International Tables for Crystallography (2006). Vol. D, Figure 1.7.3.3, p. 186.

with $-\rho^{+}(\theta, \omega)$ for the positive class and $+\rho^{-}(\theta, \omega)$ for the negative class. $\rho^{\pm}(\theta, \omega)$ is given by

$$\begin{aligned}\rho^{\pm}(\theta, \omega) &= \arccos(\mathbf{d}^{\pm} \cdot \mathbf{e}^{\pm}) = \arccos(\mathbf{u}^{\pm} \cdot \mathbf{s}^{\pm}) \\ &= \arccos\left\{\left[\frac{\cos^2\theta}{n_o^2(\omega)} + \frac{\sin^2\theta}{n_e^2(\omega)}\right]\left[\frac{\cos^2\theta}{n_o^4(\omega)} + \frac{\sin^2\theta}{n_e^4(\omega)}\right]^{-1/2}\right\}.\end{aligned} \quad (1.7.3.13)$$

Note that the extraordinary walk-off angle is nil for a propagation along the optic axis ($\theta = 0$) and everywhere in the xy plane ($\theta = \pi/2$).

1.7.3.1.4. *Biaxial class*

In a biaxial crystal, the three principal refractive indices are all different. The graphical representations of the index surfaces are given in Fig. 1.7.3.3 for the positive biaxial class ($n_x < n_y < n_z$) and for the negative one ($n_x > n_y > n_z$), both with the usual conventional orientation of the optical frame. If this is not the case, the appropriate permutation of the principal refractive indices is required.

In the orthorhombic system, the three principal axes are fixed by the symmetry; one is fixed in the monoclinic system; and none are fixed in the triclinic system. The index surface of the biaxial class has two umbilici contained in the xz plane, making an angle V with the z axis:

$$\sin^2 V(\omega) = \frac{n_y^{-2}(\omega) - n_x^{-2}(\omega)}{n_z^{-2}(\omega) - n_x^{-2}(\omega)}. \quad (1.7.3.14)$$

The propagation along the optic axes leads to the internal conical refraction effect (Schell & Bloembergen, 1978; Fève *et al.*, 1994).

1.7.3.1.4.1. *Propagation in the principal planes*

It is possible to define ordinary and extraordinary waves, but only in the principal planes of the biaxial crystal: the ordinary electric field vector is perpendicular to the z axis and to the extraordinary one. The walk-off properties of the waves are not the same in the xy plane as in the xz and yz planes.

(1) In the xy plane, the extraordinary wave has no walk-off, in contrast to the ordinary wave. The components of the electric field vectors can be established easily with the same considerations as for the uniaxial class:

$$\begin{aligned} e_x^o &= -\sin[\varphi \pm \rho^{\mp}(\varphi, \omega)] \\ e_y^o &= \cos[\varphi \pm \rho^{\mp}(\varphi, \omega)] \\ e_z^o &= 0, \end{aligned} \quad (1.7.3.15)$$

with $+\rho^{-}(\varphi, \omega)$ for the positive class and $-\rho^{+}(\varphi, \omega)$ for the negative class. $\rho^{\pm}(\varphi, \omega)$ is the walk-off angle given by (1.7.3.13), where θ is replaced by φ, n_o by n_y and n_e by n_x:

$$e_x^e = 0 \quad e_y^e = 0 \quad e_z^e = 1. \quad (1.7.3.16)$$

(2) The yz plane of a biaxial crystal has exactly the same characteristics as any plane containing the optic axis of a uniaxial crystal. The electric field vector components are given by (1.7.3.11) and (1.7.3.12) with $\varphi = \pi/2$. The ordinary walk-off is nil and the extraordinary one is given by (1.7.3.13) with $n_o = n_y$ and $n_e = n_z$.

