groups generated by $\bar{1}$, $1'$ and $\{\bar{1}, 1'\}$ are given in Tables 1.2.6.6(*a*), (*b*) and (*c*), respectively.

Table 1.2.6.7. The representations of a point group are also representations of their double groups. In addition, there are extra representations which give projective representations of the point groups. For several cases, these are associated with an ordinary representation. As extra representations, those irreducible representations of the double point groups that give rise to projective representations of the point groups with a factor system that is not associated with the trivial one are given. These do not correspond to ordinary representations of the single group.

Table 1.2.6.8. If one chooses for each element of a point group one of the two corresponding $SU(2)$ elements, the latter form a projective representation of the point group. If one selects for the rotation $R \in K \subset SO(3)$ the element

$$u(R) = E\cos(\varphi/2) + i(\boldsymbol{\sigma}\cdot\mathbf{n})\sin(\varphi/2),$$

where φ is the rotation angle and $\mathbf{n}$ the rotation axis, and for $R \in K \subset O(3)\backslash SO(3)$ the element

$$u(R) = E\cos(\psi/2) + i(\boldsymbol{\sigma}\cdot\mathbf{n})\sin(\psi/2),$$

where ψ and $\mathbf{n}$ are the rotation angle and axis of the rotation $-R$, the matrices $u(R)$ form a projective representation:

$$u(R)u(R') = \omega_s(R, R')u(RR').$$

The factor system ω_s is the spin factor system. It is determined *via* the generators and defining relations

$$W_i(A_1, \ldots, A_p) = E$$

of the point group K. Then

$$W_i(u(A_1), \ldots, u(A_p)) = \lambda_i E,$$

and the factors λ_i fix uniquely the class of the factor system ω_s. These factors are given in the table.

Because $\bar{1}$ is represented by the unit matrix in spin space, the double groups of two isomorphic point groups obtained from each other by replacing the elements $R \in O(3)\backslash SO(3)$ by $-R$ are the same.

The projective representations with factor system ω_s may sometimes be associated with one with a trivial factor system. If this is the case, there are actually no extra representations of the

Table 1.2.6.7. *Extra representations of double groups*

222^d	E	$-E$	$\pm A$	$\pm B$	$\pm AB$				
Γ'_5	2	-2	0	0	0				
422^d	E	$-E$	$\pm A^2$	A	$-A$	$\pm B$	$\pm AB$		
Γ'_6	2	-2	0	$\sqrt{2}$	$-\sqrt{2}$	0	0		
Γ'_7	2	-2	0	$-\sqrt{2}$	$\sqrt{2}$	0	0		
622^d	E	$-E$	A^2	$-A^2$	$\pm B$	$\pm A^3$	A^5	$-A^5$	$\pm A^3B$
Γ'_8	2	-2	1	-1	0	0	$\sqrt{3}$	$-\sqrt{3}$	0
Γ'_9	2	-2	1	-1	0	0	$-\sqrt{3}$	$\sqrt{3}$	0
Γ'_7	2	-2	-2	2	0	0	0	0	0
23^d	E	$-E$	A	$-A$	A^2	$-A^2$	$\pm B$		
Γ'_5	2	-2	1	-1	1	-1	0		
Γ'_6	2	-2	ω	ω^4	ω^2	ω^5	0		
Γ'_7	2	-2	ω^5	ω^2	ω^4	ω	0		
432^d	E	$-E$	B	$-B$	$\pm A^2$	A	$-A$	$\pm AB$	
Γ'_6	2	-2	1	-1	0	$\sqrt{2}$	$-\sqrt{2}$	0	
Γ'_7	2	-2	1	-1	0	$-\sqrt{2}$	$\sqrt{2}$	0	
Γ'_8	4	-4	-1	1	0	0	0	0	

Table 1.2.6.8. *Projective spin representations of the 32 crystallographic point groups*

Point group	Relations giving λ_i	Double group	Extra representations
1 $\bar{1}$	$A = E$ $A^2 = E$	1^d	No No
2, m $2/m$	$A^2 = -E$ $A^2 = B^2 = -E$, $(AB)^2 = E$	2^d	No
222, $2mm$ mmm	$A^2 = B^2 = (AB)^2 = -E$ $A^2 = B^2 = (AB)^2 = -E$ $C^2 = E$, $AC = CA$, $BC = CB$	222^d	Yes
4, $\bar{4}$ $4/m$	$A^4 = -E$ $A^4 = B^2 = -E$, $AB = BA$	4^d	No
422, $4mm$, $\bar{4}2m$ $4/mmm$	$A^4 = B^2 = (AB)^2 = -E$ As above, plus $C^2 = E$, $AC = CA$, $BC = CB$	422^d	Yes
3 $\bar{3}$	$A^3 = -E$ $A^6 = E$	3^d	No
32, $3m$ $\bar{3}m$	$A^3 = B^2 = (AB)^2 = -E$ $A^6 = E$, $B^2 = (AB)^2 = -E$	32^d	No
6, $\bar{6}$ $6/m$	$A^6 = -E$ $A^6 = B^2 = -E$, $AB = BA$	6^d	No
622, $6mm$, $\bar{6}2m$ $6/mmm$	$A^6 = B^2 = (AB)^2 = -E$ As above, plus $C^2 = E$, $AC = CA$, $BC = CB$	622^d	Yes
23 $m3$	$A^3 = B^2 = (AB)^3 = -E$ As above, plus $C^2 = E$, $AC = CA$, $BC = CB$	23^d	Yes
432, $\bar{4}3m$ $m\bar{3}m$	$A^4 = B^3 = (AB)^2 = -E$ As above, plus $C^2 = E$, $AC = CA$, $BC = CB$	432^d	Yes