International Tables for Crystallography (2006). Vol. D, Section 1.10.5, pp. 255–264.

$$S_e = \begin{pmatrix} \frac{1}{2} & -\alpha & 0 & \frac{1}{2} & 0 & 0 \\ \alpha & 0 & 0 & -\alpha & 0 & 0 \\ 0 & 0 & \alpha & 0 & -\alpha & 0 \\ \frac{1}{2} & \alpha & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \alpha & 0 & \alpha & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix},$$

$$S_f = \begin{pmatrix} -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & 0 & 0 \\ -\frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} & 0 & 0 \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & -\alpha & -\alpha \\ 0 & 0 & 0 & 0 & \alpha & -\alpha \end{pmatrix}.$$

The action on the space of piezoelectric tensors is given, for the phonon and the phason part, by taking the product of these matrices with the physical part A_E. The invariant vectors under these matrices give the invariant tensors. If the second generator is taken into account, which requires that the number of indices 1 or 4 is even, this results in the independent tensor elements

$$x_3 = c_{113}, \quad x_{16} = c_{322}, \quad x_{18} = c_{333}$$

with relation $c_{223} = c_{113}$, whereas all other elements are zero for the coupling between the electric field and phonon strain. There is no nontrivial invariant vector in the second case. Therefore, all tensor elements for the coupling between the electric field and phason strain are zero.

1.10.4.6.5. *Elasticity tensor for an icosahedral quasicrystal*

The point group of an icosahedral quasicrystal is $532(5^2 32)$ with generators having components

$$A_E = \begin{pmatrix} 1 & \tau & -1-\tau \\ \tau & 1+\tau & 1 \\ 1+\tau & -1 & \tau \end{pmatrix} /2, \qquad B_E = \begin{pmatrix} -\tau & 1+\tau & -1 \\ 1+\tau & 1 & \tau \\ 1 & -\tau & -1-\tau \end{pmatrix} /2$$

in physical space and components

$$A_I = \begin{pmatrix} -1 & \tau & -1-\tau \\ -\tau & \tau^{-1} & 1 \\ 1+\tau & 1 & -\tau \end{pmatrix} /2, \quad B_I = \begin{pmatrix} 0 & 0 & -1 \\ -1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

in internal space (see Table 1.10.5.2). The phonon and phason strain tensors form a 6D, respectively 9D, vector space, in which the point group acts with matrices

$$S_e = \frac{1}{4}\begin{pmatrix} 1 & \tau\sqrt{2} & -\varphi\sqrt{2} & 1-\tau & -\sqrt{2} & 2+\tau \\ \tau\sqrt{2} & 2 & 0 & \sqrt{2} & -2 & -\varphi\sqrt{2} \\ \varphi\sqrt{2} & 0 & -2 & -\tau\sqrt{2} & 2 & -\sqrt{2} \\ 1-\tau & \sqrt{2} & \tau\sqrt{2} & 2+\tau & \varphi\sqrt{2} & 1 \\ \sqrt{2} & 2 & 2 & -\varphi\sqrt{2} & 0 & \tau\sqrt{2} \\ 2+\tau & -\varphi\sqrt{2} & \sqrt{2} & 1 & -\tau\sqrt{2} & 1-\tau \end{pmatrix},$$

$$S_f = \frac{1}{4}\begin{pmatrix} -\tau & \varphi & -1 & \tau-1 & 1 & -\tau & 1 & -2-\tau & \varphi \\ \varphi & 1 & \tau & 1 & \tau & 1-\tau & -2-\tau & -\varphi & -1 \\ 1 & -\tau & -\varphi & \tau & \tau-1 & -1 & -\varphi & 1 & 2+\tau \\ \tau-1 & 1 & -\tau & -1 & 2+\tau & -\varphi & -\tau & \varphi & -1 \\ 1 & \tau & 1-\tau & 2+\tau & \varphi & 1 & \varphi & 1 & \tau \\ \tau & \tau-1 & -1 & \varphi & -1 & -2-\tau & 1 & -\tau & -\varphi \\ -1 & 2+\tau & -\varphi & \tau & -\varphi & 1 & \tau-1 & 1 & -\tau \\ 2+\tau & \varphi & 1 & -\varphi & -1 & -\tau & 1 & \tau & 1-\tau \\ \varphi & -1 & -2-\tau & -1 & \tau & \varphi & \tau & \tau-1 & -1 \end{pmatrix}$$

and

$$T_e = \frac{1}{4}\begin{pmatrix} 1 & -\tau\sqrt{2} & \varphi\sqrt{2} & 1-\tau & -\sqrt{2} & 2+\tau \\ \tau\sqrt{2} & -2 & 0 & \sqrt{2} & -2 & -\varphi\sqrt{2} \\ -\varphi\sqrt{2} & 0 & -2 & \tau\sqrt{2} & -2 & \sqrt{2} \\ 1-\tau & -\sqrt{2} & -\tau\sqrt{2} & 2+\tau & \varphi\sqrt{2} & 1 \\ -\sqrt{2} & 2 & 2 & \varphi\sqrt{2} & 0 & -\tau\sqrt{2} \\ 2+\tau & \varphi\sqrt{2} & -\sqrt{2} & 1 & -\tau\sqrt{2} & 1-\tau \end{pmatrix},$$

$$T_f = \frac{1}{2}\begin{pmatrix} 0 & 0 & 1 & 0 & 0 & -\tau & 0 & 0 & \varphi \\ 1 & 0 & 0 & -\tau & 0 & 0 & \varphi & 0 & 0 \\ 0 & -1 & 0 & 0 & \tau & 0 & 0 & -\varphi & 0 \\ 0 & 0 & \tau & 0 & 0 & -\varphi & 0 & 0 & -1 \\ \tau & 0 & 0 & -\varphi & 0 & 0 & -1 & 0 & 0 \\ 0 & -\tau & 0 & 0 & \varphi & 0 & 0 & 1 & 0 \\ 0 & 0 & -\varphi & 0 & 0 & -1 & 0 & 0 & \tau \\ -\varphi & 0 & 0 & -1 & 0 & 0 & \tau & 0 & 0 \\ 0 & \varphi & 0 & 0 & 1 & 0 & 0 & -\tau & 0 \end{pmatrix}.$$