(3) In the xz plane, the optic axes create a discontinuity of the shape of the internal and external sheets of the index surface leading to a discontinuity of the optic sign and of the electric field vector. The birefringence, $n_e - n_o$, is nil along the optic axis, and its sign changes on either side. Then the yz plane, xy plane and xz plane from the x axis to the optic axis have the same optic sign, the opposite of the optic sign from the optic axis to the z axis. Thus a positive biaxial crystal is negative from the optic axis to the z axis. The situation is inverted for a negative biaxial crystal. It implies the following configuration of polarization:

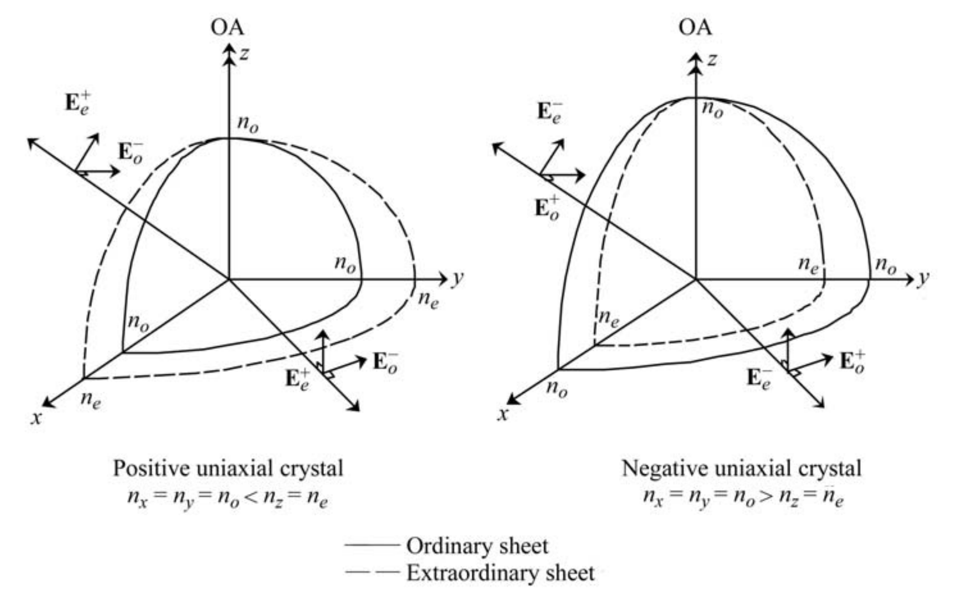

Fig. 1.7.3.2. Index surfaces of the negative and positive uniaxial classes. $\mathbf{E}_{o,e}^{\pm}$ are the ordinary (o) and extraordinary (e) electric field vectors relative to the external (+) or internal (−) sheets. OA is the optic axis.

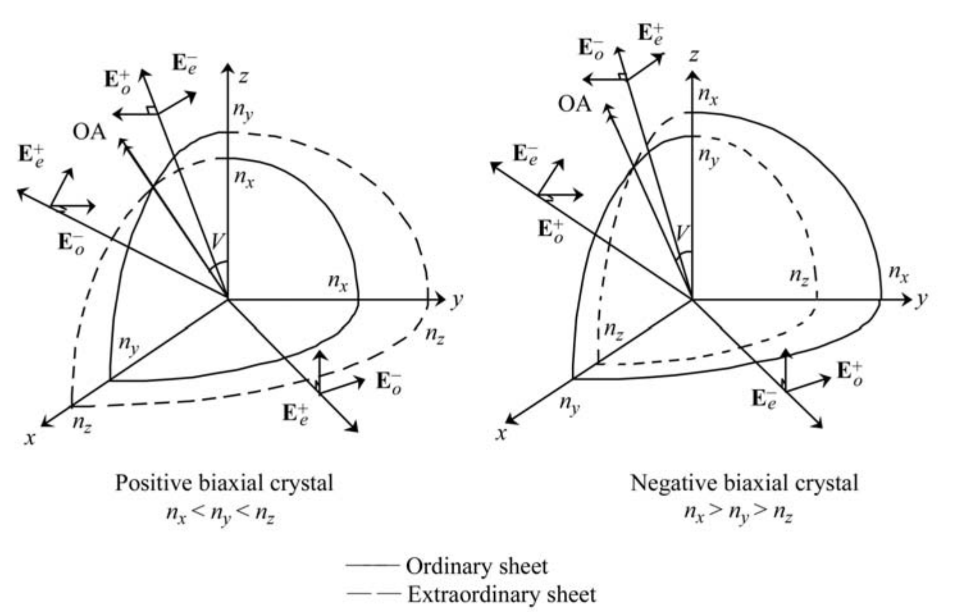

Fig. 1.7.3.3. Index surfaces of the negative and positive biaxial classes. $\mathbf{E}_{o,e}^{\pm}$ are the ordinary (o) and extraordinary (e) electric field vectors relative to the external (+) or internal (−) sheets for a propagation in the principal planes. OA is the optic axis.