International Tables for Crystallography (2006). Vol. D, Section 1.6.7, pp. 173–176.

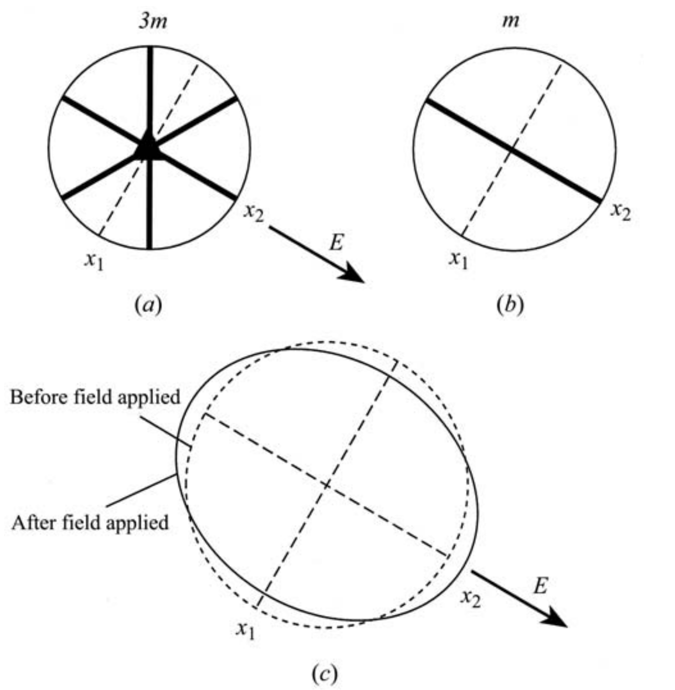

Fig. 1.6.6.1. (*a*) Symmetry elements of point group 3*m*. (*b*) Symmetry elements after field applied along x_2. (*c*) Effect on circular section of uniaxial indicatrix.

If, on the other hand, the electric field E_2^0 is applied along x_2, *i.e.* within the mirror plane, one finds

$$\begin{aligned}\Delta\eta_1 &= -r_{22}E_2^0\\ \Delta\eta_2 &= +r_{22}E_2^0\\ \Delta\eta_4 &= +r_{51}E_2^0\\ \Delta\eta_3 &= \Delta\eta_5 = \Delta\eta_6 = 0.\end{aligned} \qquad (1.6.6.9)$$

Diagonalization of the matrix

$$\begin{pmatrix}\Delta\eta_1 & 0 & 0\\ 0 & \Delta\eta_2 & \Delta\eta_4\\ 0 & \Delta\eta_4 & \Delta\eta_3\end{pmatrix} \qquad (1.6.6.10)$$

containing these terms gives three eigenvalue solutions for the changes in dielectric impermeabilities:

$$\begin{aligned}&(1) && -r_{22}E_2^0\\ &(2) && \frac{r_{22}+(r_{22}^2+4r_{51}^2)^{1/2}}{2}E_2^0\\ &(3) && \frac{r_{22}-(r_{22}^2+4r_{51}^2)^{1/2}}{2}E_2^0.\end{aligned} \qquad (1.6.6.11)$$

On calculating the eigenfunctions, it is found that solution (1) lies along x_1, thus representing a change in the value of the indicatrix axis in this direction. Solutions (2) and (3) give the other two axes of the indicatrix: these are different in length, but mutually perpendicular, and lie in the x_2x_3 plane. Thus a biaxial indicatrix is formed with one refractive index fixed along x_1 and the other two in the plane perpendicular. The effect of having the electric field imposed within the mirror plane is thus to remove the threefold axis in point group 3*m* and to form the point subgroup *m* (Fig. 1.6.6.1).

