International Tables for Crystallography (2006). Vol. D, Table 1.10.5.1, pp. 256–257.

Table 1.10.4.3. *Elastic constants for icosahedral quasicrystals*

Type	Free parameters	Relations
Phonon–phonon	2	$c_{1111} = c_{2222} = c_{3333} = x$ $c_{1122} = c_{1133} = c_{2233} = y$ $c_{1212} = c_{1313} = c_{2323} = x - y$
Phason–phason	2	$c_{1414} = c_{1616} = c_{2424} = c_{2525} = c_{3535} = c_{3636} = x$ $c_{1416} = c_{1424} = -c_{1425} = c_{1426} = -c_{1436} = y$ $c_{1524} = c_{1536} = -c_{1624} = c_{1634} = c_{1635} = -y$ $c_{1636} = c_{2425} = c_{2435} = c_{2534} = -c_{2535} = -y$ $c_{2536} = c_{2635} = c_{3536} = y, c_{1515} = c_{2626} = c_{3434} = x + 2y$ $c_{1415} = c_{1535} = -c_{1626} = -c_{2434} = -c_{2526} = -c_{3436} = y\tau$ $c_{1434} = -c_{11516} = -c_{1525} = -c_{2426} = -c_{2636} = c_{3435} = y/\tau$
Phonon–phason	1	$-c_{1114} = c_{1134} = c_{1225} = c_{2225} = c_{2336} = c_{3326} = c_{3336}$ $= -c_{1115}/\tau = c_{1125}/\tau = -c_{1135}\tau = c_{1215}\tau = c_{1226}\tau$ $= -c_{1334}\tau = -c_{2226}\tau = c_{2236}\tau = c_{2326}\tau = c_{2334}\tau$ $= -c_{3334}\tau = -c_{1116}\tau^2 = -c_{1126}\tau^2 = -c_{1216}\tau^2$ $= -c_{1335}\tau^2 = -c_{2224}\tau^2 = -c_{2234}\tau^2 = c_{3335}\tau^2$ $= -c_{1136}\tau = c_{1224}\tau = -c_{1316}\tau = -c_{2335}\tau$ $= -c_{1124}/\tau^3 = -c_{1336}/\tau^3 = c_{2235}/\tau^3 = c_{2325}/\tau^3$

Table 1.10.5.1. *Character tables of some point groups for quasicrystals*

(*a*) C_5 [$\omega = \exp(2\pi i/5)$].

C_5	ε	α	α^2	α^3	α^4
n	1	1	1	1	1
Order	1	5	5	5	5
Γ_1	1	1	1	1	1
Γ_2	1	ω	ω^2	ω^3	ω^4
Γ_3	1	ω^2	ω^4	ω	ω^3
Γ_4	1	ω^3	ω	ω^4	ω^2
Γ_5	1	ω^4	ω^3	ω^2	ω

	Generators	Vector representation	Perpendicular representation
5	$\alpha = C_{5z}$	$\Gamma_1 \oplus \Gamma_2 \oplus \Gamma_5$	$\Gamma_3 \oplus \Gamma_4$

(*b*) D_5 [$\tau = (\sqrt{5} - 1)/2$].

D_5	ε	α	α^2	β
n	1	2	2	5
Order	1	5	5	2
Γ_1	1	1	1	1
Γ_2	1	1	1	−1
Γ_3	2	τ	$-1 - \tau$	0
Γ_4	2	$-1 - \tau$	τ	0

	Generators	Vector representation	Perpendicular representation
52	$\alpha = C_{5z}$ $\beta = C_{2x}$	$\Gamma_2 \oplus \Gamma_3$	Γ_4
$5m$	$\alpha = C_{5z}$ $\beta = m_x$	$\Gamma_1 \oplus \Gamma_3$	Γ_4
$\bar{5}m$	$\sim 52 \times \mathbb{Z}_2$	$\Gamma_{1u} \oplus \Gamma_{3u}$	Γ_{4u}

(*c*) C_8 [$\omega = \exp(\pi i/4) = (1 + i)/\sqrt{2}$].

