

1.4. ARITHMETIC CRYSTAL CLASSES AND SYMMORPHIC SPACE GROUPS

1.4.2.1. *Symmorphic space groups*

The 73 space groups known as ‘symmorphic’ are in one-to-one correspondence with the arithmetic crystal classes, and their standard ‘short’ symbols (Bertaut, 1995) are obtained by interchanging the order of the geometric crystal class and the Bravais cell in the symbol for the arithmetic space group. In fact, conventional crystallographic symbolism did not distinguish between arithmetic crystal classes and symmorphic space groups until recently (de Wolff *et al.*, 1985); the symbol of the symmorphic group was used also for the arithmetic class.

This relationship between the symbols, and the equivalent rule-of-thumb *symmorphic space groups are those whose standard (short) symbols do not contain glide planes or screw axes*, reveal nothing fundamental about the nature of symmorphism; they are simply a consequence of the conventions governing the construction of symbols in *International Tables for Crystallography*.*

Although the *standard* symbols of the symmorphic space groups do not contain screw axes or glide planes, this is a result of the manner in which the space-group symbols have been devised. Most symmorphic space groups do in fact contain screw axes and/or glide planes. This is immediately obvious for the symmorphic space groups based on centred cells; $C2$ contains equal numbers of diad rotation axes and diad screw axes, and Cm contains equal numbers of reflection planes and glide planes. This is recognized in the ‘extended’ space-group symbols (Bertaut, 1995), but these are clumsy and not commonly used; those for $C2$ and Cm are $C1_{21}^2$ and $C1_a^m$, respectively. In the more symmetric crystal systems, even symmorphic space groups with primitive cells contain screw axes and/or glide planes; $P422$ ($P42_2^2$) contains many diad screw axes and $P4/mmm$ ($P4/m2/m2_1^2/m2_1^2$) contains both screw axes and glide planes.

*Three examples of informative definitions are:

1. The space group corresponding to the zero solution of the Frobenius congruences is called a symmorphic space group (Engel, 1986, p. 155).

2. A space group F is called *symmorphic* if one of its finite subgroups (and therefore an infinity of them) is of an order equal to the order of the point group R_r (Opechowski, 1986, p. 255).

3. A space group is called *symmorphic* if the coset representatives W_j can be chosen in such a way that they leave one common point fixed (Wondratschek, 1995, p. 717).

Even in context, these are pretty opaque.

The balance of symmetry elements within the symmorphic space groups is discussed in more detail in Subsection 9.7.1.2.

1.4.3. Effect of dispersion on diffraction symmetry

In the absence of dispersion (‘anomalous scattering’), the intensities of the reflections hkl and $\bar{h}\bar{k}\bar{l}$ are equal (Friedel’s law), and statements about the symmetry of the weighted reciprocal lattice and quantities derived from it often rest on the tacit or explicit assumption of this law – the condition underlying it being forgotten. In particular, if dispersion is appreciable, the symmetry of the Patterson synthesis and the ‘Laue’ symmetry are altered.

1.4.3.1. *Symmetry of the Patterson function*

In Volume A of *International Tables*, the symmetry of the Patterson synthesis is derived in two stages. First, any glide planes and screw axes are replaced by mirror planes and the corresponding rotation axes, giving a symmorphic space group (Subsection 1.4.2.1). Second, a centre of symmetry is added. This second step involves the tacit assumption of Friedel’s law, and should not be taken if any atomic scattering factors have appreciable imaginary components. In such cases, the symmetry of the Patterson synthesis will not be that of one of the 24 centrosymmetric symmorphic space groups, as given in Volume A, but will be that of the symmorphic space group belonging to the arithmetic crystal class to which the space group of the structure belongs. There are thus 73 possible Patterson symmetries.

An equivalent description of such symmetries, in terms of 73 of the 1651 dichromatic colour groups, has been given by Fischer & Knop (1987); see also Wilson (1993).

1.4.3.2. ‘Laue’ symmetry

Similarly, the eleven conventional ‘Laue’ symmetries [*International Tables for Crystallography* (1995), Volume A, p. 40 and elsewhere] involve the explicit assumption of Friedel’s law. If dispersion is appreciable, the ‘Laue’ symmetry may be that of any of the 32 point groups. The point group, in correct orientation, is obtained by dropping the Bravais-lattice symbol from the symbol of the arithmetic crystal class or of the Patterson symmetry.

References

1.1–1.3

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