Relationship between linear electro-optic coefficients r_{ijk} *and the susceptibility tensor* $\chi_{ijk}^{(2)}$. It is instructive to repeat the above calculation using the normal susceptibility tensor and equation (1.6.3.14). Consider, again, a static electric field along x_3 and light propagating along x_1. As before, the only coefficients that need to be considered with the static field along x_3 are $\chi_{113} = \chi_{223}$ and χ_{333}. Equation (1.6.3.14) can then be written as

$$\begin{pmatrix}\varepsilon_1+\varepsilon_o\chi_{13}E_3^0+n^2 & 0 & 0\\ 0 & \varepsilon_1+\varepsilon_o\chi_{13}E_3^0 & 0\\ 0 & 0 & \varepsilon_3+\varepsilon_o\chi_{33}E_3^0\end{pmatrix}\begin{pmatrix}E_1\\E_2\\E_3\end{pmatrix} = \begin{pmatrix}n^2 & 0 & 0\\ 0 & n^2 & 0\\ 0 & 0 & n^2\end{pmatrix}\begin{pmatrix}E_1\\E_2\\E_3\end{pmatrix}, \qquad (1.6.6.12)$$

where for simplicity the Voigt notation has been used. The first line of the matrix equation gives

$$(\varepsilon_1+\varepsilon_o\chi_{13}E_3^0+n^2)E_1 = n^2E_1. \qquad (1.6.6.13)$$

Since only a transverse electric field is relevant for an optical wave (plasma waves are not considered here), it can be assumed that the longitudinal field $E_1 = 0$. The remaining two equations can be solved by forming the determinantal equation

$$\begin{vmatrix}\varepsilon_1+\varepsilon_o\chi_{13}E_3^0-n^2 & 0\\ 0 & \varepsilon_3+\varepsilon_o\chi_{33}E_3^0-n^2\end{vmatrix} = 0, \qquad (1.6.6.14)$$

which leads to the results

$$n_1^2 = \varepsilon_1+\varepsilon_o\chi_{13}E_3^0 \text{ and } n_2^2 = \varepsilon_3+\varepsilon_o\chi_{33}E_3^0. \qquad (1.6.6.15)$$

Thus

$$n_1^2 = n_o^2+\varepsilon_o\chi_{13}E_3^0 \text{ and } n_2^2 = n_e^2+\varepsilon_o\chi_{33}E_3^0, \qquad (1.6.6.16)$$

and so

$$(n_1-n_o)(n_1+n_o) = \varepsilon_o\chi_{13}E_3^0 \text{ and } (n_2-n_e)(n_2+n_e) = \varepsilon_o\chi_{33}E_3^0, \qquad (1.6.6.17)$$

and since $n_1 \simeq n_o$ and $n_2 \simeq n_e$,

$$n_1-n_o = \frac{\varepsilon_o\chi_{13}E_3^0}{2n_o} \text{ and } n_2-n_e = \frac{\varepsilon_o\chi_{33}E_3^0}{2n_e}. \qquad (1.6.6.18)$$

Subtracting these two results, the induced birefringence is found:

$$(n_e-n_o)-\tfrac{1}{2}\left(\frac{\varepsilon_o\chi_{33}}{n_e}-\frac{\varepsilon_o\chi_{13}}{n_o}\right)E_3^0. \qquad (1.6.6.19)$$

Comparing with the equation (1.6.6.8) calculated for the linear electro-optic coefficients,

$$(n_e-n_o)-\tfrac{1}{2}\left(n_e^3r_{33}-n_o^3r_{13}\right)E_3^0, \qquad (1.6.6.20)$$

one finds the following relationships between the linear electro-optic coefficients and the susceptibilities $\chi^{(2)}$:

$$r_{13} = \frac{\varepsilon_o\chi_{13}}{n_o^4} \text{ and } r_{33} = \frac{\varepsilon_o\chi_{33}}{n_e^4}. \qquad (1.6.6.21)$$