This implies that the phonon elasticity tensors form a 21D space, the phason elasticity tensors a 45D space and the phonon–phason coupling a 54D space. The invariant vectors under these orthogonal transformations correspond to invariant elastic tensors. Their coordinates are the elastic constants. For the given presentation of the point group, these are given in Table 1.10.4.3. The tensor elements are expressed in parameters x and y where there are two independent tensor elements. The tensor elements that are not given are zero or equal to that given by the permutation symmetry. If bases for the phonon and phason strain are introduced by

$$[1] = 11, [2] = 12, [3] = 13, [4] = 22, [5] = 23, [6] = 33$$

for the phonon part and

$$[1] = 14, [2] = 15, [3] = 16, [4] = 24, [5] = 25, [6] = 26,$$
$$[7] = 34, [8] = 35, [9] = 36$$

for the phason part, the elastic tensors may be given in matrix form as

$$c^{ee} = \begin{pmatrix} x+y & 0 & 0 & x & 0 & x \\ 0 & y & 0 & 0 & 0 & 0 \\ 0 & 0 & y & 0 & 0 & 0 \\ x & 0 & 0 & x+y & 0 & x \\ 0 & 0 & 0 & 0 & y & 0 \\ x & 0 & 0 & x & 0 & x+y \end{pmatrix},$$

$$c^{ff} = \begin{pmatrix} z & \tau^2 u & -\tau u & -\tau u & \tau u & -\tau u & -u & 0 & \tau u \\ \tau^2 u & z-2\tau u & u & \tau u & u & 0 & 0 & \tau^2 u & \tau u \\ -\tau u & u & z & -\tau u & 0 & -\tau^2 u & \tau u & \tau u & \tau u \\ -\tau u & \tau u & -\tau u & z & \tau u & u & -\tau^2 u & \tau u & 0 \\ \tau u & u & 0 & \tau u & z & -\tau^2 u & \tau u & -\tau u & -\tau u \\ -\tau u & 0 & -\tau^2 u & u & -\tau^2 u & z-2\tau u & 0 & -\tau u & u \\ -u & 0 & \tau u & -\tau^2 u & \tau u & 0 & z-2\tau u & -u & -\tau^2 u \\ 0 & \tau^2 u & \tau u & \tau u & -\tau u & -\tau u & -u & z & -\tau u \\ \tau u & \tau u & \tau u & 0 & -\tau u & u & -\tau^2 u & -\tau u & z \end{pmatrix},$$

$$c^{ef} = \begin{pmatrix} -v & -\tau v & -\tau^2 v & -\tau^3 v & \tau v & -\tau^2 v & v & -\tau v & -\tau^{-1} v \\ -\tau^3 v & \tau v & -\tau^2 v & \tau^2 v & v & \tau v & 0 & 0 & 0 \\ v & -\tau v & -\tau^{-1} v & 0 & 0 & 0 & -\tau v & -\tau^2 v & -\tau^3 v \\ \tau^{-1} v & v & \tau v & -\tau^2 v & v & -\tau v & -\tau^2 v & -\tau^3 v & \tau v \\ 0 & 0 & 0 & -\tau^2 v & \tau^3 v & \tau v & \tau v & -\tau^{-1} v & v \\ -\tau v & -\tau^2 v & -\tau^3 v & \tau v & -\tau^{-1} v & v & -\tau v & \tau^2 v & v \end{pmatrix}.$$

The parameters x, y, z, u, v are the five independent elastic constants.

1.10.5. Tables

In this section are presented the irreducible representations of point groups of quasiperiodic structures up to rank six that do not occur as three-dimensional crystallographic point groups.

Table 1.10.4.3. *Elastic constants for icosahedral quasicrystals*

Type	Free parameters	Relations
Phonon–phonon	2	$c_{1111} = c_{2222} = c_{3333} = x$ $c_{1122} = c_{1133} = c_{2233} = y$ $c_{1212} = c_{1313} = c_{2323} = x - y$
Phason–phason	2	$c_{1414} = c_{1616} = c_{2424} = c_{2525} = c_{3535} = c_{3636} = x$ $c_{1416} = c_{1424} = -c_{1425} = c_{1426} = -c_{1436} = y$ $c_{1524} = c_{1536} = -c_{1624} = c_{1634} = c_{1635} = -y$ $c_{1636} = c_{2425} = c_{2435} = c_{2534} = -c_{2535} = -y$ $c_{2536} = c_{2635} = c_{3536} = y, c_{1515} = c_{2626} = c_{3434} = x + 2y$ $c_{1415} = c_{1535} = -c_{1626} = -c_{2434} = -c_{2526} = -c_{3436} = y\tau$ $c_{1434} = -c_{11516} = -c_{1525} = -c_{2426} = -c_{2636} = c_{3435} = y/\tau$
Phonon–phason	1	$-c_{1114} = c_{1134} = c_{1225} = c_{2225} = c_{2336} = c_{3326} = c_{3336}$ $= -c_{1115}/\tau = c_{1125}/\tau = -c_{1135}\tau = c_{1215}\tau = c_{1226}\tau$ $= -c_{1334}\tau = -c_{2226}\tau = c_{2236}\tau = c_{2326}\tau = c_{2334}\tau$ $= -c_{3334}\tau = -c_{1116}\tau^2 = -c_{1126}\tau^2 = -c_{1216}\tau^2$ $= -c_{1335}\tau^2 = -c_{2224}\tau^2 = -c_{2234}\tau^2 = c_{3335}\tau^2$ $= -c_{1136}\tau = c_{1224}\tau = -c_{1316}\tau = -c_{2335}\tau$ $= -c_{1124}/\tau^3 = -c_{1336}/\tau^3 = c_{2235}/\tau^3 = c_{2325}/\tau^3$

Table 1.10.5.1. *Character tables of some point groups for quasicrystals*

(*a*) C_5 [$\omega = \exp(2\pi i/5)$].

C_5	ε	α	α^2	α^3	α^4
n	1	1	1	1	1
Order	1	5	5	5	5
Γ_1	1	1	1	1	1
Γ_2	1	ω	ω^2	ω^3	ω^4
Γ_3	1	ω^2	ω^4	ω	ω^3
Γ_4	1	ω^3	ω	ω^4	ω^2
Γ_5	1	ω^4	ω^3	ω^2	ω

	Generators	Vector representation	Perpendicular representation
5	$\alpha = C_{5z}$	$\Gamma_1 \oplus \Gamma_2 \oplus \Gamma_5$	$\Gamma_3 \oplus \Gamma_4$

(*b*) D_5 [$\tau = (\sqrt{5} - 1)/2$].

D_5	ε	α	α^2	β
n	1	2	2	5
Order	1	5	5	2
Γ_1	1	1	1	1
Γ_2	1	1	1	-1
Γ_3	2	τ	$-1 - \tau$	0
Γ_4	2	$-1 - \tau$	τ	0

	Generators	Vector representation	Perpendicular representation
52	$\alpha = C_{5z}$ $\beta = C_{2x}$	$\Gamma_2 \oplus \Gamma_3$	Γ_4
$5m$	$\alpha = C_{5z}$ $\beta = m_x$	$\Gamma_1 \oplus \Gamma_3$	Γ_4
$\bar{5}m$	$\sim 52 \times \mathbb{Z}_2$	$\Gamma_{1u} \oplus \Gamma_{3u}$	Γ_{4u}

(*c*) C_8 [$\omega = \exp(\pi i/4) = (1 + i)/\sqrt{2}$].

