

## 1. TENSORIAL ASPECTS OF PHYSICAL PROPERTIES

$$\rho(T) = \rho_i(1 + BT^2) + AT^2. \quad (1.8.3.14)$$

The term  $\rho_i$  is the constant due to the impurity scattering. There is also a term proportional to  $BT^2$ , which is proportional to the impurity resistance. This factor is due to the Koshino–Taylor effect (Koshino, 1960; Taylor, 1964), which has been treated rigorously by Mahan & Wang (1989). It is the inelastic scattering of electrons by impurities. The impurity is part of the lattice and phonons can be excited when the impurity scatters the electrons. The term  $AT^2$  is due to electron–electron interactions. The Coulomb interaction between electrons is highly screened and makes only a small contribution to  $A$ . The largest contribution to  $A$  is caused by phonons. MacDonald *et al.* (1981) showed that electrons can interact by exchanging phonons. There are also terms due to boundary scattering, which is important in thin films: see Bruls *et al.* (1985).

Note that (1.8.3.14) has no term from phonons of  $O(T^5)$ . Such a term is lacking in simple metals, contrary to the assertion in most textbooks. Its absence is due to *phonon drag*. For a review and explanation of this behaviour, see Wisser (1984). The  $T^5$  term is found in the noble metals, where phonon drag is less important owing to the complexities of the Fermi surface.

## 1.8.3.2. Metal alloys

Alloys are solids composed of a mixture of two or more elements that do not form a stoichiometric compound. An example is  $\text{Cu}_x\text{Ni}_{1-x}$ , in which  $x$  can have any value. For small values of  $x$ , or of  $(1-x)$ , the atoms of one element just serve as impurities in the other element. This results in the type of behaviour described above. However, in the range  $0.2 < x < 0.8$ , a different type of resistivity is found. This was first summarized by Mooij (1973), who found a remarkable range of behaviours. He measured the resistivity of hundreds of alloys and also surveyed the published literature for additional results. He represented the resistivity at  $T = 300$  K by two values: the resistivity itself,  $\rho(T = 300)$ , and its logarithmic derivative,  $\alpha = d \ln(\rho)/dT$ . He produced the graph shown in Fig. 1.8.3.2, where these two values are plotted against each other. Each point is one sample as represented by these two numbers. He found that all of the results fit within a band of numbers, in which larger values of  $\rho(T = 300)$  are accompanied by negative values of  $\alpha$ . Alloys with very high values of resistivity generally have a resistivity  $\rho(T)$  that decreases with increasing temperature. The region where  $\alpha = 0$  corresponds to a resistivity of  $\rho^* = 150 \mu\Omega \text{ cm}$ , which appears to be a fixed point. As the temperature is increased, the resistivities of alloys with  $\rho > \rho^*$  decrease to this value, while the resistivities of alloys with  $\rho < \rho^*$  increase to this value.

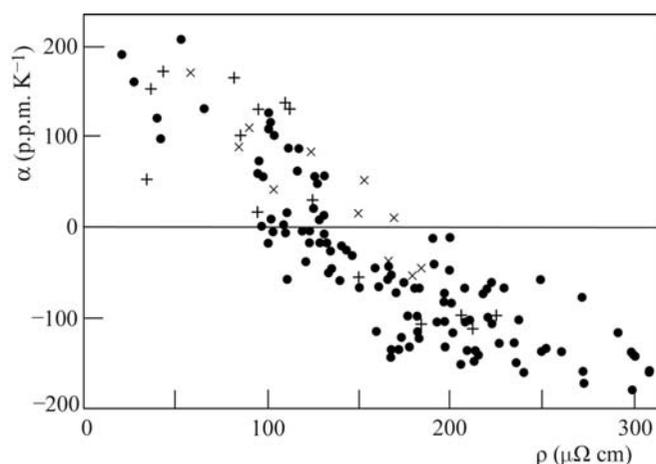


Fig. 1.8.3.2. The temperature coefficient of resistance *versus* resistivity for alloys according to Mooij (1973). Data are shown for bulk alloys (+), thin films (•) and amorphous alloys (x).