1.6.7. The linear photoelastic effect

1.6.7.1. *Introduction*

The linear photoelastic (or piezo-optic) effect (Narasimhamurty, 1981) is given by $P_i^\omega = \varepsilon_o\chi_{ijk\ell}E_j^\omega S_{k\ell}^0$, and, like the electro-optic effect, it can be discussed in terms of the change in dielectric impermeability caused by a static (or low-frequency) field, in this case a stress, applied to the crystal. This can be written in the form

$$\Delta\eta_{ij} = \pi_{ijk\ell}T_{k\ell}^0. \qquad (1.6.7.1)$$

The coefficients $\pi_{ijk\ell}$ form a fourth-rank tensor known as the *linear piezo-optic* tensor. Typically, the piezo-optic coefficients are of the order of 10^{-12} m^2 N^{-1}. It is, however, more usual to express the effect as an *elasto-optic effect* by making use of the relationship between stress and strain (see Section 1.3.3.2), thus

$$T_{k\ell} = c_{k\ell mn} S_{mn}, \qquad (1.6.7.2)$$

where the $c_{k\ell mn}$ are the elastic stiffness coefficients. Therefore equation (1.6.7.2) can be rewritten in the form

$$\Delta\eta_{ij} = \pi_{ijk\ell} c_{k\ell mn} S_{mn} = p_{ijmn} S_{mn} \qquad (1.6.7.3)$$

or, in contracted notation,

$$\Delta\eta_i = p_{ij} S_j, \qquad (1.6.7.4)$$

where, for convenience, the superscript 0 has been dropped, the elastic strain being considered as essentially static or of low frequency compared with the natural mechanical resonances of the crystal. The p_{ijmn} are coefficients that form the *linear elasto-optic* (or *strain-optic*) *tensor* (Table 1.6.7.1). Note that these coefficients are dimensionless, and typically of order 10^{-1}, showing that the change to the optical indicatrix is roughly one-tenth of the strain.

The elasto-optic effect can arise in several ways. The most obvious way is through application of an external stress, applied to the surfaces of the crystal. However, strains, and hence changes to the refractive indices, can arise in a crystal through other ways that are less obvious. Thus, it is a common finding that crystals can be twinned, and thus the boundary between twin domains, which corresponds to a mismatch between the crystal structures either side of the domain boundary, will exhibit a strain. Such a crystal, when viewed between crossed polars under a microscope will produce birefringence colours that will highlight the contrast between the domains. This is known as *strain birefringence*. Similarly, when a crystal undergoes a phase transition involving a change in crystal system, a so-called *ferroelastic transition*, there will be a change in strain owing to the difference in unit-cell shapes. Hence there will be a corresponding change in the optical indicatrix. Often the phase transition is one going from a high-temperature optically isotropic section to a low-temperature optically anisotropic section. In this case, the high-temperature section has no internal strain, but the low-temperature phase acquires a strain, which is often called the *spontaneous strain* (by analogy with the term spontaneous polarization in ferroelectrics).

1.6.7.2. *Spontaneous strain in* $BaTiO_3$

As an example of the calculation of the relationship between spontaneous strain and linear birefringence, consider the high-temperature phase transition of the well known perovskite $BaTiO_3$. This substance undergoes a transition at around 403 K on cooling from its high-temperature $Pm\bar{3}m$ phase to the room-temperature $P4mm$ phase. The $P4mm$ phase is both ferroelectric and ferroelastic. In this tetragonal phase, there is a small distortion of the unit cell along [001] and a contraction along ⟨100⟩ compared with the unit cell of the high-temperature cubic phase, and so the room-temperature phase can be expected to have a uniaxial optical indicatrix.

The elasto-optic tensor for the $m\bar{3}m$ phase is (Table 1.6.7.1)

$$\begin{pmatrix} p_{11} & p_{12} & p_{12} & 0 & 0 & 0 \\ p_{12} & p_{11} & p_{12} & 0 & 0 & 0 \\ p_{12} & p_{12} & p_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & p_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & p_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & p_{44} \end{pmatrix}. \qquad (1.6.7.5)$$

