1.1.4.1. *Introduction – Neumann's principle*

We saw in Section 1.1.1 that physical properties express in general the response of a medium to an impetus. It has been known for a long time that symmetry considerations play an important role in the study of physical phenomena. These considerations are often very fruitful and have led, for instance, to the discovery of piezoelectricity by the Curie brothers in 1880 (Curie & Curie, 1880, 1881). It is not unusual for physical properties to be related to asymmetries. This is the case in electrical polarization, optical activity *etc.* The first to codify this role was the German physicist and crystallographer F. E. Neumann, who expressed in 1833 the *symmetry principle*, now called *Neumann's principle*: *if a crystal is invariant with respect to certain symmetry elements, any of its physical properties must also be invariant with respect to the same symmetry elements* (Neumann, 1885).

This principle may be illustrated by considering the optical properties of a crystal. In an anisotropic medium, the index of refraction depends on direction. For a given wave normal, two waves may propagate, with different velocities; this is the double refraction effect. The indices of refraction of the two waves vary with direction and can be found by using the index ellipsoid known as the *optical indicatrix* (see Section 1.6.3.2). Consider the central section of the ellipsoid perpendicular to the direction of propagation of the wave. It is an ellipse. The indices of the two waves that may propagate along this direction are equal to the semi-axes of that ellipse. There are two directions for which the central section is circular, and therefore two wave directions for which there is no double refraction. These directions are called optic axes, and the medium is said to be *biaxial*. If the medium is invariant with respect to a threefold, a fourfold or a sixfold axis (as in a trigonal, tetragonal or hexagonal crystal, for instance), its ellipsoid must also be invariant with respect to the same axis, according to Neumann's principle. As an ellipsoid can only be ordinary or of revolution, the indicatrix of a trigonal, tetragonal or hexagonal crystal is necessarily an ellipsoid of revolution that has only one circular central section and one optic axis. These crystals are said to be *uniaxial*. In a cubic crystal that has four threefold axes, the indicatrix must have several axes of revolution, it is therefore a sphere, and cubic media behave as isotropic media for properties represented by a tensor of rank 2.

1.1.4.2. *Curie laws*

The example given above shows that the symmetry of the property may possess a higher symmetry than the medium. The property is represented in that case by the indicatrix. The symmetry of an ellipsoid is

$$\frac{A_2}{M}\frac{A_2'}{M'}\frac{A_2''}{M''}C = mmm \text{ for any ellipsoid}$$
(orthorhombic symmetry)

$$\frac{A_\infty}{M}\frac{\infty A_2}{\infty M}C = \frac{\infty}{m}m \text{ for an ellipsoid of revolution}$$
(cylindrical symmetry)

$$\infty\frac{A_\infty}{M}C = \infty\frac{\infty}{m} \text{ for a sphere}$$
(spherical symmetry).

[Axes A_∞ are axes of revolution, or *axes of isotropy*, introduced by Curie (1884, 1894), *cf. International Tables for Crystallography* (2002), Vol. A, Table 10.1.4.2.]

The symmetry of the indicatrix is identical to that of the medium if the crystal belongs to the orthorhombic holohedry and is higher in all other cases.

This remark is the basis of the generalization of the symmetry principle by P. Curie (1859–1906). He stated that (Curie, 1894):

(i) *the symmetry characteristic of a phenomenon is the highest compatible with the existence of the phenomenon*;

(ii) *the phenomenon may exist in a medium that possesses that symmetry or that of a subgroup of that symmetry*;

and concludes that some symmetry elements may coexist with the phenomenon but that their presence is not necessary. On the contrary, what is necessary is the *absence* of certain symmetry elements: 'asymmetry creates the phenomenon' ('*C'est la dissymétrie qui crée le phénomène*'; Curie, 1894, p. 400). Noting that physical phenomena usually express relations between a cause and an effect (an influence and a response), P. Curie restated the two above propositions in the following way, now known as Curie laws, although they are not, properly speaking, laws:

(i) *the asymmetry of the effects must pre-exist in the causes*;

(ii) *the effects may be more symmetric than the causes*.

The application of the Curie laws enable one to determine the symmetry characteristic of a phenomenon. Let us consider the phenomenon first as an effect. If Φ is the symmetry of the phenomenon and C the symmetry of the cause that produces it,

$$C \leq \Phi.$$

Let us now consider the phenomenon as a cause producing a certain effect with symmetry E:

$$\Phi \leq E.$$

We can therefore conclude that

$$C \leq \Phi \leq E.$$

If we choose among the various possible causes the most symmetric one, and among the various possible effects the one with the lowest symmetry, we can then determine the symmetry that characterizes the phenomenon.

As an example, let us determine the symmetry associated with a mechanical force. A force can be considered as the result of a traction effort, the symmetry of which is $A_\infty \infty M$. If considered as a cause, its effect may be the motion of a sphere in a given direction (for example, a spherical ball falling under its own weight). Again, the symmetry is $A_\infty \infty M$. The symmetries associated with the force considered as a cause and as an effect being the same, we may conclude that $A_\infty \infty M$ is its characteristic symmetry.

1.1.4.3. *Symmetries associated with an electric field and with magnetic induction (flux density)*

1.1.4.3.1. *Symmetry of an electric field*

Considered as an effect, an electric field may have been produced by two circular coaxial electrodes, the first one carrying positive electric charges, the other one negative charges (Fig. 1.1.4.1). The cause possesses an axis of revolution and an infinity of mirrors parallel to it, $A_\infty \infty M$. Considered as a cause, the electric field induces for instance the motion of a spherical electric charge parallel to itself. The associated symmetry is the same in each case, and the symmetry of the electric field is identical to that of a force, $A_\infty \infty M$. The electric polarization or the electric displacement have the same symmetry.

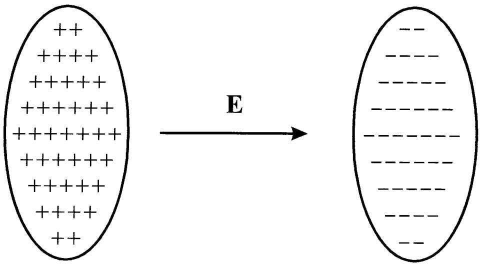

Fig. 1.1.4.1. Symmetry of an electric field.