C_8	ε	α	α^2	α^3	α^4	α^5	α^6	α^7
n	1	1	1	1	1	1	1	1
Order	1	8	4	8	2	8	6	8
Γ_1	1	1	1	1	1	1	1	1
Γ_2	1	ω	i	ω^3	-1	ω^5	$-i$	ω^7
Γ_3	1	i	-1	$-i$	1	i	-1	$-i$
Γ_4	1	ω^3	$-i$	ω	-1	ω^7	i	ω^5
Γ_5	1	-1	1	-1	1	-1	1	-1
Γ_6	1	ω^5	i	ω^7	-1	ω	$-i$	ω^3
Γ_7	1	$-i$	-1	i	1	$-i$	-1	i
Γ_8	1	ω^7	$-i$	ω^5	-1	ω^3	i	ω

	Generators	Vector representation	Perpendicular representation
8 $\bar{8}$	$\alpha = C_{8z}$ $\alpha = S_{8z}$	$\Gamma_1 \oplus \Gamma_2 \oplus \Gamma_8$ $\Gamma_4 \oplus \Gamma_5 \oplus \Gamma_6$	$\Gamma_4 \oplus \Gamma_6$ $\Gamma_2 \oplus \Gamma_8$
$8/m$	$\sim 8 \times \mathbb{Z}_2$	$\Gamma_{1u} \oplus \Gamma_{2u} \oplus \Gamma_{8u}$	$\Gamma_{4u} \oplus \Gamma_{6u}$

(*d*) D_8

D_8	ε	α	α^2	α^3	α^4	β	$\alpha\beta$
n	1	2	2	2	1	4	4
Order	1	8	4	8	2	2	2
Γ_1	1	1	1	1	1	1	1
Γ_2	1	1	1	1	1	-1	-1
Γ_3	1	-1	1	-1	1	1	-1
Γ_4	1	-1	1	-1	1	-1	1
Γ_5	2	$\sqrt{2}$	0	$-\sqrt{2}$	-2	0	0
Γ_6	2	0	-2	0	2	0	0
Γ_7	2	$-\sqrt{2}$	0	$\sqrt{2}$	-2	0	0

	Generators	Vector representation	Perpendicular representation
822	$\alpha = C_{8z}$ $\beta = C_{2x}$	$\Gamma_2 \oplus \Gamma_5$	Γ_7
$8mm$	$\alpha = C_{8z}$ $\beta = m_x$	$\Gamma_1 \oplus \Gamma_5$	Γ_7
$\bar{8}2m$	$\alpha = S_{8z}$ $\beta = C_{2x}$	$\Gamma_3 \oplus \Gamma_7$	Γ_5
$8/mmm$	$\sim 822 \times \mathbb{Z}_2$	$\Gamma_{2u} \oplus \Gamma_{5u}$	Γ_{7u}

(*e*) C_{10} [$\omega = exp(2\pi i/5)$].

C_{10}	ε	α^2	α^4	α^6	α^8
n	1	1	1	1	1
Order	1	5	5	5	5
Γ_1	1	1	1	1	1
Γ_2	1	ω	ω^2	ω^3	ω^4
Γ_3	1	ω^2	ω^4	ω	ω^3
Γ_4	1	ω^3	ω	ω^4	ω^2
Γ_5	1	ω^4	ω^3	ω^2	ω
Γ_6	1	1	1	1	1
Γ_7	1	ω	ω^2	ω^3	ω^4
Γ_8	1	ω^2	ω^4	ω	ω^3
Γ_9	1	ω^3	ω	ω^4	ω^2
Γ_{10}	1	ω^4	ω^3	ω^2	ω

C_{10}	α^5	α^7	α^9	α	α^3
n	1	1	1	1	1
Order	2	10	10	10	10
Γ_1	1	1	1	1	1
Γ_2	1	ω	ω^2	ω^3	ω^4
Γ_3	1	ω^2	ω^4	ω	ω^3
Γ_4	1	ω^3	ω	ω^4	ω^2
Γ_5	1	ω^4	ω^3	ω^2	ω
Γ_6	-1	-1	-1	-1	-1
Γ_7	-1	$-\omega$	$-\omega^2$	$-\omega^3$	$-\omega^4$
Γ_8	-1	$-\omega^2$	$-\omega^4$	$-\omega$	$-\omega^3$
Γ_9	-1	$-\omega^3$	$-\omega$	$-\omega^4$	$-\omega^2$
Γ_{10}	-1	$-\omega^4$	$-\omega^3$	$-\omega^2$	$-\omega$

Table 1.10.5.1 (*cont.*)

	Generators	Vector representation	Perpendicular representation
10	$\alpha = C_{10z}$	$\Gamma_1 \oplus \Gamma_7 \oplus \Gamma_{10}$	$\Gamma_8 \oplus \Gamma_9$
$\bar{5}$	$\alpha = S_{5z}$	$\Gamma_6 \oplus \Gamma_8 \oplus \Gamma_9$	$\Gamma_7 \oplus \Gamma_{10}$
$\overline{10}$	$\alpha = S_{10z}$	$\Gamma_2 \oplus \Gamma_4 \oplus \Gamma_6$	$\Gamma_3 \oplus \Gamma_5$
$10/m$	$\sim 10 \times \mathbb{Z}_2$	$\Gamma_{1u} \oplus \Gamma_{7u} \oplus \Gamma_{10u}$	$\Gamma_{8u} \oplus \Gamma_{9u}$

(*f*) D_{10} $[\tau = (\sqrt{5} - 1)/2]$.