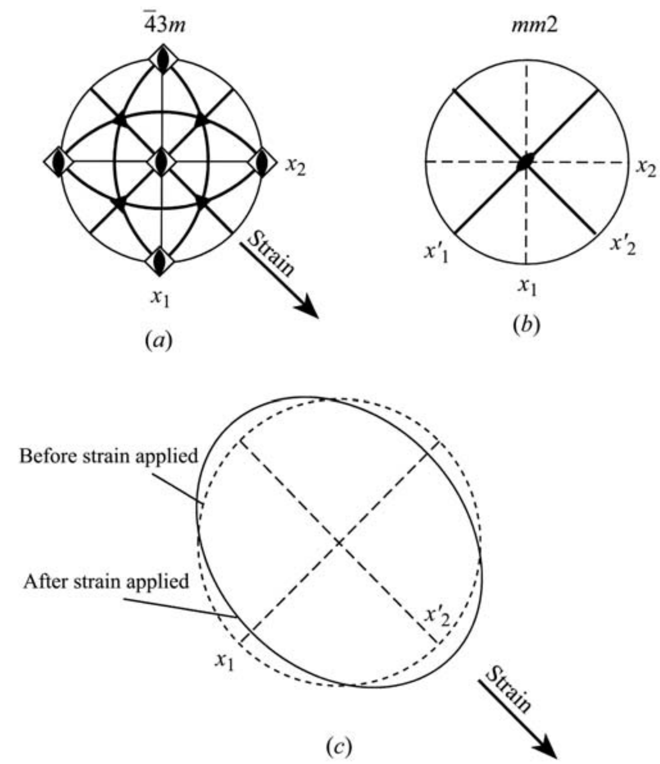

Fig. 1.6.7.1. (*a*) Symmetry elements of point group $\bar{4}3m$. (*b*) Symmetry elements after strain applied along [110]. (*c*) Effect on spherical indicatrix.

Consider the low-temperature tetragonal phase to arise as a small distortion of this cubic phase, with a spontaneous strain S_3^s given by the lattice parameters of the tetragonal phase:

$$S_3^s = [(c-a)/a]. \qquad (1.6.7.6)$$

Therefore, the equations (1.6.7.4) for the dielectric impermeability in terms of the spontaneous strain component are given in matrix form as

$$\begin{pmatrix} \Delta\eta_1 \\ \Delta\eta_2 \\ \Delta\eta_3 \\ \Delta\eta_4 \\ \Delta\eta_5 \\ \Delta\eta_6 \end{pmatrix} = \begin{pmatrix} p_{11} & p_{12} & p_{12} & 0 & 0 & 0 \\ p_{12} & p_{11} & p_{12} & 0 & 0 & 0 \\ p_{12} & p_{12} & p_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & p_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & p_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & p_{44} \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ S_3^s \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} p_{12}S_3^s \\ p_{12}S_3^s \\ p_{11}S_3^s \\ 0 \\ 0 \\ 0 \end{pmatrix} \qquad (1.6.7.7)$$

so that

$$\begin{aligned} \Delta\eta_1 &= \Delta\eta_2 = p_{12}S_3^s \\ \Delta\eta_3 &= p_{11}S_3^s \\ \Delta\eta_4 &= \Delta\eta_5 = \Delta\eta_6 = 0. \end{aligned} \qquad (1.6.7.8)$$

By analogy with equations (1.6.6.5) and (1.6.6.6), the induced changes in refractive index are then

Table 1.6.7.1. *Symmetry constraints on the linear elasto-optic (strain-optic) tensor p_{ij} (contracted notation) (see Section 1.1.4.10.6)*

