

## 1. TENSORIAL ASPECTS OF PHYSICAL PROPERTIES

The matrix coefficients  $A_{li}$  are the direction cosines of  $Ox'_1$  with respect to the axes  $Ox_1$ ,  $Ox_2$  and  $Ox_3$ . In spherical coordinates, they are given by (Fig. 1.3.3.3)

$$A_{11} = \cos \theta \sin \varphi, \quad A_{12} = \sin \theta \sin \varphi, \quad A_{13} = \cos \varphi,$$

where  $\theta$  is the angle between  $Ox'_1$  and  $Ox_1$ , and  $\varphi$  is the angle between  $Ox'_1$  and  $Ox_3$ . Using the reduction of  $s_{ijkl}$  for the various crystal classes (Section 1.1.4.9.9), we find, in terms of the reduced two-index components, the following.

(i) *Triclinic system* (groups 1,  $\bar{1}$ ):

$$\begin{aligned} s'_{11} = & [s_{11} \cos^4 \theta + s_{22} \sin^4 \theta + (2s_{12} + s_{66}) \sin^2 2\theta/4 \\ & + (s_{16} \cos \theta + s_{26} \sin \theta) \sin 2\theta] \sin^4 \varphi \\ & + 2\{[(s_{25} + s_{46}) \sin \theta + (s_{14} + s_{56}) \cos \theta] \sin 2\theta/2 \\ & + s_{15} \cos^3 \theta + s_{24} \sin^3 \theta\} \cos \varphi \sin^3 \varphi \\ & + [(2s_{23} + s_{44}) \sin^2 \theta + (2s_{13} + s_{55}) \cos^2 \theta \\ & + (s_{36} + s_{45}) \sin 2\theta] \sin^2 2\varphi/4 \\ & + 2(s_{35} \cos \theta + s_{34} \sin \theta) \cos^3 \varphi \sin \varphi + s_{33} \cos^4 \varphi. \end{aligned}$$

(ii) *Monoclinic system* (groups 2,  $m$ ,  $2/m$ ):

$$\begin{aligned} s'_{11} = & [s_{11} \cos^4 \theta + s_{22} \sin^4 \theta + (2s_{12} + s_{66}) \sin^2 2\theta/4] \sin^4 \varphi \\ & + 2[(s_{25} + s_{46}) \sin^2 \theta + s_{15} \cos^2 \theta] \cos \varphi \sin^3 \varphi \cos \theta \\ & + [(2s_{23} + s_{44}) \sin^2 \theta + (2s_{13} + s_{55}) \cos^2 \theta] \sin^2 2\varphi/4 \\ & + 2s_{35} \cos^3 \varphi \sin \varphi \cos \theta + s_{33} \cos^4 \varphi. \end{aligned}$$

(iii) *Orthorhombic system* (groups 222,  $2mm$ ,  $mmm$ ):

$$\begin{aligned} s'_{11} = & [s_{11} \cos^4 \theta + s_{22} \sin^4 \theta + (2s_{12} + s_{66}) \sin^2 2\theta/4] \sin^4 \varphi \\ & + [(2s_{23} + s_{44}) \sin^2 \theta + (2s_{13} + s_{55}) \cos^2 \theta] \sin^2 2\varphi/4 \\ & + s_{33} \cos^4 \varphi. \end{aligned}$$

(iv) *Trigonal system* (groups 3,  $\bar{3}$ ):

$$\begin{aligned} s'_{11} = & s_{11} \sin^4 \varphi + s_{33} \cos^4 \varphi + (2s_{13} + s_{44}) \sin^2 2\varphi/4 \\ & + (s_{14} \sin 3\theta - s_{25} \cos 3\theta) \sin 2\varphi \sin^2 \varphi. \end{aligned}$$

(v) *Trigonal system* (groups 32,  $3m$ ,  $\bar{3}m$ ):

$$\begin{aligned} s'_{11} = & s_{11} \sin^4 \varphi + s_{33} \cos^4 \varphi + (2s_{13} + s_{44}) \sin^2 2\varphi/4 \\ & + s_{14} \sin 3\theta \sin 2\varphi \sin^2 \varphi. \end{aligned}$$

(vi) *Tetragonal system* (groups 4,  $\bar{4}$ ,  $4/m$ ):

$$\begin{aligned} s'_{11} = & \{s_{11} + [s_{66} - 2(s_{11} - s_{12})] \sin^2 2\theta/4\} \sin^4 \varphi + s_{33} \cos^4 \varphi \\ & + (2s_{13} + s_{44}) \sin^2 2\varphi/4 + s_{16} \sin 4\theta \sin^4 \varphi/2. \end{aligned}$$