D_{10}	ε	α	α^2	α^3
n	1	2	2	2
Order	1	10	5	10
Γ_1	1	1	1	1
Γ_2	1	1	1	1
Γ_3	1	-1	1	-1
Γ_4	1	-1	1	-1
Γ_5	2	$1+\tau$	τ	$-\tau$
Γ_6	2	τ	$-1-\tau$	$-1-\tau$
Γ_7	2	$-\tau$	$-1-\tau$	$1+\tau$
Γ_8	2	$-1-\tau$	τ	τ

D_{10}	α^4	α^5	β	$\alpha\beta$
n	2	1	5	5
Order	5	2	2	2
Γ_1	1	1	1	1
Γ_2	1	1	-1	-1
Γ_3	1	-1	1	-1
Γ_4	1	-1	-1	1
Γ_5	$-1-\tau$	-2	0	0
Γ_6	τ	2	0	0
Γ_7	τ	-2	0	0
Γ_8	$-1-\tau$	2	0	0

	Generators	Vector representation	Perpendicular representation
$10\,22$	$\alpha = C_{10z}$ $\beta = C_{2x}$	$\Gamma_2 \oplus \Gamma_5$	Γ_7
$10\,mm$	$\alpha = C_{10z}$ $\beta = m_x$	$\Gamma_1 \oplus \Gamma_5$	Γ_7
$\overline{10}\,2m$	$\alpha = S_{10z}$ $\beta = C_{2x}$	$\Gamma_4 \oplus \Gamma_8$	Γ_6
$\bar{5}m$	$\alpha = S_{5z}$ $\beta = C_{2x}$	$\Gamma_3 \oplus \Gamma_7$	Γ_5
$10/mmm$	$\sim 10\,22 \times \mathbb{Z}_2$	$\Gamma_{2u} \oplus \Gamma_{5u}$	Γ_{7u}

(*g*) C_{12} $[\omega = \exp(\pi i/6)]$.

C_{12}	ε	α	α^2	α^3	α^4	α^5
n	1	1	1	1	1	1
Order	1	12	6	4	3	12
Γ_1	1	1	1	1	1	1
Γ_2	1	ω	ω^2	i	ω^4	ω^5
Γ_3	1	ω^2	ω^4	-1	$-\omega^2$	$-\omega^4$
Γ_4	1	i	-1	$-i$	1	i
Γ_5	1	ω^4	$-\omega^2$	1	ω^4	$-\omega^2$
Γ_6	1	ω^5	$-\omega^4$	i	$-\omega^2$	ω
Γ_7	1	-1	1	-1	1	-1
Γ_8	1	$-\omega$	ω^2	$-i$	ω^4	$-\omega^5$
Γ_9	1	$-\omega^2$	ω^4	1	$-\omega^2$	ω^4
Γ_{10}	1	$-i$	-1	i	1	$-i$
Γ_{11}	1	$-\omega^4$	$-\omega^2$	-1	ω^4	ω^2
Γ_{12}	1	$-\omega^5$	$-\omega^4$	$-i$	$-\omega^2$	$-\omega$

C_{12}	α^6	α^7	α^8	α^9	α^{10}	α^{11}
n	1	1	1	1	1	1
Order	2	12	3	4	6	12
Γ_1	1	1	1	1	1	1
Γ_2	-1	$-\omega$	$-\omega^2$	$-i$	$-\omega^4$	$-\omega^5$
Γ_3	1	ω^2	ω^4	-1	$-\omega^2$	$-\omega^4$
Γ_4	-1	$-i$	1	i	-1	$-i$
Γ_5	1	ω^4	$-\omega^2$	1	ω^4	$-\omega^2$
Γ_6	-1	$-\omega^5$	ω^4	$-i$	ω^2	$-\omega$
Γ_7	1	-1	1	-1	1	-1
Γ_8	-1	ω	$-\omega^2$	i	$-\omega^4$	ω^5
Γ_9	1	$-\omega^2$	ω^4	1	$-\omega^2$	ω^4
Γ_{10}	-1	i	1	$-i$	-1	i
Γ_{11}	1	$-\omega^4$	$-\omega^2$	-1	ω^4	ω^2
Γ_{12}	-1	ω^5	ω^4	i	ω^2	ω

	Generators	Vector representation	Perpendicular representation
12	$\alpha = C_{12z}$	$\Gamma_1 \oplus \Gamma_2 \oplus \Gamma_{12}$	$\Gamma_6 \oplus \Gamma_8$
$\overline{12}$	$\alpha = S_{12z}$	$\Gamma_6 \oplus \Gamma_7 \oplus \Gamma_8$	$\Gamma_2 \oplus \Gamma_{12}$
$12/m$	$\sim 12 \times \mathbb{Z}_2$	$\Gamma_{1u} \oplus \Gamma_{2u} \oplus \Gamma_{12u}$	$\Gamma_{6u} \oplus \Gamma_{8u}$

(*h*) D_{12}

D_{12}	ε	α	α^2	α^3
n	1	2	2	2
Order	1	12	6	4
Γ_1	1	1	1	1
Γ_2	1	1	1	1
Γ_3	1	-1	1	-1
Γ_4	1	-1	1	-1
Γ_5	2	$\sqrt{3}$	1	0
Γ_6	2	1	-1	-2
Γ_7	2	0	-2	0
Γ_8	2	-1	-1	2
Γ_9	2	$-\sqrt{3}$	1	0

D_{12}	α^4	α^5	α^6	β	$\alpha\beta$
n	2	2	1	6	6
Order	3	12	2	2	2
Γ_1	1	1	1	1	1
Γ_2	1	1	1	-1	-1
Γ_3	1	-1	1	1	-1
Γ_4	1	-1	1	-1	1
Γ_5	-1	$-\sqrt{3}$	-2	0	0
Γ_6	-1	1	2	0	0
Γ_7	2	0	-2	0	0
Γ_8	-1	-1	2	0	0
Γ_9	-1	$\sqrt{3}$	-2	0	0

	Generators	Vector representation	Perpendicular representation
$12\,22$	$\alpha = C_{12z}$ $\beta = C_{2x}$	$\Gamma_2 \oplus \Gamma_5$	Γ_9
$12\,mm$	$\alpha = C_{12z}$ $\beta = m_x$	$\Gamma_1 \oplus \Gamma_5$	Γ_9
$\overline{12}\,2m$	$\alpha = S_{12z}$ $\beta = C_{2x}$	$\Gamma_4 \oplus \Gamma_9$	Γ_5
$12/mmm$	$\sim 12\,22 \times \mathbb{Z}_2$	$\Gamma_{2u} \oplus \Gamma_{5u}$	Γ_{9u}

(*i*) I $[\tau = (\sqrt{5} - 1)/2]$.

