

1. TENSORIAL ASPECTS OF PHYSICAL PROPERTIES

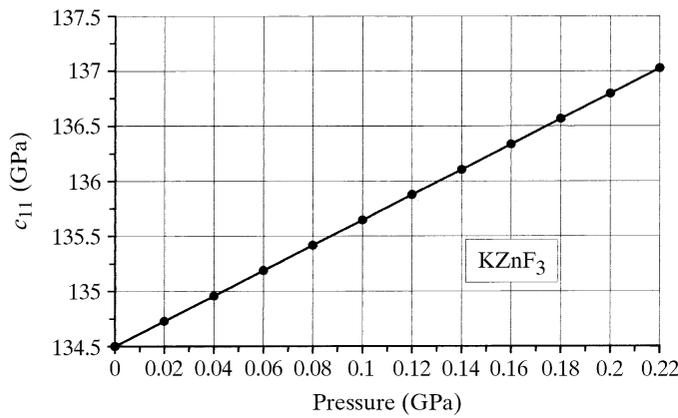


Fig. 1.3.5.2. Pressure dependence of the elastic stiffness c_{11} of a KZnF_3 crystal. Reproduced with permission from *Ultrasonics Symposium Proc. IEEE* (Fischer *et al.*, 1980). Copyright (1980) IEEE.

of magnitude for c_{11} in crystals is about 100 GPa and, within the experimental uncertainty, the pressure dependence of c_{11} does not depart from a linear behaviour up to at least 0.2 GPa.

These observations can be quantitatively justified on the basis of an equation of state of a solid:

$$f(T_{ij}, S_{ij}, X, \Theta) = 0,$$

where T_{ij} represents the stress tensor, S_{ij} the strain tensor, X the position of the elementary elements of the solid and Θ the temperature.

Different equations of state of solids have been proposed. They correspond to different degrees of approximation that can only be discussed and understood in a microscopic theory of lattice dynamics. The different steps in the development of lattice dynamics, the Einstein model, the Debye model and the Grüneisen model, will be presented in Section 2.1.2.7. Concerning the temperature and the pressure dependences of the elastic constants, we may notice that rather sophisticated models are needed to describe correctly the general trends mentioned above:

(a) In the *Einstein model*, where the N atoms of a crystal are considered as $3N$ independent harmonic oscillators, the temperature increase affects only the amplitude of the oscillations and not the average positions of the atoms; consequently, this model can explain neither the thermal expansion nor the *temperature dependence* of the elastic constants. In addition, this theory ignores the difference between isothermal and adiabatic elastic constants. Similarly, if the oscillators are harmonic, the stiffness of the ‘springs’ connecting atoms does not depend on the distances between atoms and the model cannot therefore explain the pressure dependence of the elastic constants, which requires anharmonic ‘springs’ or, more accurately, anharmonic potentials.

(b) In the *Debye model*, the $3N$ oscillators are not independent but they are still harmonic. The result is that here again the elastic constants are pressure and temperature independent.

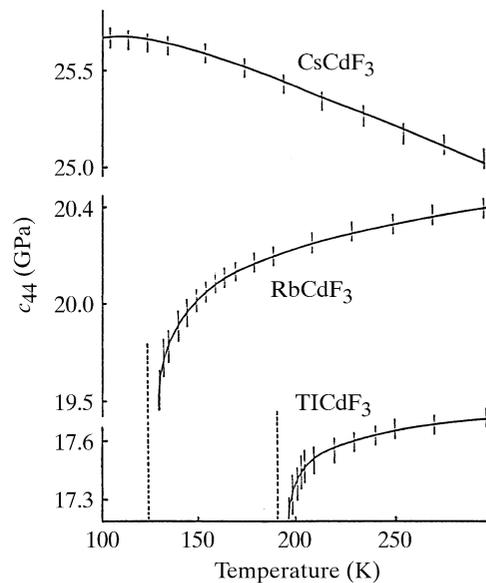


Fig. 1.3.5.3. Temperature dependence of the elastic constant c_{44} in RbCdF_3 , CsCdF_3 and TiCdF_3 crystals; the crystals of RbCdF_3 and TiCdF_3 undergo structural phase transitions (after Rousseau *et al.*, 1975).

(c) In the *Grüneisen model*, the frequencies of the oscillators are volume-dependent; this so-called ‘quasiharmonic approximation’ can justify the standard behaviour shown in Figs. 1.3.5.1 and 1.3.5.2.

1.3.5.2. Temperature dependence of the elastic constants

Table 1.3.5.1 gives typical values of $(\partial \ln c_{ij} / \partial \Theta)_p$ for some cubic crystals considered within their stability domain. In column 6, the ‘elastic Debye temperature’ of the crystal, $(\Theta_D)_{el}$, has been calculated according to the formula

$$(\Theta_D)_{el} = hv/k_B(3n/4\pi)^{1/3},$$

where h is the Planck constant, k_B is the Boltzmann constant, v is an average velocity (see for instance De Launay, 1956) and n is the number of atoms per unit volume.

