

9.1. SPHERE PACKINGS AND PACKINGS OF ELLIPSOIDS

Two other types of homogeneous sphere packings (15 and 16) with contact number  $k = 10$  also refer to densest layers of spheres. In these cases, each sphere has three contacts to one neighbouring layer and one contact to the other layer that is stacked directly above or below the original layer.

Cubic closest packings may also be regarded as built up from square layers  $4^4$  stacked in such a way that each sphere has four neighbouring spheres in the same layer and four neighbours each from the layers above and below (cf. Fig. 9.1.1.3). If square layers are stacked such that each sphere has contact to four spheres of one neighbouring layer and to two spheres of the other layer (cf. Fig. 9.1.1.4), sphere packings with contact number 10 result. In total, two types of homogeneous packings (17 and 18) with this kind of stacking exist. Sphere packings of type 9 may also be decomposed into  $4^4$  layers parallel to (101) or (011) in a five-layer sequence. These nets are made up from parallel rhombi and stacked such that each sphere has contact with three other spheres from the layer above and from the layer below. If such layers are stacked in a two-layer sequence, sphere packings of type 13

with symmetry  $Cmcm$  result (O’Keeffe, 1998). Sphere packings of type 14 are also built up from  $4^4$  layers, but here the rhombi occur in two different orientations (O’Keeffe, 1998). Sphere packings with high contact numbers may also be derived by stacking of other layers. Type 20, for example, refers to  $3^46$  layers where each sphere is in contact with three spheres of one neighbouring net and two spheres of the other one (Sowa & Koch, 1999). Such a sphere packing may alternatively be derived from the cubic closest packing by omitting systematically  $1/7$  of the spheres in each of the  $3^6$  nets.

Sphere packings of types 8 and 19 (cf. Figs. 9.1.1.5 and 9.1.1.6) cannot be built up from plane layers of spheres in contact although their contact numbers are also high.

Table 9.1.1.2 contains complete information on homogeneous sphere packings with  $k = 10, 11,$  and  $12$  and with cubic or tetragonal symmetry.

The least dense (most open) homogeneous sphere packings known so far have already been described by Heesch & Laves (1933). Sphere packings of that type (24) cannot be stable because their contact number is 3 (cf. Fig. 9.1.1.7). As discussed

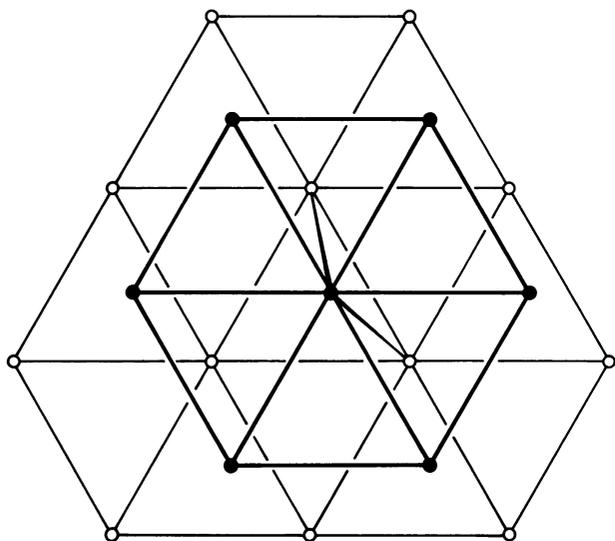


Fig. 9.1.1.2. Two triangular nets representing two densest packed layers of spheres. The layers are stacked in such a way that each sphere is in contact with two spheres of the other layer.

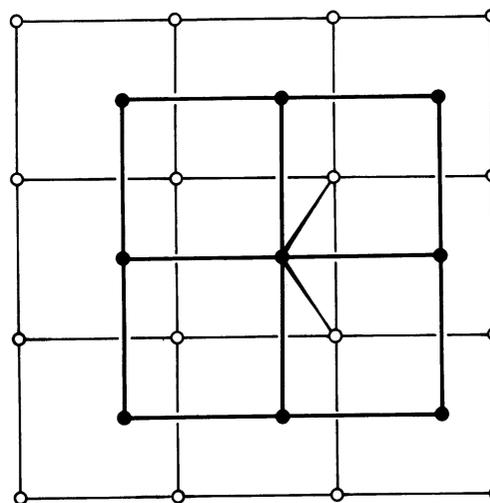


Fig. 9.1.1.4. Two square nets representing two layers of spheres stacked in such a way that each sphere is in contact with two spheres of the other layer.

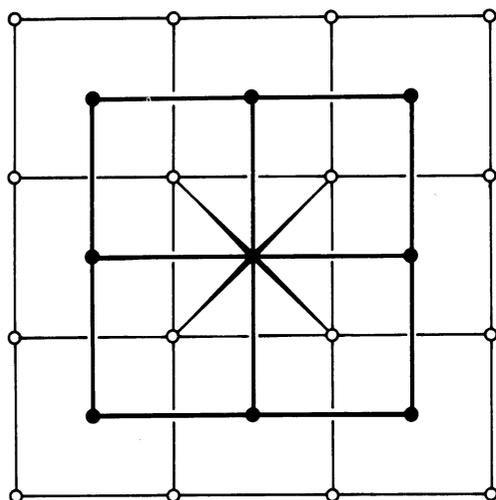


Fig. 9.1.1.3. Two square nets representing two layers of spheres stacked in such a way that each sphere is in contact with four spheres of the other layer.

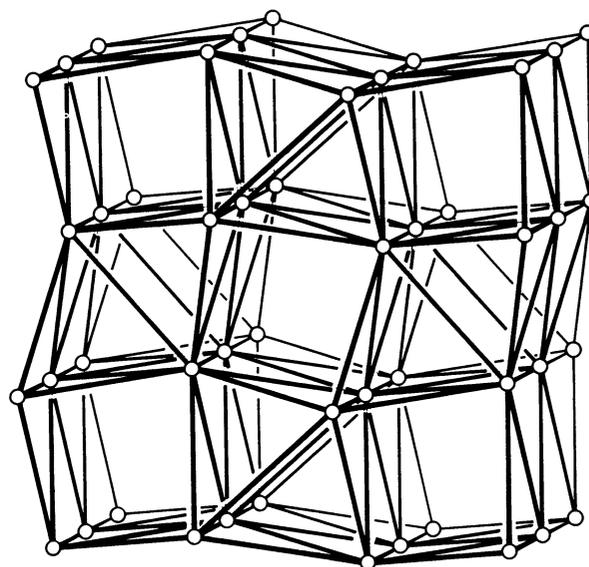


Fig. 9.1.1.5. Sphere packing of type 8 (Table 9.1.1.2) represented by a graph:  $k = 11, P4_2/mmm, 4(f), xx0$ .