The extra gain with respect to the multidimensional Cooley–Tukey method is that *there are no twiddle factors between p-primary pieces corresponding to different primes p*.

The case where **N** is not diagonal has been examined by Guessoum & Mersereau (1986).

*1.3.3.3.2.3. Nesting of Winograd small FFTs*

Suppose that the CRT has been used as above to map an $n$-dimensional DFT to a $\mu$-dimensional DFT. For each $\kappa = 1, \ldots, \mu$ [$\kappa$ runs over those pairs $(i, j)$ such that $\nu(i, j) > 0$], the Rader/Winograd procedure may be applied to put the matrix of the $\kappa$th 1D DFT in the **CBA** normal form of a Winograd small FFT. The full DFT matrix may then be written, up to permutation of data and results, as

$$\bigotimes_{\kappa=1}^{\mu} (\mathbf{C}_\kappa \mathbf{B}_\kappa \mathbf{A}_\kappa).$$

A well known property of the tensor product of matrices allows this to be rewritten as

$$\left(\bigotimes_{\gamma=1}^{\mu} \mathbf{C}_\gamma\right) \times \left(\bigotimes_{\beta=1}^{\mu} \mathbf{B}_\beta\right) \times \left(\bigotimes_{\alpha=1}^{\mu} \mathbf{A}_\alpha\right)$$

and thus to form a matrix in which the *combined* pre-addition, multiplication and post-addition matrices have been *precomputed*. This procedure, called *nesting*, can be shown to afford a reduction of the arithmetic operation count compared to the row–column method (Morris, 1978).

Clearly, the nesting rearrangement need not be applied to all $\mu$ dimensions, but can be restricted to any desired subset of them.

*1.3.3.3.2.4. The Nussbaumer–Quandalle algorithm*

Nussbaumer's approach views the DFT as the evaluation of certain polynomials constructed from the data (as in Section 1.3.3.2.4). For instance, putting $\omega = e(1/N)$, the 1D $N$-point DFT

$$X^*(k^*) = \sum_{k=0}^{N-1} X(k) \omega^{k^* k}$$

may be written

$$X^*(k^*) = Q(\omega^{k^*}),$$

where the polynomial $Q$ is defined by

$$Q(z) = \sum_{k=0}^{N-1} X(k) z^k.$$

Let us consider (Nussbaumer & Quandalle, 1979) a 2D transform of size $N \times N$:

$$X^*(k_1^*, k_2^*) = \sum_{k_1=0}^{N-1} \sum_{k_2=0}^{N-1} X(k_1, k_2) \omega^{k_1^* k_1 + k_2^* k_2}.$$

By introduction of the polynomials

$$Q_{k_2}(z) = \sum_{k_1} X(k_1, k_2) z^{k_1}$$
$$R_{k_2^*}(z) = \sum_{k_2} \omega^{k_2^* k_2} Q_{k_2}(z),$$

this may be rewritten:

$$X^*(k_1^*, k_2^*) = R_{k_2^*}(\omega^{k_1^*}) = \sum_{k_2} \omega^{k_2^* k_2} Q_{k_2}(\omega^{k_1^*}).$$

Let us now suppose that $k_1^*$ is coprime to $N$. Then $k_1^*$ has a unique inverse modulo $N$ (denoted by $1/k_1^*$), so that multiplication by $k_1^*$ simply permutes the elements of $\mathbb{Z}/N\mathbb{Z}$ and hence

$$\sum_{k_2=0}^{N-1} f(k_2) = \sum_{k_2=0}^{N-1} f(k_1^* k_2)$$

for any function $f$ over $\mathbb{Z}/N\mathbb{Z}$. We may thus write:

$$\begin{aligned} X^*(k_1^*, k_2^*) &= \sum_{k_2} \omega^{k_1^* k_2^* k_2} Q_{k_1^* k_2}(\omega^{k_1^*}) \\ &= S_{k_1^* k_2}(\omega^{k_1^*}) \end{aligned}$$

where

$$S_{k^*}(z) = \sum_{k_2} z^{k^* k_2} Q_{k_2}(z).$$

Since only the value of polynomial $S_{k^*}(z)$ at $z = \omega^{k_1^*}$ is involved in the result, the computation of $S_{k^*}$ may be carried out modulo the unique cyclotomic polynomial $P(z)$ such that $P(\omega^{k_1^*}) = 0$. Thus, if we define:

$$T_{k^*}(z) = \sum_{k_2} z^{k^* k_2} Q_{k_2}(z) \bmod P(z)$$

we may write:

$$X^*(k_1^*, k_2^*) = T_{k_1^* k_2^*}(\omega^{k_1^*})$$

or equivalently

$$X^*\left(k_1^*, \frac{k_2^*}{k_1^*}\right) = T_{k_2^*}(\omega^{k_1^*}).$$

For $N$ an odd prime $p$, all nonzero values of $k_1^*$ are coprime with $p$ so that the $p \times p$-point DFT may be calculated as follows:

(1) form the polynomials

$$T_{k_2^*}(z) = \sum_{k_1} \sum_{k_2} X(k_1, k_2) z^{k_1 + k_2^* k_2} \bmod P(z)$$

for $k_2^* = 0, \ldots, p - 1$;

(2) evaluate $T_{k_2^*}(\omega^{k_1^*})$ for $k_1^* = 0, \ldots, p - 1$;

(3) put $X^*(k_1^*, k_2^*/k_1^*) = T_{k_2^*}(\omega^{k_1^*})$;

(4) calculate the terms for $k_1^* = 0$ separately by