It is interesting to compare $(\Theta_D)_{el}$, the ‘elastic Debye temperature’, with $(\Theta_D)_{cal}$, the ‘calorimetric Debye temperature’. The definition of $(\Theta_D)_{cal}$ will be given in Section 2.1.2.7. It results from the attempt at founding a universal description for the thermal properties of solids when the temperature is expressed as a reduced temperature, $\Theta/(\Theta_D)_{cal}$; $(\Theta_D)_{cal}$ is obtained from calorimetric measurements at low temperature. It is worth noting that accurate values of low-temperature elastic constants and low-temperature calorimetric measurements lead to an excellent agreement between $(\Theta_D)_{el}$ and $(\Theta_D)_{cal}$ [better than 2 or 3 K (De Launay, 1956)]. This agreement demonstrates the validity of the Debye model in the vicinity of 0 K. From Table 1.3.5.1, we can observe that for ionic crystals $(\partial \ln c_{11} / \partial \Theta)_p$ is, in general, greater than $(\partial \ln c_{44} / \partial \Theta)_p$. This remark is not valid for covalent and metallic crystals. Typical orders of magnitude are given in Table 1.3.5.2. These statements concern only general trends valid for stable crystals.

In the case of *temperature-induced phase transitions*, some elastic constants are softened in the vicinity and sometimes far from the critical temperature. As an example, Fig. 1.3.5.3 shows the temperature dependence of c_{44} in RbCdF_3 , CsCdF_3 and TiCdF_3 single crystals. RbCdF_3 and TiCdF_3 undergo structural phase transitions at 124 and 191 K, respectively, while CsCdF_3 remains stable in this temperature range.

Table 1.3.5.1. Temperature dependence of the elastic stiffnesses for some cubic crystals

Material	Temperature range (K)	$(\partial \ln c_{11} / \partial \Theta)_p$ (10^{-4} K^{-1})	$(\partial \ln c_{44} / \partial \Theta)_p$ (10^{-4} K^{-1})	$(\partial \ln c_{12} / \partial \Theta)_p$ (10^{-4} K^{-1})	Θ^{el} (K)
Al	80–300	−3.1	−4.45	−1.3	430
Cu	80–300	−2.01	−3.33	−1.24	344
Ag	50–300	−2.3	−4.0	−1.5	226
Pb	100–300	−4.4	−1.5	−2.8	105
Si	80–300	−0.81	−10.6	−1.10	648
Ge	150–1000	−1.2	−1.15	−1.10	374
ZnS	100–300	−1.2	−0.65	−0.8	347
NaCl	100–300	−7.8	−2.2	−4.7	321
KCl	80–300	−8.3	−2.1	−3.6	236
KBr	80–300	−7.6	−2.1	7	172

1.3. ELASTIC PROPERTIES

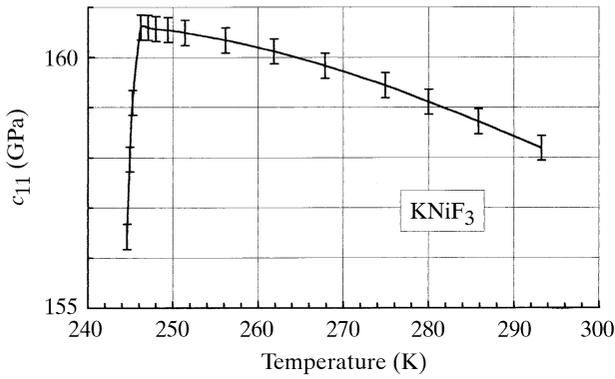


Fig. 1.3.5.4. Temperature dependence of the elastic constant c_{11} in KNiF_3 , which undergoes a para-antiferromagnetic phase transition. Reprinted with permission from *Appl. Phys. Lett.* (Nouet *et al.*, 1972). Copyright (1972) American Institute of Physics.

The softening of c_{44} when the temperature decreases starts more than 100 K before the critical temperature, Θ_c . In contrast, Fig. 1.3.5.4 shows the temperature dependence of c_{11} in KNiF_3 , a crystal that undergoes a para-antiferromagnetic phase transition at 246 K; the coupling between the elastic and the magnetic energy is weak, consequently c_{11} decreases abruptly only a few degrees before the critical temperature. We can generalize this observation and state that the softening of an elastic constant occurs over a large domain of temperature when this constant is the order parameter or is strongly coupled to the order parameter of the transformation; for instance, in the cooperative Jahn-Teller phase transition in DyVO_4 , $(c_{11} - c_{12})/2$ is the soft acoustic phonon mode leading to the phase transition and this parameter anticipates the phase transition 300 K before it occurs (Fig. 1.3.5.5).

