

## 2.4. Brillouin scattering

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### 2.4.1. Introduction

Brillouin scattering originates from the interaction of an incident radiation with thermal acoustic vibrations in matter. The phenomenon was predicted by Brillouin in 1922 (Brillouin, 1922) and first observed in light scattering by Gross (Gross, 1930*a,b*). However, owing to specific spectrometric difficulties, precise experimental studies of Brillouin lines in crystals were not performed until the 1960s (Cecchi, 1964; Benedek & Fritsch, 1966; Gornall & Stoicheff, 1970) and Brillouin scattering became commonly used for the investigation of elastic properties of condensed matter with the advent of laser sources and multipass Fabry–Perot interferometers (Hariharan & Sen, 1961; Sandercock, 1971). More recently, Brillouin scattering of neutrons (Egelstaff *et al.*, 1989) and X-rays (Sette *et al.*, 1998) has been observed.

Brillouin scattering of light probes long-wavelength acoustic phonons. Thus, the detailed atomic structure is irrelevant and the vibrations of the scattering medium are determined by macroscopic parameters, in particular the density  $\rho$  and the elastic coefficients  $c_{ijkl}$ . For this reason, Brillouin scattering is observed in gases, in liquids and in crystals as well as in disordered solids.

Vacher & Boyer (1972) and Cummins & Schoen (1972) have performed a detailed investigation of the selection rules for Brillouin scattering in materials of various symmetries. In this chapter, calculations of the sound velocities and scattered intensities for the most commonly investigated vibrational modes in bulk condensed matter are presented. Brillouin scattering from surfaces will not be discussed. The current state of the art for Brillouin spectroscopy is also briefly summarized.

### 2.4.2. Elastic waves

#### 2.4.2.1. Non-piezoelectric media

The fundamental equation of dynamics (see Section 1.3.4.2), applied to the displacement  $\mathbf{u}$  of an elementary volume at  $\mathbf{r}$  in a homogeneous material is

$$\rho \ddot{\mathbf{u}}_i = \frac{\partial T_{ij}}{\partial x_j}. \quad (2.4.2.1)$$

Summation over repeated indices will always be implied, and  $\mathbf{T}$  is the stress tensor. In non-piezoelectric media, the constitutive equation for small strains  $\mathbf{S}$  is simply

$$T_{ij} = c_{ijkl} S_{kl}. \quad (2.4.2.2)$$

The strain being the symmetrized spatial derivative of  $\mathbf{u}$ , and  $\mathbf{c}$  being symmetric upon interchange of  $k$  and  $\ell$ , the introduction of (2.4.2.2) in (2.4.2.1) gives (see also Section 1.3.4.2)

$$\rho \ddot{\mathbf{u}}_i = c_{ijkl} \frac{\partial^2 \mathbf{u}_k}{\partial x_j \partial x_\ell}. \quad (2.4.2.3)$$

One considers harmonic plane-wave solutions of wavevector  $\mathbf{Q}$  and frequency  $\omega$ ,

$$\mathbf{u}(\mathbf{r}, t) = \mathbf{u}_0 \exp i(\mathbf{Q} \cdot \mathbf{r} - \omega t). \quad (2.4.2.4)$$

For  $\mathbf{u}_0$  small compared with the wavelength  $2\pi/Q$ , the total derivative  $\ddot{\mathbf{u}}$  can be replaced by the partial  $\partial^2 \mathbf{u} / \partial t^2$  in (2.4.2.3). Introducing (2.4.2.4) into (2.4.2.3), one obtains

$$c_{ijkl} \hat{Q}_j \hat{Q}_\ell \mathbf{u}_{0k} = C \delta_{ik} \mathbf{u}_{0k}, \quad (2.4.2.5)$$

where  $\hat{\mathbf{Q}} = \mathbf{Q}/|\mathbf{Q}|$  is the unit vector in the propagation direction,  $\delta_{ik}$  is the unit tensor and  $C \equiv \rho V^2$ , where  $V = \omega/|\mathbf{Q}|$  is the phase velocity of the wave. This shows that  $\mathbf{u}_0$  is an eigenvector of the tensor  $c_{ijkl} \hat{Q}_j \hat{Q}_\ell$ . For a given propagation direction  $\mathbf{Q}$ , the three eigenvalues  $C^{(s)}$  are obtained by solving

$$\left| c_{ijkl} \hat{Q}_j \hat{Q}_\ell - C \delta_{ik} \right| = 0. \quad (2.4.2.6)$$