C_8	ε	α	α^2	α^3	α^4	α^5	α^6	α^7
n	1	1	1	1	1	1	1	1
Order	1	8	4	8	2	8	6	8
Γ_1	1	1	1	1	1	1	1	1
Γ_2	1	ω	i	ω^3	−1	ω^5	$-i$	ω^7
Γ_3	1	i	−1	$-i$	1	i	−1	$-i$
Γ_4	1	ω^3	$-i$	ω	−1	ω^7	i	ω^5
Γ_5	1	−1	1	−1	1	−1	1	−1
Γ_6	1	ω^5	i	ω^7	−1	ω	$-i$	ω^3
Γ_7	1	$-i$	−1	i	1	$-i$	−1	i
Γ_8	1	ω^7	$-i$	ω^5	−1	ω^3	i	ω

	Generators	Vector representation	Perpendicular representation
8 $\bar{8}$	$\alpha = C_{8z}$ $\alpha = S_{8z}$	$\Gamma_1 \oplus \Gamma_2 \oplus \Gamma_8$ $\Gamma_4 \oplus \Gamma_5 \oplus \Gamma_6$	$\Gamma_4 \oplus \Gamma_6$ $\Gamma_2 \oplus \Gamma_8$
$8/m$	$\sim 8 \times \mathbb{Z}_2$	$\Gamma_{1u} \oplus \Gamma_{2u} \oplus \Gamma_{8u}$	$\Gamma_{4u} \oplus \Gamma_{6u}$

(*d*) D_8

D_8	ε	α	α^2	α^3	α^4	β	$\alpha\beta$
n	1	2	2	2	1	4	4
Order	1	8	4	8	2	2	2
Γ_1	1	1	1	1	1	1	1
Γ_2	1	1	1	1	1	−1	−1
Γ_3	1	−1	1	−1	1	1	−1
Γ_4	1	−1	1	−1	1	−1	1
Γ_5	2	$\sqrt{2}$	0	$-\sqrt{2}$	−2	0	0
Γ_6	2	0	−2	0	2	0	0
Γ_7	2	$-\sqrt{2}$	0	$\sqrt{2}$	−2	0	0

	Generators	Vector representation	Perpendicular representation
822	$\alpha = C_{8z}$ $\beta = C_{2x}$	$\Gamma_2 \oplus \Gamma_5$	Γ_7
$8mm$	$\alpha = C_{8z}$ $\beta = m_x$	$\Gamma_1 \oplus \Gamma_5$	Γ_7
$\bar{8}2m$	$\alpha = S_{8z}$ $\beta = C_{2x}$	$\Gamma_3 \oplus \Gamma_7$	Γ_5
$8/mmm$	$\sim 822 \times \mathbb{Z}_2$	$\Gamma_{2u} \oplus \Gamma_{5u}$	Γ_{7u}

(*e*) C_{10} [$\omega = exp(2\pi i/5)$].

C_{10}	ε	α^2	α^4	α^6	α^8
n	1	1	1	1	1
Order	1	5	5	5	5
Γ_1	1	1	1	1	1
Γ_2	1	ω	ω^2	ω^3	ω^4
Γ_3	1	ω^2	ω^4	ω	ω^3
Γ_4	1	ω^3	ω	ω^4	ω^2
Γ_5	1	ω^4	ω^3	ω^2	ω
Γ_6	1	1	1	1	1
Γ_7	1	ω	ω^2	ω^3	ω^4
Γ_8	1	ω^2	ω^4	ω	ω^3
Γ_9	1	ω^3	ω	ω^4	ω^2
Γ_{10}	1	ω^4	ω^3	ω^2	ω

C_{10}	α^5	α^7	α^9	α	α^3
n	1	1	1	1	1
Order	2	10	10	10	10
Γ_1	1	1	1	1	1
Γ_2	1	ω	ω^2	ω^3	ω^4
Γ_3	1	ω^2	ω^4	ω	ω^3
Γ_4	1	ω^3	ω	ω^4	ω^2
Γ_5	1	ω^4	ω^3	ω^2	ω
Γ_6	−1	−1	−1	−1	−1
Γ_7	−1	$-\omega$	$-\omega^2$	$-\omega^3$	$-\omega^4$
Γ_8	−1	$-\omega^2$	$-\omega^4$	$-\omega$	$-\omega^3$
Γ_9	−1	$-\omega^3$	$-\omega$	$-\omega^4$	$-\omega^2$
Γ_{10}	−1	$-\omega^4$	$-\omega^3$	$-\omega^2$	$-\omega$

