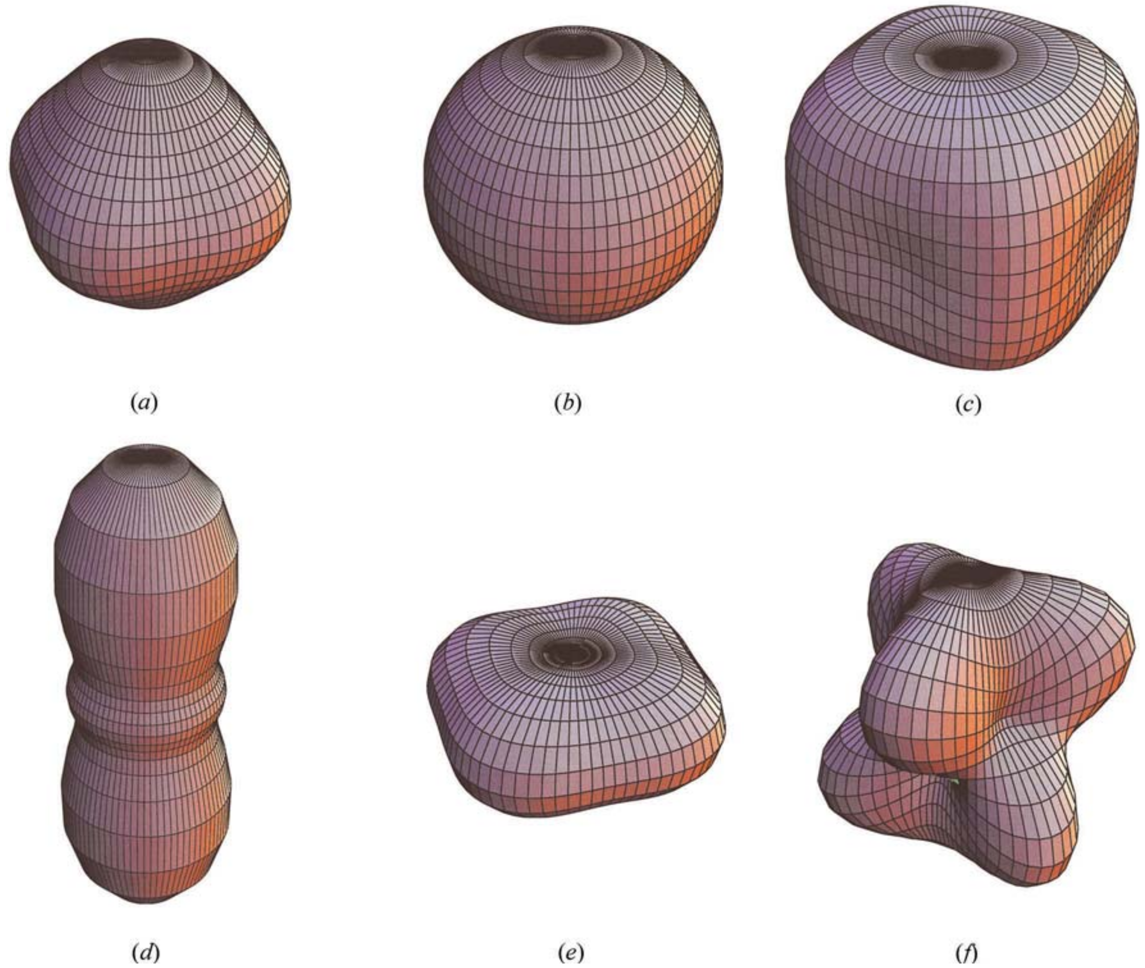

Fig. 1.3.3.4. Representation surface of the inverse of Young's modulus. (*a*) NaCl, cubic, anisotropy factor > 1; (*b*) W, cubic, anisotropy factor = 1; (*c*) Al, cubic, anisotropy factor < 1; (*d*) Zn, hexagonal; (*e*) Sn, tetragonal; (*f*) calcite, trigonal.

If one introduces the Lamé constants,

$$\mu = (1/2)(c_{11} - c_{12}) = c_{44}$$
$$\lambda = c_{12},$$

the equations may be written in the form often used in mechanics:

$$\begin{aligned} T_1 &= 2\mu S_1 + \lambda(S_1 + S_2 + S_3) \\ T_2 &= 2\mu S_2 + \lambda(S_1 + S_2 + S_3) \\ T_3 &= 2\mu S_3 + \lambda(S_1 + S_2 + S_3). \end{aligned} \quad (1.3.3.16)$$

Two coefficients suffice to define the elastic properties of an isotropic material, s_{11} and s_{12}, c_{11} and c_{12}, μ and λ, μ and ν, *etc.* Table 1.3.3.3 gives the relations between the more common elastic coefficients.

1.3.3.6. *Equilibrium conditions of elasticity for isotropic media*

We saw in Section 1.3.2.3 that the condition of equilibrium is

$$\partial T_{ij}/\partial x_i + \rho F_j = 0.$$

If we use the relations of elasticity, equation (1.3.3.2), this condition can be rewritten as a condition on the components of the strain tensor:

$$c_{ijkl}\frac{\partial S_{kl}}{\partial x_j} + \rho F_i = 0.$$

Recalling that

$$S_{kl} = \tfrac{1}{2}\left[\frac{\partial u_k}{\partial x_l} + \frac{\partial u_l}{\partial x_k}\right],$$

the condition becomes a condition on the displacement vector, $\mathbf{u}(\mathbf{r})$:

$$c_{ijkl}\frac{\partial^2}{\partial x_l \partial x_j} + \rho F_i = 0.$$

In an isotropic orthonormal medium, this equation, projected on the axis $0x_1$, can be written with the aid of relations (1.3.3.5) and (1.3.3.9):

$$\begin{aligned} &c_{11}\frac{\partial^2 u_1}{(\partial x_1)^2} + c_{12}\left[\frac{\partial^2 u_2}{\partial x_1 \partial x_2} + \frac{\partial^2 u_3}{\partial x_1 \partial x_3}\right] \\ &\quad + \tfrac{1}{2}(c_{11} - c_{12})\left[\frac{\partial^2 u_1}{(\partial x_2)^2} + \frac{\partial^2 u_3}{\partial x_1 \partial x_3} + \frac{\partial^2 u_1}{(\partial x_3)^2}\right] + \rho F_i \\ &\quad = 0. \end{aligned}$$

This equation can finally be rearranged in one of the three following forms with the aid of Table 1.3.3.3.

$$\begin{aligned} \tfrac{1}{2}(c_{11} - c_{12})\Delta\mathbf{u} + \tfrac{1}{2}(c_{11} + c_{12})\nabla(\nabla\mathbf{u}) + \rho\mathbf{F} &= 0 \\ \mu\Delta\mathbf{u} + (\mu + \lambda)\nabla(\nabla\mathbf{u}) + \rho\mathbf{F} &= 0 \\ \mu\left[\Delta\mathbf{u} + \frac{1}{1 - 2\nu}\nabla(\nabla\mathbf{u})\right] + \rho\mathbf{F} &= 0. \end{aligned} \quad (1.3.3.17)$$

references