Triclinic	Orthorhombic	Tetragonal	Trigonal
Point group 1	Point groups 222, $mm2$, mmm	Point groups 4, $\bar{4}$, $4/m$	Point groups 3, $\bar{3}$
$\begin{pmatrix} p_{11} & p_{12} & p_{13} & p_{14} & p_{15} & p_{16} \\ p_{21} & p_{22} & p_{23} & p_{24} & p_{25} & p_{26} \\ p_{31} & p_{32} & p_{33} & p_{34} & p_{35} & p_{36} \\ p_{41} & p_{42} & p_{43} & p_{44} & p_{45} & p_{46} \\ p_{51} & p_{52} & p_{53} & p_{54} & p_{55} & p_{56} \\ p_{61} & p_{62} & p_{63} & p_{64} & p_{65} & p_{66} \end{pmatrix}$	$\begin{pmatrix} p_{11} & p_{12} & p_{13} & 0 & 0 & 0 \\ p_{21} & p_{22} & p_{23} & 0 & 0 & 0 \\ p_{31} & p_{32} & p_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & p_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & p_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & p_{66} \end{pmatrix}$	$\begin{pmatrix} p_{11} & p_{12} & p_{13} & 0 & 0 & p_{16} \\ p_{12} & p_{22} & p_{13} & 0 & 0 & -p_{16} \\ p_{31} & p_{31} & p_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & p_{44} & p_{45} & 0 \\ 0 & 0 & 0 & -p_{45} & p_{55} & 0 \\ p_{61} & -p_{61} & 0 & {} & 0 & p_{66} \end{pmatrix}$	$\begin{pmatrix} p_{11} & p_{12} & p_{13} & p_{14} & p_{15} & p_{16} \\ p_{12} & p_{11} & p_{13} & -p_{14} & -p_{15} & -p_{16} \\ p_{31} & p_{31} & p_{33} & 0 & 0 & 0 \\ p_{41} & -p_{41} & 0 & p_{44} & p_{45} & -p_{51} \\ p_{51} & -p_{51} & 0 & -p_{44} & p_{44} & p_{41} \\ -p_{16} & p_{16} & 0 & -p_{15} & p_{14} & \frac{1}{2}(p_{11}-p_{12}) \end{pmatrix}$
Monoclinic		Point groups $4mm$, $\bar{4}2m$, 422, $4/mmm$	Point groups $3m$, $\bar{3}m$, 32
Point groups 2, m, $2/m$ ($2 \parallel x_2$)	Point groups 2, m, $2/m$ ($2 \parallel x_3$)	$\begin{pmatrix} p_{11} & p_{12} & p_{13} & 0 & 0 & 0 \\ p_{12} & p_{22} & p_{13} & 0 & 0 & 0 \\ p_{31} & p_{31} & p_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & p_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & p_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & p_{66} \end{pmatrix}$	$\begin{pmatrix} p_{11} & p_{12} & p_{13} & p_{14} & 0 & 0 \\ p_{12} & p_{11} & p_{13} & -p_{14} & 0 & 0 \\ p_{13} & p_{13} & p_{33} & 0 & 0 & 0 \\ p_{41} & -p_{41} & 0 & p_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & p_{44} & p_{41} \\ 0 & 0 & 0 & 0 & p_{14} & \frac{1}{2}(p_{11}-p_{12}) \end{pmatrix}$
$\begin{pmatrix} p_{11} & p_{12} & p_{13} & 0 & p_{15} & 0 \\ p_{21} & p_{22} & p_{23} & 0 & p_{25} & 0 \\ p_{31} & p_{32} & p_{33} & 0 & p_{35} & 0 \\ 0 & 0 & 0 & p_{44} & 0 & p_{46} \\ p_{51} & p_{52} & p_{53} & 0 & p_{55} & 0 \\ 0 & 0 & 0 & p_{64} & 0 & p_{66} \end{pmatrix}$	$\begin{pmatrix} p_{11} & p_{12} & p_{13} & 0 & 0 & p_{16} \\ p_{21} & p_{22} & p_{23} & 0 & 0 & p_{26} \\ p_{31} & p_{32} & p_{33} & 0 & 0 & p_{36} \\ 0 & 0 & 0 & p_{44} & p_{45} & 0 \\ 0 & 0 & 0 & p_{54} & p_{55} & 0 \\ p_{61} & p_{62} & p_{63} & 0 & 0 & p_{66} \end{pmatrix}$		