Table 1.10.5.1 (*cont.*)

	Generators	Vector representation	Perpendicular representation
10 $\bar{5}$ $\overline{10}$	$\alpha = C_{10z}$ $\alpha = S_{5z}$ $\alpha = S_{10z}$	$\Gamma_1 \oplus \Gamma_7 \oplus \Gamma_{10}$ $\Gamma_6 \oplus \Gamma_8 \oplus \Gamma_9$ $\Gamma_2 \oplus \Gamma_4 \oplus \Gamma_6$	$\Gamma_8 \oplus \Gamma_9$ $\Gamma_7 \oplus \Gamma_{10}$ $\Gamma_3 \oplus \Gamma_5$
$10/m$	$\sim 10 \times \mathbb{Z}_2$	$\Gamma_{1u} \oplus \Gamma_{7u} \oplus \Gamma_{10u}$	$\Gamma_{8u} \oplus \Gamma_{9u}$

(*f*) D_{10} $[\tau = (\sqrt{5} - 1)/2]$.

D_{10}	ε	α	α^2	α^3
n	1	2	2	2
Order	1	10	5	10
Γ_1	1	1	1	1
Γ_2	1	1	1	1
Γ_3	1	-1	1	-1
Γ_4	1	-1	1	-1
Γ_5	2	$1 + \tau$	τ	$-\tau$
Γ_6	2	τ	$-1 - \tau$	$-1 - \tau$
Γ_7	2	$-\tau$	$-1 - \tau$	$1 + \tau$
Γ_8	2	$-1 - \tau$	τ	τ

D_{10}	α^4	α^5	β	$\alpha\beta$
n	2	1	5	5
Order	5	2	2	2
Γ_1	1	1	1	1
Γ_2	1	1	-1	-1
Γ_3	1	-1	1	-1
Γ_4	1	-1	-1	1
Γ_5	$-1 - \tau$	-2	0	0
Γ_6	τ	2	0	0
Γ_7	τ	-2	0	0
Γ_8	$-1 - \tau$	2	0	0

	Generators	Vector representation	Perpendicular representation
$10\,22$	$\alpha = C_{10z}$ $\beta = C_{2x}$	$\Gamma_2 \oplus \Gamma_5$	Γ_7
$10\,mm$	$\alpha = C_{10z}$ $\beta = m_x$	$\Gamma_1 \oplus \Gamma_5$	Γ_7
$\overline{10}\,2m$	$\alpha = S_{10z}$ $\beta = C_{2x}$	$\Gamma_4 \oplus \Gamma_8$	Γ_6
$\bar{5}m$	$\alpha = S_{5z}$ $\beta = C_{2x}$	$\Gamma_3 \oplus \Gamma_7$	Γ_5
$10/mmm$	$\sim 10\,22 \times \mathbb{Z}_2$	$\Gamma_{2u} \oplus \Gamma_{5u}$	Γ_{7u}

(*g*) C_{12} $[\omega = \exp(\pi i/6)]$.

C_{12}	ε	α	α^2	α^3	α^4	α^5
n	1	1	1	1	1	1
Order	1	12	6	4	3	12
Γ_1	1	1	1	1	1	1
Γ_2	1	ω	ω^2	i	ω^4	ω^5
Γ_3	1	ω^2	ω^4	-1	$-\omega^2$	$-\omega^4$
Γ_4	1	i	-1	$-i$	1	i
Γ_5	1	ω^4	$-\omega^2$	1	ω^4	$-\omega^2$
Γ_6	1	ω^5	$-\omega^4$	i	$-\omega^2$	ω
Γ_7	1	-1	1	-1	1	-1
Γ_8	1	$-\omega$	ω^2	$-i$	ω^4	$-\omega^5$
Γ_9	1	$-\omega^2$	ω^4	1	$-\omega^2$	ω^4
Γ_{10}	1	$-i$	-1	i	1	$-i$
Γ_{11}	1	$-\omega^4$	$-\omega^2$	-1	ω^4	ω^2
Γ_{12}	1	$-\omega^5$	$-\omega^4$	$-i$	$-\omega^2$	$-\omega$