I	ε	α	α^2	β	$\alpha\beta$
n	1	12	12	20	15
Order	1	5	5	3	2
Γ_1	1	1	1	1	1
Γ_2	3	$1+\tau$	$-\tau$	0	-1
Γ_3	3	$-\tau$	$1+\tau$	0	-1
Γ_4	4	-1	-1	1	0
Γ_5	5	0	0	-1	1

	Generators	Vector representation	Perpendicular representation
532	$\alpha = C_5$ $\beta = C_{3d}$	Γ_2	Γ_3
$\bar{5}\bar{3}m$	$\sim 532 \times \mathbb{Z}_2$	Γ_{2u}	Γ_{3u}

Table 1.10.5.2. *Matrices of the irreducible representations of dimension $d \geq 2$ corresponding to the irreps of Table 1.10.5.1*

(*a*) D_5

Representation	$D(\alpha^p)$	$D(\beta)$
Γ_3	$\begin{pmatrix} \cos(2\pi p/5) & -\sin(2\pi p/5) \\ \sin(2\pi p/5) & \cos(2\pi p/5) \end{pmatrix}$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
Γ_4	$\begin{pmatrix} \cos(4\pi p/5) & -\sin(4\pi p/5) \\ \sin(4\pi p/5) & \cos(4\pi p/5) \end{pmatrix}$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

(*b*) D_8

Representation	$D(\alpha^p)$	$D(\beta)$
Γ_5	$\begin{pmatrix} \cos(\pi p/4) & -\sin(\pi p/4) \\ \sin(\pi p/4) & \cos(\pi p/4) \end{pmatrix}$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
Γ_6	$\begin{pmatrix} \cos(\pi p/2) & -\sin(\pi p/2) \\ \sin(\pi p/2) & \cos(\pi p/2) \end{pmatrix}$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
Γ_7	$\begin{pmatrix} \cos(3\pi p/4) & -\sin(3\pi p/4) \\ \sin(3\pi p/4) & \cos(3\pi p/4) \end{pmatrix}$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

(*c*) D_{10}

Representation	$D(\alpha^p)$	$D(\beta)$
Γ_5	$\begin{pmatrix} \cos(\pi p/5) & -\sin(\pi p/5) \\ \sin(\pi p/5) & \cos(\pi p/5) \end{pmatrix}$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
Γ_6	$\begin{pmatrix} \cos(2\pi p/5) & -\sin(2\pi p/5) \\ \sin(2\pi p/5) & \cos(2\pi p/5) \end{pmatrix}$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
Γ_7	$\begin{pmatrix} \cos(3\pi p/5) & -\sin(3\pi p/5) \\ \sin(3\pi p/5) & \cos(3\pi p/5) \end{pmatrix}$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
Γ_8	$\begin{pmatrix} \cos(4\pi p/5) & -\sin(4\pi p/5) \\ \sin(4\pi p/5) & \cos(4\pi p/5) \end{pmatrix}$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

(*d*) D_{12}

Representation	$D(\alpha^p)$	$D(\beta)$
Γ_5	$\begin{pmatrix} \cos(\pi p/6) & -\sin(\pi p/6) \\ \sin(\pi p/6) & \cos(\pi p/6) \end{pmatrix}$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
Γ_6	$\begin{pmatrix} \cos(\pi p/3) & -\sin(\pi p/3) \\ \sin(\pi p/3) & \cos(\pi p/3) \end{pmatrix}$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
Γ_7	$\begin{pmatrix} \cos(\pi p/2) & -\sin(\pi p/2) \\ \sin(\pi p/2) & \cos(\pi p/2) \end{pmatrix}$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
Γ_8	$\begin{pmatrix} \cos(2\pi p/3) & -\sin(2\pi p/3) \\ \sin(2\pi p/3) & \cos(2\pi p/3) \end{pmatrix}$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
Γ_9	$\begin{pmatrix} \cos(5\pi p/6) & -\sin(5\pi p/6) \\ \sin(5\pi p/6) & \cos(5\pi p/6) \end{pmatrix}$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

Table 1.10.5.1 gives the characters of the point groups C_n with $n = 5, 8, 10, 12$, D_n with $n = 5, 8, 10, 12$, and the icosahedral group I. The direct products with $\mathbb{Z}_2$ then follow easily. Although these direct products of a group K with $\mathbb{Z}_2$ do not belong to the isomorphism class of K, their irreducible representations are nevertheless given in the table for K because these irreducible representations have the same labels as those for K apart from an additional subindex u. The reresentations of the subgroup K of $K \times \mathbb{Z}_2$ are the same as for K itself, those for the cosets get an additional minus sign. In the tables, the characters for the groups $K \times \mathbb{Z}_2$ are separated from those for K by a horizontal rule. In addition to the characters are given the realizations of crystallographic point groups, and the irreducible components of the vector representations in direct space V_E and internal space V_I for these realizations. The vector representation in V_I is called the perpendicular representation.

In Table 1.10.5.2 the representation matrices for the irreducible representations in more than one dimension are given (one-dimensional representations are just the characters). For the cyclic groups there are only one-dimensional representations, for the dihedral groups there are one- and two-dimensional irreducible representations. There are four irreducible representations of I of dimension larger than one. The four- and five-dimensional ones are given as integer representations. They form

Table 1.10.5.2 (*cont.*)

(*e*) *I*. First column: numbering of the elements. $f = (1+\sqrt{5})/2$, $t = (\sqrt{5}-1)/2$. Horizontal rules separate conjugation classes.