1.3.5.3. Pressure dependence of the elastic constants

As mentioned above, anharmonic potentials are needed to explain the stress dependence of the elastic constants of a crystal. Thus, if the strain-energy density is developed in a polynomial in terms of the strain, only the first and the second elastic constants are used in linear elasticity (harmonic potentials), whereas higher-order elastic constants are also needed for nonlinear elasticity (anharmonic potentials).

Concerning the pressure dependence of the elastic constants (nonlinear elastic effect), considerable attention has been paid to their experimental determination since they are a unique source of significant information in many fields:

(i) In *geophysics*, a large part of the knowledge we have on the interior of the earth comes from the measurement of the transit time of elastic bursts propagating in the mantle and in the core (in the upper mantle, the average pressure is estimated to be about a few hundred GPa, a value which is comparable to that of the elastic stiffnesses of many materials).

(ii) In *solid-state physics*, the pressure dependence of the elastic constants gives significant indications concerning the stability of crystals. For example, Fig. 1.3.5.2 shows the pressure dependence of the elastic constants of KZnF_3 , a cubic crystal belonging to the perovskite family. As mentioned previously, this crystal is known to be stable over a wide range of temperature and the elastic stiffnesses c_{ij} depend linearly on pressure. It may be noted that, consequently, the third-order elastic constants

Table 1.3.5.2. Order of magnitude of the temperature dependence of the elastic stiffnesses for different types of crystals

Type of crystal	$(\partial \ln c_{11} / \partial \Theta)_p$ (K^{-1})	$(\partial \ln c_{44} / \partial \Theta)_p$ (K^{-1})
Ionic	-10^{-3}	-3×10^{-4}
Covalent	-10^{-4}	-8×10^{-5}
Metallic	-2×10^{-4}	-3×10^{-4}

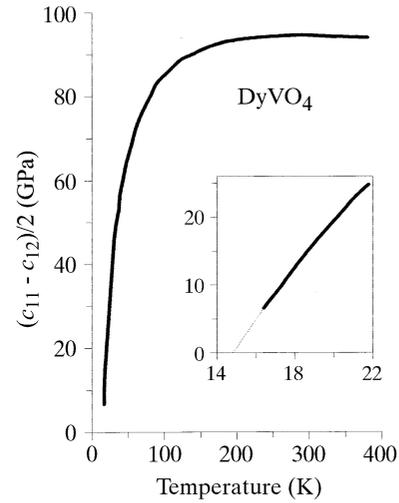


Fig. 1.3.5.5. Temperature dependence of $(c_{11} - c_{12})/2$ in DyVO_4 , which undergoes a cooperative Jahn-Teller phase transition (after Melcher & Scott, 1972).

(TOECs) are constant. On the contrary, we observe in Fig. 1.3.5.6 that the pressure dependence of the elastic constants of TlCdF_3 , a cubic crystal belonging to the same family but which is known to become unstable when the temperature is decreased to 191 K (Fischer, 1982), is nonlinear even at low pressures. In this case, the development of the strain-energy density in terms of strains cannot be stopped after the terms containing the third-order elastic constants; the contributions of the fourth- and fifth-order elastic constants are not negligible.

(iii) For practical use in the case of technical materials such as concrete or worked metals, the pressure dependence of the elastic moduli is also required for examining the effect of applied stresses or of an applied hydrostatic pressure, and for studying residual stresses resulting from loading (heating) and unloading (cooling) the materials.

1.3.6. Nonlinear elasticity

1.3.6.1. Introduction

In a solid body, the relation between the stress tensor T and the strain tensor S is usually described by Hooke's law, which postulates linear relations between the components of T and S (Section 1.3.3.1). Such relations can be summarized by (see equation 1.3.3.2)

$$T_{ij} = c_{ijkl} S_{kl},$$

where the c_{ijkl} 's are the elastic stiffnesses.

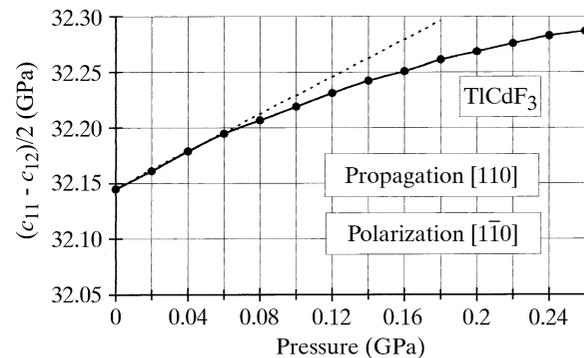


Fig. 1.3.5.6. Pressure dependence of the elastic constants $(c_{11} - c_{12})/2$ in TlCdF_3 . Reproduced with permission from *Ultrasonics Symposium Proc. IEEE* (Fischer *et al.*, 1980). Copyright (1980) IEEE.