To each  $C^{(s)}$  there is an eigenvector  $\mathbf{u}^{(s)}$  given by (2.4.2.5) and an associated phase velocity

$$V^{(s)} = \sqrt{C^{(s)}/\rho}. \quad (2.4.2.7)$$

The tensor  $c_{ijkl} \hat{Q}_j \hat{Q}_\ell$  is symmetric upon interchange of the indices ( $i, k$ ) because  $c_{ijkl} = c_{klij}$ . Its eigenvalues are real positive, and the three directions of vibration  $\hat{\mathbf{u}}^{(s)}$  are mutually perpendicular. The notation  $\hat{\mathbf{u}}^{(s)}$  indicates a unit vector. The tensor  $c_{ijkl} \hat{Q}_j \hat{Q}_\ell$  is also invariant upon a change of sign of the propagation direction. This implies that the solution of (2.4.2.5) is the same for all symmetry classes belonging to the same Laue class.

For a general direction  $\hat{\mathbf{Q}}$ , and for a symmetry lower than isotropic,  $\hat{\mathbf{u}}^{(s)}$  is neither parallel nor perpendicular to  $\hat{\mathbf{Q}}$ , so that the modes are neither purely longitudinal nor purely transverse. In this case (2.4.2.6) is also difficult to solve. The situation is much simpler when  $\hat{\mathbf{Q}}$  is parallel to a symmetry axis of the Laue class. Then, one of the vibrations is purely longitudinal (LA), while the other two are purely transverse (TA). A pure mode also exists when  $\hat{\mathbf{Q}}$  belongs to a symmetry plane of the Laue class, in which case there is a transverse vibration with  $\hat{\mathbf{u}}$  perpendicular to the symmetry plane. For all these *pure mode directions*, (2.4.2.6) can be factorized to obtain simple analytical solutions. In this chapter, only pure mode directions are considered.

#### 2.4.2.2. Piezoelectric media

In piezoelectric crystals, a stress component is also produced by the internal electric field  $\mathbf{E}$ , so that the constitutive equation (2.4.2.2) has an additional term (see Section 1.1.5.2),

$$T_{ij} = c_{ijkl} S_{kl} - e_{mij} E_m, \quad (2.4.2.8)$$

where  $\mathbf{e}$  is the piezoelectric tensor at constant strain.

The electrical displacement vector  $\mathbf{D}$ , related to  $\mathbf{E}$  by the dielectric tensor  $\boldsymbol{\varepsilon}$ , also contains a contribution from the strain,

$$D_m = \varepsilon_{mn} E_n + e_{mkl} S_{kl}, \quad (2.4.2.9)$$

where  $\boldsymbol{\varepsilon}$  is at the frequency of the elastic wave.

In the absence of free charges,  $\text{div } \mathbf{D} = 0$ , and (2.4.2.9) provides a relation between  $\mathbf{E}$  and  $\mathbf{S}$ ,

$$\varepsilon_{mn} Q_n E_m + e_{mkl} Q_m S_{kl} = 0. \quad (2.4.2.10)$$

For long waves, it can be shown that  $\mathbf{E}$  and  $\mathbf{Q}$  are parallel. (2.4.2.10) can then be solved for  $\mathbf{E}$ , and this value is replaced in (2.4.2.8) to give

$$T_{ij} = \left[ c_{ijkl} + \frac{e_{mij} e_{nkl} \hat{Q}_m \hat{Q}_n}{\varepsilon_{gh} \hat{Q}_g \hat{Q}_h} \right] S_{kl} \equiv c_{ijkl}^{(e)} S_{kl}. \quad (2.4.2.11)$$

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Comparing (2.4.2.11) and (2.4.2.2), one sees that the effective elastic tensor  $\mathbf{c}^{(e)}$  now depends on the propagation direction  $\mathbf{Q}$ . Otherwise, all considerations of the previous section, starting from (2.4.2.6), remain, with  $\mathbf{c}$  simply replaced by  $\mathbf{c}^{(e)}$ .