Hexagonal	Cubic	Isotropic
Point groups 6, $\bar{6}$, $6/m$	Point groups $m\bar{3}$, 23	
$\begin{pmatrix} p_{11} & p_{12} & p_{13} & 0 & 0 & p_{16} \\ p_{12} & p_{11} & p_{13} & 0 & 0 & -p_{16} \\ p_{31} & p_{31} & p_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & p_{44} & p_{45} & 0 \\ 0 & 0 & 0 & -p_{45} & p_{44} & 0 \\ -p_{16} & p_{16} & 0 & 0 & 0 & \frac{1}{2}(p_{11}-p_{12}) \end{pmatrix}$	$\begin{pmatrix} p_{11} & p_{12} & p_{21} & 0 & 0 & 0 \\ p_{21} & p_{11} & p_{12} & 0 & 0 & 0 \\ p_{12} & p_{21} & p_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & p_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & p_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & p_{44} \end{pmatrix}$	$\begin{pmatrix} p_{11} & p_{12} & p_{12} & 0 & 0 & 0 \\ p_{12} & p_{11} & p_{13} & 0 & 0 & 0 \\ p_{12} & p_{12} & p_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2}(p_{11}-p_{12}) & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2}(p_{11}-p_{12}) & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2}(p_{11}-p_{12}) \end{pmatrix}$
Point groups $6mm$, $\bar{6}m2$, 622, $6/mmm$	Point groups $\bar{4}3m$, 432, $m\bar{3}m$	
$\begin{pmatrix} p_{11} & p_{12} & p_{13} & 0 & 0 & 0 \\ p_{12} & p_{11} & p_{13} & 0 & 0 & 0 \\ p_{13} & p_{13} & p_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & p_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & p_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2}(p_{11}-p_{12}) \end{pmatrix}$	$\begin{pmatrix} p_{11} & p_{12} & p_{12} & 0 & 0 & 0 \\ p_{12} & p_{11} & p_{13} & 0 & 0 & 0 \\ p_{12} & p_{12} & p_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & p_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & p_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & p_{44} \end{pmatrix}$	

$$\Delta n_1 = \Delta n_2 = -\frac{n_{\rm cub}^3}{2} p_{12} S_3^s$$
$$\Delta n_3 = -\frac{n_{\rm cub}^3}{2} p_{11} S_3^s, \qquad (1.6.7.9)$$

where $n_{\rm cub}$ is the refractive index of the cubic phase. Thus the birefringence in the tetragonal phase as seen by light travelling along x_1 is given by

$$\Delta n_3 - \Delta n_2 = -\frac{n_{\rm cub}^3}{2}(p_{11} - p_{12}) S_3^s. \qquad (1.6.7.10)$$

Thus a direct connection is made between the birefringence of the tetragonal phase of $BaTiO_3$ and its lattice parameters *via* the spontaneous strain. As in the case of the linear electro-optic effect, the calculation can be repeated using equation (1.6.3.14) with the susceptibilities χ_{11} and χ_{12} to yield the relationship

$$p_{11} = \frac{c_{1111}\varepsilon_o \chi_{11}}{n_o^4}; \quad p_{12} = \frac{c_{1122}\varepsilon_o \chi_{12}}{n_o^4}. \qquad (1.6.7.11)$$

1.6.7.3. *The acousto-optic effect*

The acousto-optic effect (Sapriel, 1976) is really a variant of the elasto-optic effect, in that the strain field is created by the passage of a sound wave through the crystal. If this wave has frequency ω_1, the resulting polarization in the presence of a light wave of frequency ω_2 is given by $P_i^{\omega} = \chi_{ijk\ell} E_j^{\omega_2} S_{k\ell}^{\omega_1}$, where $\omega = \omega_1 \pm \omega_2$. However, since the sound-wave frequency is very small compared with that of the light, to all intents and purposes the change in frequency of the light field can be ignored. The effect then of the sound wave is to produce within an acousto-optic crystal a spatially modulated change in refractive index: a beam of light can then be diffracted by this spatial modulation, the resulting optical diffraction pattern thus changing with the changing sound signal. Acousto-optic materials therefore can be used as transducers for converting sound signals into optical signals for transmission down optical fibres in communications systems. Consider, for instance, a sound wave propagating along the [110] direction in gallium arsenide (GaAs), which crystallizes in point group $\bar{4}3m$. Suppose that this sound wave is longitudinally polarized. With respect to the cube axes, this corresponds to an oscillatory shear strain $S_{12}\sin(\omega t - k\xi)$, where ξ is a distance along the [110] direction (Fig. 1.6.7.1). Then one can write