No.	Order	Γ_2	Γ_4	Γ_5
1	1	$\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}$	$\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{pmatrix}$	$\begin{pmatrix}1&0&0&0&0\\0&1&0&0&0\\0&0&1&0&0\\0&0&0&1&0\\0&0&0&0&1\end{pmatrix}$
2	5	$\begin{pmatrix}1/2&t/2&-f/2\\t/2&f/2&1/2\\f/2&-1/2&t/2\end{pmatrix}$	$\begin{pmatrix}0&0&0&-1\\1&0&0&-1\\0&1&0&-1\\0&0&1&-1\end{pmatrix}$	$\begin{pmatrix}1&0&0&0&-1\\0&0&0&0&-1\\0&1&0&0&-1\\0&0&1&0&-1\\0&0&0&1&-1\end{pmatrix}$
3	5	$\begin{pmatrix}1/2&-t/2&f/2\\-t/2&f/2&1/2\\f/2&-1/2&t/2\end{pmatrix}$	$\begin{pmatrix}0&0&1&-1\\1&0&0&-1\\0&0&0&-1\\0&1&0&-1\end{pmatrix}$	$\begin{pmatrix}0&-1&1&0&0\\0&-1&0&0&1\\0&-1&0&0&0\\0&-1&0&1&0\\1&-1&0&0&0\end{pmatrix}$
4	5	$\begin{pmatrix}1/2&t/2&f/2\\t/2&f/2&-1/2\\-f/2&1/2&t/2\end{pmatrix}$	$\begin{pmatrix}-1&1&0&0\\-1&0&1&0\\-1&0&0&1\\-1&0&0&0\end{pmatrix}$	$\begin{pmatrix}1&-1&0&0&0\\0&-1&1&0&0\\0&-1&0&1&0\\0&-1&0&0&1\\0&-1&0&0&0\end{pmatrix}$
5	5	$\begin{pmatrix}t/2&-f/2&1/2\\f/2&1/2&t/2\\-1/2&t/2&f/2\end{pmatrix}$	$\begin{pmatrix}0&-1&0&0\\0&-1&0&1\\1&-1&0&0\\0&-1&1&0\end{pmatrix}$	$\begin{pmatrix}0&1&0&-1&0\\0&0&0&-1&1\\0&0&1&-1&0\\1&0&0&-1&0\\0&0&0&-1&0\end{pmatrix}$
6	5	$\begin{pmatrix}f/2&-1/2&-t/2\\1/2&t/2&f/2\\-t/2&-f/2&1/2\end{pmatrix}$	$\begin{pmatrix}0&1&-1&0\\0&0&-1&0\\0&0&-1&1\\1&0&-1&0\end{pmatrix}$	$\begin{pmatrix}0&1&0&0&0\\0&0&0&1&0\\0&0&0&0&1\\0&0&1&0&0\\1&0&0&0&0\end{pmatrix}$
7	5	$\begin{pmatrix}f/2&1/2&t/2\\-1/2&t/2&f/2\\t/2&-f/2&1/2\end{pmatrix}$	$\begin{pmatrix}0&-1&0&1\\0&-1&1&0\\1&-1&0&0\\0&-1&0&0\end{pmatrix}$	$\begin{pmatrix}-1&0&1&0&0\\-1&1&0&0&0\\-1&0&0&0&1\\-1&0&0&0&0\\-1&0&0&1&0\end{pmatrix}$
8	5	$\begin{pmatrix}t/2&f/2&-1/2\\-f/2&1/2&t/2\\1/2&t/2&f/2\end{pmatrix}$	$\begin{pmatrix}-1&0&1&0\\-1&0&0&0\\-1&0&0&1\\-1&1&0&0\end{pmatrix}$	$\begin{pmatrix}0&0&0&1&-1\\1&0&0&0&-1\\0&0&1&0&-1\\0&0&0&0&-1\\0&1&0&0&-1\end{pmatrix}$
9	5	$\begin{pmatrix}t/2&f/2&1/2\\-f/2&1/2&-t/2\\-1/2&-t/2&f/2\end{pmatrix}$	$\begin{pmatrix}0&0&-1&0\\0&0&-1&1\\0&1&-1&0\\1&0&-1&0\end{pmatrix}$	$\begin{pmatrix}-1&0&0&1&0\\-1&0&1&0&0\\-1&0&0&0&0\\-1&1&0&0&0\\-1&0&0&0&1\end{pmatrix}$
10	5	$\begin{pmatrix}f/2&1/2&-t/2\\-1/2&t/2&-f/2\\-t/2&f/2&1/2\end{pmatrix}$	$\begin{pmatrix}0&-1&0&1\\1&-1&0&0\\0&-1&0&0\\0&-1&1&0\end{pmatrix}$	$\begin{pmatrix}0&0&0&0&1\\1&0&0&0&0\\0&0&0&1&0\\0&1&0&0&0\\0&0&1&0&0\end{pmatrix}$
11	5	$\begin{pmatrix}1/2&-t/2&-f/2\\-t/2&f/2&-1/2\\f/2&1/2&t/2\end{pmatrix}$	$\begin{pmatrix}0&1&-1&0\\0&0&-1&1\\1&0&-1&0\\0&0&-1&0\end{pmatrix}$	$\begin{pmatrix}0&0&-1&0&1\\0&0&-1&0&0\\1&0&-1&0&0\\0&0&-1&1&0\\0&1&-1&0&0\end{pmatrix}$
12	5	$\begin{pmatrix}f/2&-1/2&t/1\\1/2&t/2&-f/2\\t/2&f/2&1/2\end{pmatrix}$	$\begin{pmatrix}0&0&1&-1\\0&0&0&-1\\0&1&0&-1\\1&0&0&-1\end{pmatrix}$	$\begin{pmatrix}0&0&0&-1&0\\0&1&0&-1&0\\1&0&0&-1&0\\0&0&0&-1&1\\0&0&1&-1&0\end{pmatrix}$

Table 1.10.5.2 (*cont.*)

No.	Order	Γ_2	Γ_4	Γ_5
13	5	$\begin{pmatrix} t/2 & -f/2 & -1/2 \\ f/2 & 1/2 & -t/2 \\ 1/2 & -t/2 & f/2 \end{pmatrix}$	$\begin{pmatrix} -1 & 0 & 0 & 1 \\ -1 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 1 \end{pmatrix}$
14	5	$\begin{pmatrix} -t/2 & f/2 & -1/2 \\ f/2 & 1/2 & t/2 \\ 1/2 & -t/2 & -f/2 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & -1 & 1 \\ 0 & 0 & -1 & 0 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & -1 & 0 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 1 & -1 & 0 \end{pmatrix}$
15	5	$\begin{pmatrix} -t/2 & f/2 & 1/2 \\ f/2 & 1/2 & -t/2 \\ -1/2 & t/2 & -f/2 \end{pmatrix}$	$\begin{pmatrix} 0 & -1 & 1 & 0 \\ 0 & -1 & 0 & 1 \\ 0 & -1 & 0 & 0 \\ 1 & -1 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 0 & 1 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \end{pmatrix}$
16	5	$\begin{pmatrix} -f/2 & 1/2 & -t/2 \\ -1/2 & -t/2 & f/2 \\ t/2 & f/2 & 1/2 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & -1 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & -1 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 & 0 \\ 0 & -1 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 \\ 1 & -1 & 0 & 0 & 0 \end{pmatrix}$
17	5	$\begin{pmatrix} -t/2 & -f/2 & 1/2 \\ -f/2 & 1/2 & t/2 \\ -1/2 & -t/2 & -f/2 \end{pmatrix}$	$\begin{pmatrix} 0 & -1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & -1 & 0 & 1 \\ 1 & -1 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & 0 & -1 \end{pmatrix}$
18	5	$\begin{pmatrix} -t/2 & -f/2 & -1/2 \\ -f/2 & 1/2 & -t/2 \\ 1/2 & t/2 & -f/2 \end{pmatrix}$	$\begin{pmatrix} -1 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 \end{pmatrix}$	$\begin{pmatrix} -1 & 1 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 & 0 \end{pmatrix}$
19	5	$\begin{pmatrix} -f/2 & -1/2 & t/2 \\ 1/2 & -t/2 & f/2 \\ -t/2 & f/2 & 1/2 \end{pmatrix}$	$\begin{pmatrix} 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & -1 \end{pmatrix}$	$\begin{pmatrix} -1 & 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 \end{pmatrix}$
20	5	$\begin{pmatrix} -f/2 & 1/2 & t/2 \\ -1/2 & -t/2 & -f/2 \\ -t/2 & -f/2 & 1/2 \end{pmatrix}$	$\begin{pmatrix} 0 & -1 & 1 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & -1 & 0 & 1 \\ 0 & -1 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & -1 & 1 \end{pmatrix}$
21	5	$\begin{pmatrix} 1/2 & -t/2 & f/2 \\ t/2 & -f/2 & -1/2 \\ f/2 & 1/2 & -t/2 \end{pmatrix}$	$\begin{pmatrix} -1 & 1 & 0 & 0 \\ -1 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 & 1 & -1 \\ 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & -1 \\ 1 & 0 & 0 & 0 & -1 \end{pmatrix}$
22	5	$\begin{pmatrix} 1/2 & -t/2 & -f/2 \\ t/2 & -f/2 & 1/2 \\ -f/2 & -1/2 & -t/2 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & -1 \\ 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -1 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \end{pmatrix}$
23	5	$\begin{pmatrix} 1/2 & t/2 & f/2 \\ -t/2 & -f/2 & 1/2 \\ f/2 & -1/2 & -t/2 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & -1 & 0 \\ 1 & 0 & -1 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & 1 & -1 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & -1 & 0 & 1 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 \\ 1 & 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \end{pmatrix}$
24	5	$\begin{pmatrix} 1/2 & t/2 & -f/2 \\ -t/2 & -f/2 & -1/2 \\ -f/2 & 1/2 & -t/2 \end{pmatrix}$	$\begin{pmatrix} -1 & 0 & 1 & 0 \\ -1 & 0 & 0 & 1 \\ -1 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix}$

