10^{-15}$ m) is the classical radius of the electron.

A5.1.1.2. *Maxwell's equations*

The electromagnetic field is represented by two vectors, $\mathbf{E}$ and $\mathbf{B}$, which are the electric field and the magnetic induction, respectively. To describe the interaction of the field with matter, three other vectors must be taken into account, the electrical current density, $\mathbf{j}$, the electric displacement, $\mathbf{D}$, and the magnetic field, $\mathbf{H}$. The space and time derivatives of these vectors are related in a continuous

references