### 2.4.3. Coupling of light with elastic waves

#### 2.4.3.1. Direct coupling to displacements

The change in the relative optical dielectric tensor  $\boldsymbol{\kappa}$  produced by an elastic wave is usually expressed in terms of the strain, using the Pockels piezo-optic tensor  $\mathbf{p}$ , as

$$(\Delta\kappa^{-1})_{ij} = p_{ijk\ell} S_{k\ell}. \quad (2.4.3.1)$$

The elastic wave should, however, be characterized by both strain  $\mathbf{S}$  and rotation  $\mathbf{A}$  (Nelson & Lax, 1971; see also Section 1.3.1.3):

$$A_{[k\ell]} = \frac{1}{2} \left( \frac{\partial u_k}{\partial x_\ell} - \frac{\partial u_\ell}{\partial x_k} \right). \quad (2.4.3.2)$$

The square brackets on the left-hand side are there to emphasize that the component is antisymmetric upon interchange of the indices,  $A_{[k\ell]} = -A_{[\ell k]}$ . For birefringent crystals, the rotations induce a change of the local  $\boldsymbol{\kappa}$  in the laboratory frame. In this case, (2.4.3.1) must be replaced by

$$(\Delta\kappa^{-1})_{ij} = p'_{ijk\ell} \frac{\partial u_k}{\partial x_\ell}, \quad (2.4.3.3)$$

where  $\mathbf{p}'$  is the new piezo-optic tensor given by

$$p'_{ijk\ell} = p_{ijk\ell} + p_{ij[\ell k]}. \quad (2.4.3.4)$$

One finds for the rotational part

$$p_{ij[\ell k]} = \frac{1}{2} [(\kappa^{-1})_{i\ell} \delta_{kj} + (\kappa^{-1})_{\ell j} \delta_{ik} - (\kappa^{-1})_{ik} \delta_{\ell j} - (\kappa^{-1})_{kj} \delta_{i\ell}]. \quad (2.4.3.5)$$

If the principal axes of the dielectric tensor coincide with the crystallographic axes, this gives

$$p_{ij[\ell k]} = \frac{1}{2} (\delta_{i\ell} \delta_{kj} - \delta_{ik} \delta_{\ell j}) (1/n_i^2 - 1/n_j^2). \quad (2.4.3.6)$$

This is the expression used in this chapter, as monoclinic and triclinic groups are not listed in the tables below.

For the calculation of the Brillouin scattering, it is more convenient to use

$$(\Delta\kappa)_{mn} = -\kappa_{mi} \kappa_{nj} p'_{ijk\ell} \frac{\partial u_k}{\partial x_\ell}, \quad (2.4.3.7)$$

which is valid for small  $\Delta\kappa$ .

#### 2.4.3.2. Coupling via the electro-optic effect

Piezoelectric media also exhibit an electro-optic effect linear in the applied electric field or in the field-induced crystal polarization. This effect is described in terms of the third-rank electro-optic tensor  $\mathbf{r}$  defined by

$$(\Delta\kappa^{-1})_{ij} = r_{ijm} E_m. \quad (2.4.3.8)$$

Using the same approach as in (2.4.2.10), for long waves  $E_m$  can be expressed in terms of  $S_{k\ell}$ , and (2.4.3.8) leads to an effective Pockels tensor  $\mathbf{p}^e$  accounting for both the piezo-optic and the electro-optic effects:

$$p_{ijk\ell}^e = p_{ijk\ell} - \frac{r_{ijm} e_{nkl} \hat{Q}_m \hat{Q}_n}{\varepsilon_{gh} \hat{Q}_g \hat{Q}_h}. \quad (2.4.3.9)$$

The total change in the inverse dielectric tensor is then

$$(\Delta\kappa^{-1})_{ij} = (p_{ijk\ell}^e + p_{ij[\ell k]}) \frac{\partial u_k}{\partial x_\ell} = p'_{ijk\ell} \frac{\partial u_k}{\partial x_\ell}. \quad (2.4.3.10)$$

The same equation (2.4.3.7) applies.

### 2.4.4. Brillouin scattering in crystals

#### 2.4.4.1. Kinematics

Brillouin scattering occurs when an incident photon at frequency  $\nu_i$  interacts with the crystal to either produce or absorb an acoustic phonon at  $\delta\nu$ , while a scattered photon at  $\nu_s$  is simultaneously emitted. Conservation of energy gives

$$\delta\nu = \nu_s - \nu_i, \quad (2.4.4.1)$$

where positive  $\delta\nu$  corresponds to the anti-Stokes process. Conservation of momentum can be written