Table 1.10.5.2 (*cont.*)

No.	Order	Γ_2	Γ_4	Γ_5
25	5	$\begin{pmatrix} -f/2 & -1/2 & -t/2 \\ 1/2 & -t/2 & -f/2 \\ t/2 & -f/2 & 1/2 \end{pmatrix}$	$\begin{pmatrix} 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & -1 \end{pmatrix}$	$\begin{pmatrix} 0 & -1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 \end{pmatrix}$
26	3	$\begin{pmatrix} -1/2 & t/2 & -f/2 \\ -t/2 & f/2 & 1/2 \\ f/2 & 1/2 & -t/2 \end{pmatrix}$	$\begin{pmatrix} 1 & -1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 & -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 & 1 \\ 1 & -1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \end{pmatrix}$
27	3	$\begin{pmatrix} -1/2 & -t/2 & f/2 \\ t/2 & f/2 & 1/2 \\ -f/2 & 1/2 & -t/2 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & -1 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 & -1 \\ 1 & 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 & -1 \end{pmatrix}$
28	3	$\begin{pmatrix} -1/2 & t/2 & f/1 \\ -t/2 & f/2 & -1/2 \\ -f/2 & -1/2 & -t/2 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 1 \\ 1 & 0 & -1 & 0 & 0 \end{pmatrix}$
29	3	$\begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & -1 \\ 1 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 & -1 \end{pmatrix}$
30	3	$\begin{pmatrix} -1/2 & -t/2 & -f/2 \\ t/2 & f/2 & -1/2 \\ f/2 & -1/2 & -t/2 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & -1 & 0 & 0 & 1 \\ 0 & -1 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 & 0 \end{pmatrix}$
31	3	$\begin{pmatrix} 0 & 0 & -1 \\ 1 & 0 & 0 \\ 0 & -1 & 0 \end{pmatrix}$	$\begin{pmatrix} 1 & -1 & 0 & 0 \\ 0 & -1 & 0 & 1 \\ 0 & -1 & 1 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & -1 \\ 0 & 1 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 & -1 \end{pmatrix}$
32	3	$\begin{pmatrix} f/2 & 1/2 & -t/2 \\ 1/2 & -t/2 & f/2 \\ t/2 & -f/2 & -1/2 \end{pmatrix}$	$\begin{pmatrix} -1 & 0 & 1 & 0 \\ -1 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} -1 & 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 & 1 \\ -1 & 0 & 1 & 0 & 0 \end{pmatrix}$
33	3	$\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 1 & -1 & 0 \\ 1 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & -1 & 0 \end{pmatrix}$
34	3	$\begin{pmatrix} 0 & 0 & -1 \\ -1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \\ 1 & 0 & 0 & -1 \end{pmatrix}$	$\begin{pmatrix} 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 1 \\ 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 \end{pmatrix}$
35	3	$\begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & 1 \\ -1 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} -1 & 0 & 0 & 1 \\ -1 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 & -1 & 1 \\ 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 1 & 0 & -1 & 0 \end{pmatrix}$
36	3	$\begin{pmatrix} f/2 & -1/2 & t/2 \\ -1/2 & -t/2 & f/2 \\ -t/2 & -f/2 & -1/2 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & -1 & 0 & 0 & 1 \\ 0 & -1 & 0 & 1 & 0 \\ 1 & -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 \end{pmatrix}$

Table 1.10.5.2 (*cont.*)

