One may note that

$$S_1 = S_{11}; \quad S_2 = S_{22}; \quad S_3 = S_{33};$$
$$S_4 = S_{23} + S_{32}; \quad S_5 = S_{31} + S_{13}; \quad S_6 = S_{12} + S_{21}.$$

The Voigt strain matrix S is of the form

$$\begin{pmatrix} S_1 & S_6 & S_5 \\ S_6 & S_2 & S_4 \\ S_5 & S_4 & S_3 \end{pmatrix}.$$

1.3.1.3.2. *Geometrical interpretation of the coefficients of the strain tensor*

Let us consider an orthonormal system of axes with centre P. We remove nothing from the generality of the following by limiting ourselves to a planar problem and assuming that point P' to which P goes in the deformation lies in the plane x_1Px_2 (Fig. 1.3.1.4). Let us consider two neighbouring points, Q and R, lying on axes Px_1 and Px_2, respectively ($PQ = \mathrm{d}x_1$, $PR = \mathrm{d}x_2$). In the deformation, they go to points Q' and R' defined by

$$\mathbf{QQ}': \begin{cases} \mathrm{d}x'_1 = \mathrm{d}x_1 + (\partial u_1/\partial x_1)\mathrm{d}x_1 \\ \mathrm{d}x'_2 = (\partial u_2/\partial x_1)\mathrm{d}x_1 \\ \mathrm{d}x'_3 = 0 \end{cases}$$

$$\mathbf{RR}': \begin{cases} \mathrm{d}x'_1 = (\partial u_1/\partial x_2)\mathrm{d}x_2 \\ \mathrm{d}x'_2 = \mathrm{d}x_2 + (\partial u_2/\partial x_2)\mathrm{d}x_2 \\ \mathrm{d}x'_3 = 0. \end{cases}$$

As the coefficients $\partial u_i/\partial x_j$ are small, the lengths of $P'Q'$ and $P'R'$ are hardly different from PQ and PR, respectively, and the elongations in the directions Px_1 and Px_2 are

$$\frac{P'Q' - PQ}{PQ} = \frac{\mathrm{d}x'_1 - \mathrm{d}x_1}{\mathrm{d}x_1} = \frac{\partial u_1}{\partial x_1} = S_1$$
$$\frac{P'R' - PR}{PR} = \frac{\mathrm{d}x'_2 - \mathrm{d}x_2}{\mathrm{d}x_2} = \frac{\partial u_2}{\partial x_2} = S_2.$$

The components S_1, S_2, S_3 of the principal diagonal of the Voigt matrix can then be interpreted as the elongations in the three directions Px_1, Px_2 and Px_3. The angles α and β between **PQ** and **P′Q′**, and **PR** and **P′R′**, respectively, are given in the same way by

$$\alpha = \mathrm{d}x'_2/\mathrm{d}x_1 = \partial u_2/\partial x_1; \quad \beta = \mathrm{d}x'_1/dx_2 = \partial u_1/\partial x_2.$$

One sees that the coefficient S_6 of Voigt's matrix is therefore

$$S_6 = \frac{\partial u_2}{\partial x_1} + \frac{\partial u_1}{\partial x_2} = \alpha + \beta.$$

The angle $\alpha + \beta$ is equal to the difference between angles $\mathbf{PQ} \wedge \mathbf{PR}$ before deformation and $\mathbf{P''Q''} \wedge \mathbf{P'R'}$ after deformation. The nondiagonal terms of the Voigt matrix therefore represent the shears in the planes parallel to Px_1, Px_2 and Px_3, respectively.

To summarize, if one considers a small cube before deformation, it becomes after deformation an arbitrary parallelepiped; the relative elongations of the three sides are given by the diagonal terms of the strain tensor and the variation of the angles by its nondiagonal terms.

The cubic dilatation (1.3.1.5) is

$$\Delta(B) - 1 = S_1 + S_2 + S_3$$

(taking into account the fact that the coefficients S_{ij} are small).

1.3.1.4. *Particular components of the deformation*

1.3.1.4.1. *Simple elongation*

Matrix M has only one coefficient, e_1, and reduces to (Fig. 1.3.1.5*a*)

$$\begin{pmatrix} e_1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

The quadric of elongations is reduced to two parallel planes, perpendicular to Ox_1, with the equation $x_1 = \pm 1/\sqrt{|e|}$.

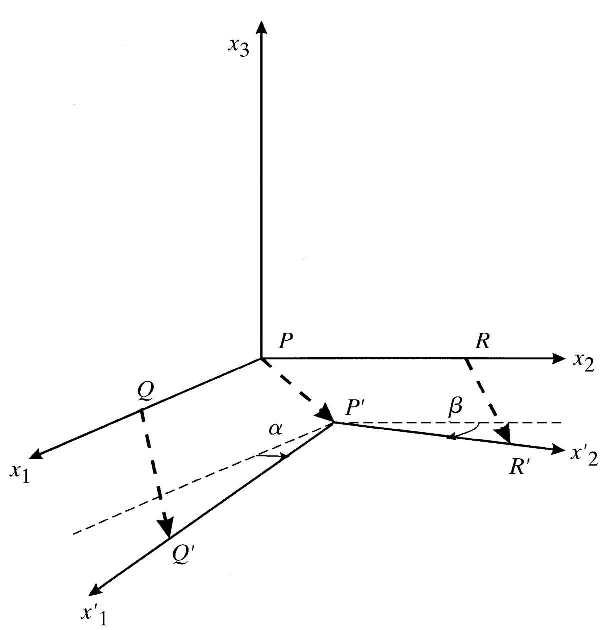

Fig. 1.3.1.4. Geometrical interpretation of the components of the strain tensor. Ox_1, Ox_2, Ox_3: axes before deformation; Ox'_1, Ox'_2, Ox'_3: axes after deformation.

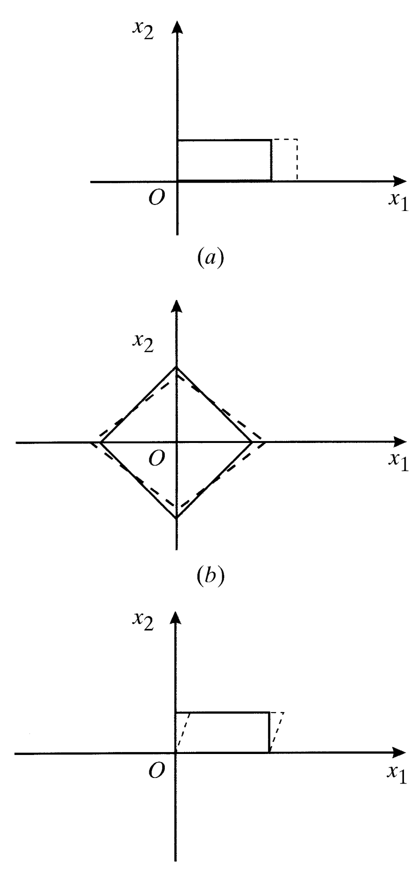

Fig. 1.3.1.5. Special deformations. The state after deformation is represented by a dashed line. (*a*) Simple elongation; (*b*) pure shear; (*c*) simple shear.