$$\Delta\eta_{ij} = p_{ij12} S_{12} \sin(\omega t - k\xi) \qquad (1.6.7.12)$$

or in contracted notation

$$\Delta\eta_i = p_{i6} S_6 \sin(\omega t - k\xi). \qquad (1.6.7.13)$$

From Table 1.6.7.1, it seen that the change in dielectric impermeability tensor is

$$\Delta\eta_6 = p_{66}S_6\sin(\omega t - k\xi) = p_{44}S_6\sin(\omega t - k\xi) \qquad (1.6.7.14)$$

since all other components are zero. This means that the original spherical indicatrix of the cubic crystal has been distorted to form a biaxial indicatrix whose axes oscillate in length according to

$$\begin{aligned} n_1 &= n_{\rm cub} + \frac{n^3_{\rm cub}}{2}p_{44}S_6\sin(\omega t - k\xi) \\ n_2 &= n_{\rm cub} - \frac{n^3_{\rm cub}}{2}p_{44}S_6\sin(\omega t - k\xi) \\ n_3 &= n_{\rm cub}, \end{aligned} \qquad (1.6.7.15)$$

thus forming an optical grating of spatial periodicity given by the $k\xi$ term. In gallium arsenide, at a wavelength of light equal to 1.15 µm, $p_{11} = -0.165$, $p_{12} = -0.140$ and $p_{44} = -0.072$. It is convenient to define a figure of merit for acousto-optic materials (Yariv & Yeh, 1983) given by

$$M = \frac{n^6p^2}{dv^3}, \qquad (1.6.7.16)$$

where v is the velocity of the sound wave and d is the density of the solid. For gallium arsenide, $d = 5340$ kg m^{-3}, and for a sound wave propagating as above $v = 5.15$ m s^{-1}. At the wavelength $\lambda = 1.15$µm, $n = 3.37$, and so it is found that $M = 104$. In practice, figures of merits can range from less than 0.001 up to as high as 4400 in the case of Te, and so the value for gallium arsenide makes it potentially useful as an acousto-optic material for infrared signals.

1.6.8. Glossary

α, β, γ	refractive indices of biaxial indicatrix, $\alpha < \beta < \gamma$
$\hat{\alpha}$	polarizability operator
B_i	ith component of magnetic induction
c	velocity of light
$c_{k\ell mn}$	$k\ell mn$th component of elastic stiffness tensor
$\chi_{ijk\ldots}$	$ijk\ldots$th component of generalized susceptibility
d	density
D_i	ith component of dielectric displacement
Δ	phase difference of light
$\hat{e}_{ijm}$	unit antisymmetric pseudotensor of rank 3
E_i	ith component of electric field
g_{ij}, G_{ij}	ijth component of gyration tensor
$\mathbf{G}$	gyration vector
$\gamma_{ij\ell}$	third-rank optical gyration susceptibility
$\mathbf{H}$	magnetic field intensity
η_{ij}	ijth component of dielectric impermeability tensor
ε_o	permittivity of free space
ε_{ij}	ijth component of dielectric tensor
κ	ellipticity of wave
$\mathbf{k}$	wavevector of light propagating in crystal ($\lvert k\rvert = 2\pi/\lambda$)
λ	wavelength of light
μ_o	vacuum magnetic permeability
n	refractive index of light
$n_\alpha, n_\beta, n_\gamma$	refractive indices for biaxial indicatrix, $n_\alpha < n_\beta < n_\gamma$
n_o	ordinary refractive index
n_e	extraordinary refractive index
Ψ_i	wavefunction of state i
P_i	ith component of electric polarization
$p_{ijk\ell}$	$ijk\ell$th component of elasto-optic (strain-optic) tensor
$\hat{p}$	electric dipole operator
ρ	optical rotatory power
$\pi_{ijk\ell}$	$ijk\ell$th component of linear piezo-optic tensor
r_{ijk}	ijkth component of linear electro-optic tensor
$\hat{s}$	unit vector in the direction of **s**, the wave normal
S_{ij}	ijth component of strain tensor
T_{ij}	ijth component of stress tensor
v	velocity of sound
V	half the angle between optic axes
ω	cyclic frequency
x_i	direction of ith Cartesian axis, $i = 1, 2, 3$

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