

## 1. TENSORIAL ASPECTS OF PHYSICAL PROPERTIES

Then one imposes a current density  $j_x$  along the  $x$  axis. One finds that the Lorentz force induces a voltage, or the equivalent electric field  $E_y$ , in the  $y$  direction. The electric field is proportional to both the current and magnetic field. The ratio  $E_y/(j_x B_z)$  is the Hall constant  $R_H$ . The inverse of  $R_H$  is just the charge  $e$  and the speed of light  $c$  multiplied by the density of electrons  $n_0$ :

$$\frac{E_y}{j_x B_z} = R_H = \frac{1}{n_0 e c}. \quad (1.8.3.19)$$

This provides a simple and accurate method of measuring the density of electrons. It works well when there is only one kind of current carrier and works well in semiconductors with a low density of carriers. A typical experiment for a semiconductor is to measure the conductivity  $\sigma$  and the Hall constant  $R_H$ ; the mobility is then  $\mu = c R_H \sigma$ . If the conducting particles are holes in a semiconductor, the Hall constant has the opposite sign, which indicates positive charge carriers.

Measurement of the Hall effect does not work well if the semiconductor contains a mixture of different carriers, such as electrons and holes, or even electrons from different kinds of conduction bands. In these cases, the constant  $R_H$  is not easily interpreted. Similarly, measuring the Hall effect is rarely useful in metals. It only works well in the alkali metals, which have all of the electrons in the first Brillouin zone on a spherical Fermi surface. In most metals, the Fermi surface extends over several Brillouin zones and has numerous pockets or regions of different curvatures. Regions of positive curvature act as electrons and give a negative Hall constant; regions of negative curvature act as holes and give a positive contribution to the Hall constant. Again, it is difficult to interpret the Hall constant when both contributions are present. In general, the Hall effect is most useful in semiconductors.

## 1.8.3.5. Insulators

Insulators are crystals that do not conduct electricity by the flow of electrons or holes. We shall not mention this case. The band gaps  $E_G$  are sufficiently large that the intrinsic mobility is very small.

## 1.8.3.6. Ionic conductors

There are many ionic solids that have an appreciable electrical conductivity from the diffusive motion of ions. Any material in which the conductivity from the motion of ions is very much larger than that from the motion of electrons is useful as a battery material. For this reason, such materials have been investigated extensively, see *e.g.* Mahan & Roth (1976) or Salamon (1979).

## 1.8.4. Thermal conductivity

## 1.8.4.1. Introduction

The thermal conductivity determines the ability of the crystal to conduct heat. Device applications of crystals usually need an extreme value of the thermal conductivity: some applications need a low thermal conductivity, while others need a high thermal conductivity. At room temperature, the largest value of thermal conductivity is for diamond, which has  $K = 1.8 \text{ kW m}^{-1} \text{ K}^{-1}$ ; see *e.g.* Spitzer (1970), Slack (1979) or Berman (1976). The lowest values are for amorphous materials, which have  $K = 0.1 \text{ W m}^{-1} \text{ K}^{-1}$ .

Heat flow can be carried by two kinds of excitations: phonons and electrons. The phonons carry most of the heat in insulators and semiconductors. Electrons carry appreciable amounts of heat only if there is a high density of conduction electrons, as in a metal. In metals, the electrons usually carry most of the heat. Of course, the heat conduction by phonons and electrons depends upon temperature. At high temperatures, the standard assumption is that the heat flows from phonons and electrons are inde-

pendent and can be calculated separately. However, there is an electron-phonon interaction, which causes a correlation between the two kinds of heat flow. This is called *phonon drag* and is an important phenomenon at low temperatures – typically less than 50 K. We are concerned mostly with higher temperatures, so will not discuss phonon drag here.

First consider the heat flow carried by phonons. As a rule of thumb, crystals with high values of thermal conductivity are those that are: (1) tetrahedrally bonded, (2) contain elements of low atomic number, and (3) lack impurities and defects (Spitzer, 1970; Berman, 1976; Slack, 1979).

The inverse of the thermal conductivity is called the thermal resistivity. There is an equivalent of Matthiessen's rule for thermal conductivity: it is a reasonable approximation to take the various contributions to the thermal resistivity and simply to add them. This is not a rigorous theorem; it is just a process that gives a reasonable answer most of the time. Here we shall discuss four contributions to the thermal resistivity: boundary scattering, impurity scattering, isotope scattering and anharmonic interactions:

$$R_K = R_B + R_i + R_l + R_A. \quad (1.8.4.1)$$

These various terms are discussed in order.

## 1.8.4.2. Boundary scattering

At low temperatures, the phonons that are thermally excited are those which have an energy near to or less than the thermal energy  $\hbar\Omega(q) \leq k_B T$ . This usually means acoustic modes of long wavelength. They tend to have a long mean free path, which can extend to the size of the crystal. In this case, the limiting process on the phonon scattering is simply bouncing off the walls of the crystal. The formula for this process is best derived from the classical formula for the thermal conductivity (see Ziman, 1962),

$$K = (1/3)C\bar{v}\Lambda, \quad (1.8.4.2)$$

where  $C$  is the heat capacity,  $\bar{v}$  is the average velocity and  $\Lambda$  is the mean free path. To apply this to the present problem, take  $\Lambda$  to be equal to the average dimension of the crystal and  $\bar{v}$  to be the speed of sound averaged over the various directions. At low temperatures, the heat capacity  $C \propto T^3$ , as given by the Debye theory. Since  $\Lambda$  and  $\bar{v}$  are constants, then  $K \propto T^3$ , which agrees well with the dependence found experimentally. Of course, the thermal resistance  $R_B$  is just the inverse of  $R_B \propto T^{-3}$ .

## 1.8.4.3. Impurity scattering

Impurities can be either point defects or extended defects such as dislocations. Here we confine our remarks to point defects. When acoustic phonons of long wavelength scatter from point defects, the process is very much like the Rayleigh scattering of light. The basic cross section varies as the fourth power of the frequency. Equivalently, at long wavelength, it varies as the fourth power of the wave number  $q$  of the phonons (see Ziman, 1962):

$$\frac{1}{\tau_i(q)} = n_i q^4 C_i, \quad (1.8.4.3)$$

where  $n_i$  is the concentration of impurities and  $C_i$  is a constant characteristic of the impurity. Of course, this lifetime for the phonon of wave number  $q$  must be averaged over all of the wavevectors in the crystal as a function of temperature. This averaging is actually mathematically delicate. At temperatures  $T$  less than the Debye temperature  $\Theta$ , the average value of  $q$  is  $q \sim k_B T / (\hbar v)$  and  $R_i \sim T^4$ . At high temperatures ( $T > \Theta$ ), the average of  $1/\tau(q)$  is a constant, since all values of  $q$  in the Brillouin zone are equally accessible. In this limit,  $R_i \sim \text{constant}$ .