C_{12}	α^6	α^7	α^8	α^9	α^{10}	α^{11}
n	1	1	1	1	1	1
Order	2	12	3	4	6	12
Γ_1	1	1	1	1	1	1
Γ_2	-1	$-\omega$	$-\omega^2$	$-i$	$-\omega^4$	$-\omega^5$
Γ_3	1	ω^2	ω^4	-1	$-\omega^2$	$-\omega^4$
Γ_4	-1	$-i$	1	i	-1	$-i$
Γ_5	1	ω^4	$-\omega^2$	1	ω^4	$-\omega^2$
Γ_6	-1	$-\omega^5$	ω^4	$-i$	ω^2	$-\omega$
Γ_7	1	-1	1	-1	1	-1
Γ_8	-1	ω	$-\omega^2$	i	$-\omega^4$	ω^5
Γ_9	1	$-\omega^2$	ω^4	1	$-\omega^2$	ω^4
Γ_{10}	-1	i	1	$-i$	-1	i
Γ_{11}	1	$-\omega^4$	$-\omega^2$	-1	ω^4	ω^2
Γ_{12}	-1	ω^5	ω^4	i	ω^2	ω

	Generators	Vector representation	Perpendicular representation
12 $\overline{12}$	$\alpha = C_{12z}$ $\alpha = S_{12z}$	$\Gamma_1 \oplus \Gamma_2 \oplus \Gamma_{12}$ $\Gamma_6 \oplus \Gamma_7 \oplus \Gamma_8$	$\Gamma_6 \oplus \Gamma_8$ $\Gamma_2 \oplus \Gamma_{12}$
$12/m$	$\sim 12 \times \mathbb{Z}_2$	$\Gamma_{1u} \oplus \Gamma_{2u} \oplus \Gamma_{12u}$	$\Gamma_{6u} \oplus \Gamma_{8u}$

(*h*) D_{12}

D_{12}	ε	α	α^2	α^3
n	1	2	2	2
Order	1	12	6	4
Γ_1	1	1	1	1
Γ_2	1	1	1	1
Γ_3	1	-1	1	-1
Γ_4	1	-1	1	-1
Γ_5	2	$\sqrt{3}$	1	0
Γ_6	2	1	-1	-2
Γ_7	2	0	-2	0
Γ_8	2	-1	-1	2
Γ_9	2	$-\sqrt{3}$	1	0

D_{12}	α^4	α^5	α^6	β	$\alpha\beta$
n	2	2	1	6	6
Order	3	12	2	2	2
Γ_1	1	1	1	1	1
Γ_2	1	1	1	-1	-1
Γ_3	1	-1	1	1	-1
Γ_4	1	-1	1	-1	1
Γ_5	-1	$-\sqrt{3}$	-2	0	0
Γ_6	-1	1	2	0	0
Γ_7	2	0	-2	0	0
Γ_8	-1	-1	2	0	0
Γ_9	-1	$\sqrt{3}$	-2	0	0

	Generators	Vector representation	Perpendicular representation
$12\,22$	$\alpha = C_{12z}$ $\beta = C_{2x}$	$\Gamma_2 \oplus \Gamma_5$	Γ_9
$12\,mm$	$\alpha = C_{12z}$ $\beta = m_x$	$\Gamma_1 \oplus \Gamma_5$	Γ_9
$\overline{12}\,2m$	$\alpha = S_{12z}$ $\beta = C_{2x}$	$\Gamma_4 \oplus \Gamma_9$	Γ_5
$12/mmm$	$\sim 12\,22 \times \mathbb{Z}_2$	$\Gamma_{2u} \oplus \Gamma_{5u}$	Γ_{9u}

(*i*) I $[\tau = (\sqrt{5} - 1)/2]$.

I	ε	α	α^2	β	$\alpha\beta$
n	1	12	12	20	15
Order	1	5	5	3	2
Γ_1	1	1	1	1	1
Γ_2	3	$1 + \tau$	$-\tau$	0	-1
Γ_3	3	$-\tau$	$1 + \tau$	0	-1
Γ_4	4	-1	-1	1	0
Γ_5	5	0	0	-1	1

	Generators	Vector representation	Perpendicular representation
532	$\alpha = C_5$ $\beta = C_{3d}$	Γ_2	Γ_3
$\bar{5}\bar{3}m$	$\sim 532 \times \mathbb{Z}_2$	Γ_{2u}	Γ_{3u}

references