(vii) *Tetragonal system* (groups 422,  $\bar{4}2m$ ,  $4mm$ ,  $4/mmm$ ):

$$\begin{aligned} s'_{11} = & \{s_{11} + [s_{66} - 2(s_{11} - s_{12})] \sin^2 2\theta/4\} \sin^4 \varphi + s_{33} \cos^4 \varphi \\ & + (2s_{13} + s_{44}) \sin^2 2\varphi/4. \end{aligned}$$

(viii) *Hexagonal system*:

$$s'_{11} = s_{11} \sin^4 \varphi + s_{33} \cos^4 \varphi + (2s_{13} + s_{44}) \sin^2 2\varphi/4.$$

(ix) *Cubic system*:

$$s'_{11} = s_{11} + [s_{44} - 2(s_{11} - s_{12})] \sin^2 \varphi [\cos^2 \varphi + \sin^2 2\theta \sin^2 \varphi/4].$$

This expression reduces to  $s_{11}$  if  $s_{44} - 2(s_{11} - s_{12}) = 0$  and we retrieve the relation between elastic compliances in an isotropic material (Sections 1.1.4.10.4 and 1.3.3.2.3).

The representation surface of  $s_{11}$ , the inverse of Young's modulus, is illustrated in Figure 1.3.3.4 for crystals of different symmetries. As predicted by the Neumann principle, the representation surface is invariant with respect to the symmetry elements of the point group of the crystal but, as stated by the Curie laws, its symmetry can be larger. In the examples of Fig. 1.3.3.4, the symmetry of the surface is the same as that of the point group for sodium chloride (Fig. 1.3.3.4a), tungsten (Fig. 1.3.3.4b) and aluminium (Fig. 1.3.3.4c), which have  $m\bar{3}m$  as point group, for tin (Fig. 1.3.3.4e,  $4/mmm$ ) and for calcite (Fig. 1.3.3.4f,  $\bar{3}m$ ). But in the case of zinc (Fig. 1.3.3.4d,  $6/mmm$ ), the surface is of revolution and has a larger symmetry. It is interesting to compare the differences in shapes of the representation surfaces for the three cubic crystals, depending on the value of the anisotropy factor, which is larger than 1 for sodium chloride, smaller than 1 for aluminium and close to 1 for tungsten (see Table 1.3.3.2). In this latter case, the crystal is pseudo-isotropic and the surface is practically a sphere.

## 1.3.3.5. Isotropic materials

The isotropy relation between elastic compliances and elastic stiffnesses is given in Section 1.3.3.2.3. For reasons of symmetry, the directions of the eigenvectors of the stress and strain tensors are necessarily the same in an isotropic medium. If we take these directions as axes, the two tensors are automatically diagonalized and the second relation (1.3.3.7) becomes

$$\begin{aligned} T_1 &= c_{11}S_1 + c_{12}(S_2 + S_3) \\ T_2 &= c_{12}S_1 + c_{11}S_2 + c_{12}S_3 \\ T_3 &= c_{12}(S_1 + S_2) + c_{11}S_3. \end{aligned}$$

These relations can equally well be written in the symmetrical form

$$\begin{aligned} T_1 &= (c_{11} - c_{12})S_1 + c_{12}(S_1 + S_2 + S_3) \\ T_2 &= (c_{11} - c_{12})S_2 + c_{12}(S_1 + S_2 + S_3) \\ T_3 &= (c_{11} - c_{12})S_3 + c_{12}(S_1 + S_2 + S_3). \end{aligned}$$

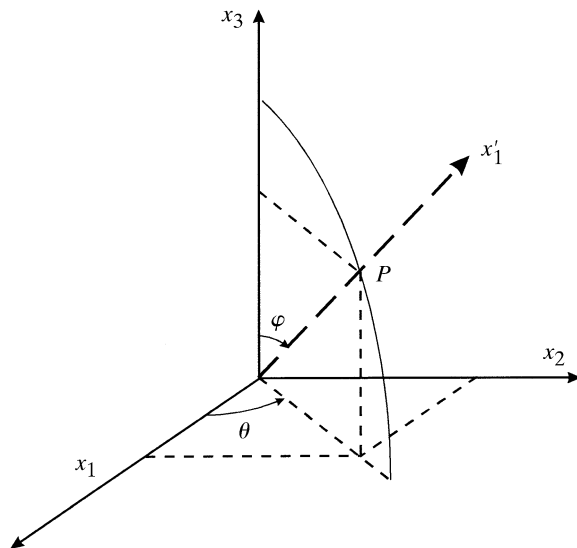


Fig. 1.3.3.3. Spherical coordinates.