Mooij's observations are obviously important, but the reason for this behaviour is not certain. Several different explanations have been proposed and all are plausible: see Jonson & Girvin (1979), Allen & Chakraborty (1981) or Tsuei (1986).

Recently, another group of alloys have been found that are called *bad metals*. The ruthenates (see Allen *et al.*, 1996; Klein *et al.*, 1996) have a resistivity  $\rho > \rho^*$  that increases at high temperatures. Their values are outliers on Mooij's plot.

## 1.8.3.3. Semiconductors

The resistivity of semiconductors varies from sample to sample, even of the same material. The conductivity can be written as  $\sigma = n_0 e \mu$ , where  $e$  is the charge on the electron,  $\mu = e\tau/m^*$  is the mobility and  $n_0$  is the density of conducting particles (electrons or holes). It is the density of particles  $n_0$  that varies from sample to sample. It depends upon the impurity content of the semiconductor as well as upon temperature. Since no two samples have exactly the same number of impurities, they do not have the same values of  $n_0$ . In semiconductors and insulators, the conducting particles are extrinsic – they come from defects, impurities or thermal excitation – in contrast to metals, where the density of the conducting electrons is usually an intrinsic property.

In semiconductors, instead of talking about the conductivity, the more fundamental transport quantity (Rode, 1975) is the mobility  $\mu$ . It is the same for each sample at high temperature if the density of impurities and defects is low. There is an intrinsic mobility, which can be calculated assuming there are no impurities and can be measured in samples with a very low density of impurities. We shall discuss the intrinsic mobility first.

Fig. 1.8.3.3 shows the intrinsic mobility of electrons in silicon, from Rode (1972), as a function of temperature. The mobility generally decreases with increasing temperature. This behaviour is found in all common semiconductors. The mobility also decreases with an increasing concentration of impurities: see Jacoboni *et al.* (1977).

The intrinsic mobility of semiconductors is due to the scattering of electrons and holes by phonons. The phonons come in various branches called TA, LA, TO and LO, where T is transverse, L is longitudinal, A is acoustic and O is optical. At long wavelengths, the acoustic modes are just the sound waves, which

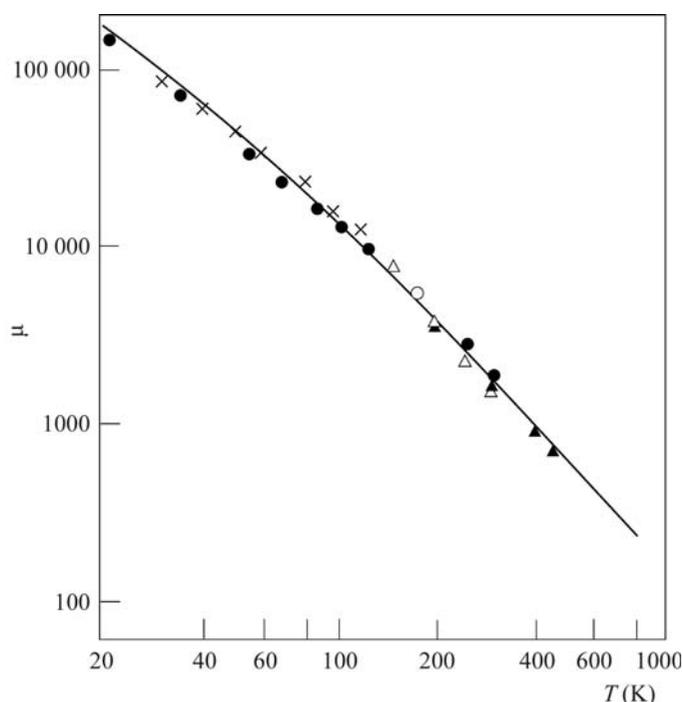


Fig. 1.8.3.3. The intrinsic mobility of electrons in silicon. Solid line: theory; points: experimental. After Rode (1972).