$$\mathbf{Q} = \mathbf{k}_s - \mathbf{k}_i, \quad (2.4.4.2)$$

where  $\mathbf{Q}$  is the wavevector of the emitted phonon, and  $\mathbf{k}_s$ ,  $\mathbf{k}_i$  are those of the scattered and incident photons, respectively. One can define unit vectors  $\mathbf{q}$  in the direction of the wavevectors  $\mathbf{k}$  by

$$\mathbf{k}_i = 2\pi\mathbf{q}n/\lambda_0, \quad (2.4.4.3a)$$

$$\mathbf{k}_s = 2\pi\mathbf{q}'n'/\lambda_0, \quad (2.4.4.3b)$$

where  $n$  and  $n'$  are the appropriate refractive indices, and  $\lambda_0$  is the vacuum wavelength of the radiation. Equation (2.4.4.3b) assumes that  $\delta\nu \ll \nu_i$  so that  $\lambda_0$  is not appreciably changed in the scattering. The incident and scattered waves have unit polarization vectors  $\mathbf{e}$  and  $\mathbf{e}'$ , respectively, and corresponding indices  $n$  and  $n'$ . The polarization vectors are the principal directions of vibration derived from the sections of the ellipsoid of indices by planes perpendicular to  $\mathbf{q}$  and  $\mathbf{q}'$ , respectively. We assume that the electric vector of the light field  $\mathbf{E}_{\text{opt}}$  is parallel to the displacement  $\mathbf{D}_{\text{opt}}$ . This is exactly true for many cases listed in the tables below. In the other cases (such as skew directions in the orthorhombic group) this assumes that the birefringence is sufficiently small for the effect of the angle between  $\mathbf{E}_{\text{opt}}$  and  $\mathbf{D}_{\text{opt}}$  to be negligible. A full treatment, including this effect, has been given by Nelson *et al.* (1972).

After substituting (2.4.4.3) in (2.4.4.2), the unit vector in the direction of the phonon wavevector is given by

$$\hat{\mathbf{Q}} = \frac{n'\mathbf{q}' - n\mathbf{q}}{|n'\mathbf{q}' - n\mathbf{q}|}. \quad (2.4.4.4)$$

The Brillouin shift  $\delta\nu$  is related to the phonon velocity  $V$  by

$$\delta\nu = VQ/2\pi. \quad (2.4.4.5)$$

Since  $\nu\lambda_0 = c$ , from (2.4.4.5) and (2.4.4.3), (2.4.4.4) one finds

$$\delta\nu \cong (V/\lambda_0)[n^2 + (n')^2 - 2nn' \cos \theta]^{1/2}, \quad (2.4.4.6)$$

where  $\theta$  is the angle between  $\mathbf{q}$  and  $\mathbf{q}'$ .

#### 2.4.4.2. Scattering cross section

The power  $dP_{\text{in}}$ , scattered from the illuminated volume  $V$  in a solid angle  $d\Omega_{\text{in}}$ , where  $P_{\text{in}}$  and  $\Omega_{\text{in}}$  are measured inside the sample, is given by

$$\frac{dP_{\text{in}}}{d\Omega_{\text{in}}} = V \frac{k_B T \pi^2 n'}{2n\lambda_0^4 C} M I_{\text{in}}, \quad (2.4.4.7)$$

where  $I_{\text{in}}$  is the incident light intensity inside the material,  $C = \rho V^2$  is the appropriate elastic constant for the observed phonon, and the factor  $k_B T$  results from taking the fluctuation-

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dissipation theorem in the classical limit for  $h\delta\nu \ll k_B T$  (Hayes & Loudon, 1978). The coupling coefficient  $M$  is given by

$$M = |e_m e'_n \kappa_{mi} \kappa_{nj} p'_{ijk\ell} \hat{u}_k \hat{Q}_\ell|^2. \quad (2.4.4.8)$$

In practice, the incident intensity is defined outside the scattering volume,  $I_{\text{out}}$ , and for normal incidence one can write

$$I_{\text{in}} = \frac{4n}{(n+1)^2} I_{\text{out}}. \quad (2.4.4.9a)$$

Similarly, the scattered power is observed outside as  $P_{\text{out}}$ , and

$$P_{\text{out}} = \frac{4n'}{(n'+1)^2} P_{\text{in}}, \quad (2.4.4.9b)$$

again for normal incidence. Finally, the approximative relation between the scattering solid angle  $\Omega_{\text{out}}$ , outside the sample, and the solid angle  $\Omega_{\text{in}}$ , in the sample, is