No.	Order	Γ_2	Γ_4	Γ_5
37	3	$\begin{pmatrix} 0 & 0 & 1 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \end{pmatrix}$	$\begin{pmatrix} -1 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ -1 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 1 \\ 1 & 0 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 \end{pmatrix}$
38	3	$\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & -1 \\ -1 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & -1 \\ 0 & 1 & 0 & -1 \end{pmatrix}$	$\begin{pmatrix} -1 & 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 1 & 0 \\ -1 & 1 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 \end{pmatrix}$
39	3	$\begin{pmatrix} f/2 & 1/2 & t/2 \\ 1/2 & -t/2 & -f/2 \\ -t/2 & f/2 & -1/2 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 1 & -1 & 0 \\ 1 & 0 & -1 & 0 \\ 0 & 0 & -1 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 & -1 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 \\ 0 & -1 & 1 & 0 & 0 \\ 0 & -1 & 0 & 1 & 0 \end{pmatrix}$
40	3	$\begin{pmatrix} -t/2 & -f/2 & 1/2 \\ f/2 & -1/2 & -t/2 \\ 1/2 & t/2 & f/2 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix}$	$\begin{pmatrix} -1 & 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 & 1 \\ -1 & 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 \end{pmatrix}$
41	3	$\begin{pmatrix} -t/2 & -f/2 & -1/2 \\ f/2 & -1/2 & t/2 \\ -1/2 & -t/2 & f/2 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & -1 & 1 & 0 \\ 1 & 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 1 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \end{pmatrix}$
42	3	$\begin{pmatrix} -t/2 & f/2 & 1/2 \\ -f/2 & -1/2 & t/2 \\ 1/2 & -t/2 & f/2 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & -1 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & -1 & 1 \\ 1 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 & 0 \end{pmatrix}$
43	3	$\begin{pmatrix} -t/2 & f/2 & -1/2 \\ -f/2 & -1/2 & -t/2 \\ -1/2 & t/2 & f/2 \end{pmatrix}$	$\begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & -1 \end{pmatrix}$
44	3	$\begin{pmatrix} f/2 & -1/2 & -t/2 \\ -1/2 & -t/2 & -f/2 \\ t/2 & f/2 & -1/2 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 & 1 \\ 0 & 1 & 0 & -1 & 0 \\ 1 & 0 & 0 & -1 & 0 \end{pmatrix}$
45	3	$\begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & -1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & -1 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} -1 & 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 1 \\ -1 & 1 & 0 & 0 & 0 \end{pmatrix}$
46	2	$\begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$	$\begin{pmatrix} 0 & 1 & 0 & -1 \\ 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & -1 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix}$
47	2	$\begin{pmatrix} -f/2 & 1/2 & t/2 \\ 1/2 & t/2 & f/2 \\ t/2 & f/2 & -1/2 \end{pmatrix}$	$\begin{pmatrix} -1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ -1 & 0 & 0 & 1 \\ -1 & 0 & 1 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$
48	2	$\begin{pmatrix} -f/2 & -1/2 & -t/2 \\ -1/2 & t/2 & f/2 \\ -t/2 & f/2 & -1/2 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} -1 & 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 \\ -1 & 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 & 1 \end{pmatrix}$

Table 1.10.5.2 (*cont.*)

No.	Order	Γ_2	Γ_4	Γ_5
49	2	$\begin{pmatrix} -f/2 & -1/2 & t/2 \\ -1/2 & t/2 & -f/2 \\ t/2 & -f/2 & -1/2 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & -1 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix}$
50	2	$\begin{pmatrix} -f/2 & 1/2 & -t/2 \\ 1/2 & t/2 & -f/2 \\ -t/2 & -f/2 & -1/2 \end{pmatrix}$	$\begin{pmatrix} -1 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ -1 & 1 & 0 & 0 \\ -1 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 & -1 & 1 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & -1 & 0 \end{pmatrix}$
51	2	$\begin{pmatrix} t/2 & f/2 & -1/2 \\ f/2 & -1/2 & -t/2 \\ -1/2 & -t/2 & -f/2 \end{pmatrix}$	$\begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$	$\begin{pmatrix} -1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 1 \\ -1 & 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 \\ -1 & 1 & 0 & 0 & 0 \end{pmatrix}$
52	2	$\begin{pmatrix} t/2 & f/2 & 1/2 \\ f/2 & -1/2 & t/2 \\ 1/2 & t/2 & -f/2 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & -1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & -1 \end{pmatrix}$	$\begin{pmatrix} 0 & 1 & -1 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 0 & 1 \end{pmatrix}$
53	2	$\begin{pmatrix} -1/2 & t/2 & f/2 \\ t/2 & -f/2 & 1/2 \\ f/2 & 1/2 & t/2 \end{pmatrix}$	$\begin{pmatrix} 0 & -1 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix}$
54	2	$\begin{pmatrix} -1/2 & t/2 & -f/2 \\ t/2 & -f/2 & -1/2 \\ -f/2 & -1/2 & t/2 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & -1 & 1 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & -1 & 0 \\ 1 & 0 & -1 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 1 & 0 & -1 \\ 0 & 1 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & -1 \end{pmatrix}$
55	2	$\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$	$\begin{pmatrix} 0 & -1 & 0 & 1 \\ 0 & -1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 1 & -1 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 \end{pmatrix}$
56	2	$\begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} -1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 \\ -1 & 0 & 1 & 0 \\ -1 & 1 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 1 & -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 \\ 0 & -1 & 0 & 1 & 0 \\ 0 & -1 & 1 & 0 & 0 \end{pmatrix}$
57	2	$\begin{pmatrix} -1/2 & -t/2 & -f/2 \\ -t/2 & -f/2 & 1/2 \\ -f/2 & 1/2 & t/2 \end{pmatrix}$	$\begin{pmatrix} 1 & -1 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & -1 & 0 & 1 \\ 0 & -1 & 1 & 0 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 1 \\ 0 & 0 & -1 & 1 & 0 \end{pmatrix}$
58	2	$\begin{pmatrix} t/2 & -f/2 & -1/2 \\ -f/2 & -1/2 & t/2 \\ -1/2 & t/2 & -f/2 \end{pmatrix}$	$\begin{pmatrix} 0 & 1 & -1 & 0 \\ 1 & 0 & -1 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & -1 & 1 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 & 1 \end{pmatrix}$
59	2	$\begin{pmatrix} t/2 & -f/2 & 1/2 \\ -f/2 & -1/2 & -t/2 \\ 1/2 & -t/2 & -f/2 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 1 & -1 \\ 0 & 1 & 0 & -1 \\ 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & -1 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & -1 \end{pmatrix}$
60	2	$\begin{pmatrix} -1/2 & -t/2 & f/2 \\ -t/2 & -f/2 & -1/2 \\ f/2 & -1/2 & t/2 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{pmatrix}$

Table 1.10.5.3. *The representation matrices for* Γ_3

The representation matrices for Γ_3 are the same as for Γ_2. Correspondences are given as pairs i, j: $\Gamma_3(R_i) = \Gamma_2(R_j)$.

i	j	i	j	i	j	i	j	i	j	i	j
1	1	11	21	21	5	31	42	41	29	51	48
2	14	12	16	22	6	32	45	42	39	52	54
3	23	13	17	23	8	33	36	43	33	53	46
4	15	14	4	24	10	34	27	44	30	54	50
5	25	15	2	25	11	35	26	45	38	55	52
6	24	16	13	26	34	36	28	46	49	56	57
7	19	17	12	27	35	37	31	47	53	57	59
8	20	18	7	28	43	38	40	48	51	58	56
9	18	19	9	29	44	39	37	49	47	59	58
10	22	20	3	30	41	40	32	50	55	60	60

crystallographic groups in 4D and 5D. The two three-dimensional representations have the same matrices. The elements, however, are connected by an outer automorphism. That means that the ith element R_i is represented by $\Gamma_2(R_i)$ in the representation Γ_2, and by $\Gamma_3(R_i) = \Gamma_2(\varphi R_i)$ in Γ_3. The element φR_i is another element R_j. The corresponding j for each i is given in Table 1.10.5.3.

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