$$\Omega_{\text{out}} = (n')^2 \Omega_{\text{in}}. \quad (2.4.4.9c)$$

Substituting (2.4.4.9a,b,c) in (2.4.4.7), one obtains (Vacher & Boyer, 1972)

$$\frac{dP_{\text{out}}}{d\Omega_{\text{out}}} = \frac{8\pi^2 k_B T}{\lambda_0^4} \frac{n^4}{(n+1)^2} \frac{(n')^4}{(n'+1)^2} \beta V I_{\text{out}}, \quad (2.4.4.10)$$

where the coupling coefficient  $\beta$  is

$$\beta = \frac{1}{n^4 (n')^4} \frac{|e_m e'_n \kappa_{mi} \kappa_{nj} p'_{ijk\ell} \hat{u}_k \hat{Q}_\ell|^2}{C}. \quad (2.4.4.11)$$

In the cases of interest here, the tensor  $\kappa$  is diagonal,  $\kappa_{ij} = n_i^2 \delta_{ij}$  without summation on  $i$ , and (2.4.4.11) can be written in the simpler form

$$\beta = \frac{1}{n^4 (n')^4} \frac{|e_i n_i^2 p'_{ijk\ell} \hat{u}_k \hat{Q}_\ell e'_j n_j^2|^2}{C}. \quad (2.4.4.12)$$

### 2.4.5. Use of the tables

The tables in this chapter give information on modes and scattering geometries that are in most common use in the study of hypersound in single crystals. Just as in the case of X-rays, Brillouin scattering is not sensitive to the presence or absence of a centre of symmetry (Friedel, 1913). Hence, the results are the same for all crystalline classes belonging to the same centric group, also called Laue class. The correspondence between the point groups and the Laue classes analysed here is shown in Table 2.4.5.1. The monoclinic and triclinic cases, being too cumbersome, will not be treated here.

For tensor components  $c_{ijk\ell}$  and  $p_{ijk\ell}$ , the tables make use of the usual contracted notation for index pairs running from 1 to 6. However, as the tensor  $p'_{ijk\ell}$  is not symmetric upon interchange of  $(k, \ell)$ , it is necessary to distinguish the order  $(k, \ell)$  and  $(\ell, k)$ . This is accomplished with the following correspondence:

$$\begin{aligned} 1, 1 &\rightarrow 1 & 2, 2 &\rightarrow 2 & 3, 3 &\rightarrow 3 \\ 1, 2 &\rightarrow 6 & 2, 3 &\rightarrow 4 & 3, 1 &\rightarrow 5 \\ 2, 1 &\rightarrow \bar{6} & 3, 2 &\rightarrow \bar{4} & 1, 3 &\rightarrow \bar{5}. \end{aligned}$$

Geometries for longitudinal modes (LA) are listed in Tables 2.4.5.2 to 2.4.5.8. The first column gives the direction of the scattering vector  $\hat{\mathbf{Q}}$  that is parallel to the displacement  $\hat{\mathbf{u}}$ . The second column gives the elastic coefficient according to (2.4.2.6). In piezoelectric materials, effective elastic coefficients defined in (2.4.2.11) must be used in this column. The third column gives the direction of the light polarizations  $\hat{\mathbf{e}}$  and  $\hat{\mathbf{e}}'$ , and the last column

gives the corresponding coupling coefficient  $\beta$  [equation (2.5.5.11)]. In general, the strongest scattering intensity is obtained for polarized scattering ( $\hat{\mathbf{e}} = \hat{\mathbf{e}}'$ ), which is the only situation listed in the tables. In this case, the coupling to light ( $\beta$ ) is independent of the scattering angle  $\theta$ , and thus the tables apply to any  $\theta$  value.

Tables 2.4.5.9 to 2.4.5.15 list the geometries usually used for the observation of TA modes in backscattering ( $\theta = 180^\circ$ ). In this case,  $\hat{\mathbf{u}}$  is always perpendicular to  $\hat{\mathbf{Q}}$  (pure transverse modes), and  $\hat{\mathbf{e}}'$  is not necessarily parallel to  $\hat{\mathbf{e}}$ . Cases where pure TA modes with  $\hat{\mathbf{u}}$  in the plane perpendicular to  $\hat{\mathbf{Q}}$  are degenerate are indicated by the symbol  $D$  in the column for  $\hat{\mathbf{u}}$ . For the Pockels tensor components, the notation is  $p_{\alpha\beta}$  if the rotational term vanishes by symmetry, and it is  $p'_{\alpha\beta}$  otherwise.

Tables 2.4.5.16 to 2.4.5.22 list the common geometries used for the observation of TA modes in  $90^\circ$  scattering. In these tables, the polarization vector  $\hat{\mathbf{e}}$  is always perpendicular to the scattering plane and  $\hat{\mathbf{e}}'$  is always parallel to the incident wavevector of light  $\mathbf{q}$ . Owing to birefringence, the scattering vector  $\hat{\mathbf{Q}}$  does not exactly bisect  $\mathbf{q}$  and  $\mathbf{q}'$  [equation (2.4.4.4)]. The tables are written for strict  $90^\circ$  scattering,  $\mathbf{q} \cdot \mathbf{q}' = 0$ , and in the case of birefringence the values of  $\mathbf{q}^{(m)}$  to be used are listed separately in Table 2.4.5.23. The latter assumes that the birefringences are not large, so that the values of  $\mathbf{q}^{(m)}$  are given only to first order in the birefringence.

### 2.4.6. Techniques of Brillouin spectroscopy

Brillouin spectroscopy with visible laser light requires observing frequency shifts falling typically in the range  $\sim 1$  to  $\sim 100$  GHz, or  $\sim 0.03$  to  $\sim 3$   $\text{cm}^{-1}$ . To achieve this with good resolution one mostly employs interferometry. For experiments at very small angles (near forward scattering), photocorrelation spectroscopy can also be used. If the observed frequency shifts are  $\geq 1$   $\text{cm}^{-1}$ , rough measurements of spectra can sometimes be obtained with modern grating instruments. Recently, it has also become possible to perform Brillouin scattering using other excitations, in particular neutrons or X-rays. In these cases, the coupling does not occur *via* the Pockels effect, and the frequency shifts that are observed are much larger. The following discussion is restricted to optical interferometry.

The most common interferometer that has been used for this purpose is the single-pass planar Fabry–Perot (Born & Wolf, 1993). Upon illumination with monochromatic light, the frequency response of this instrument is given by the Airy function, which consists of a regular comb of maxima obtained as the optical path separating the mirrors is increased. Successive maxima are separated by  $\lambda/2$ . The ratio of the maxima separation to the width of a single peak is called the finesse  $F$ , which increases as the mirror reflectivity increases. The finesse is also limited by the planarity of the mirrors. A practical limit is  $F \sim 100$ . The resolving power of such an instrument is  $R = 2\ell/\lambda$ , where  $\ell$  is the optical thickness. Values of  $R$  around  $10^6$  to  $10^7$  can be achieved. It is impractical to increase  $\ell$  above  $\sim 5$  cm because the luminosity of the instrument is proportional to  $1/\ell$ . If higher

Table 2.4.5.1. Definition of Laue classes

Crystal system	Laue class	Point groups
Cubic	$C_1$ $C_2$	432, $\bar{4}3m$ , $m\bar{3}m$ 23, $\bar{3}m$
Hexagonal	$H_1$ $H_2$	622, $6mm$ , $\bar{6}2m$ , $6/mmm$ 6, $\bar{6}$ , $6/m$
Tetragonal	$T_1$ $T_2$	422, $4mm$ , $\bar{4}2m$ , $4/mmm$ 4, $\bar{4}$ , $4/m$
Trigonal	$R_1$ $R_2$	32, $3m$ , $\bar{3}m$ 3, $\bar{3}$
Orthorhombic	$O$	$mmm$ , $2mm$ , 222

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Table 2.4.5.2. *Cubic Laue classes  $C_1$  and  $C_2$ : longitudinal modes*

This table, written for the class  $C_2$ , is also valid for the class  $C_1$  with the additional relation  $p_{12} = p_{13}$ . It can also be used for the spherical system where  $c_{44} = \frac{1}{2}(c_{11} - c_{12})$ ,  $p_{44} = \frac{1}{2}(p_{11} - p_{12})$ .

$\hat{\mathbf{Q}} = \hat{\mathbf{u}}$	$C$	$\mathbf{e} = \mathbf{e}'$	$\beta$
(1, 0, 0)	$c_{11}$	(0, 1, 0)	$p_{13}^2/c_{11}$
(1, 0, 0)	$c_{11}$	(0, 0, 1)	$p_{12}^2/c_{11}$
(1, 1, 0)/ $\sqrt{2}$	$\frac{1}{2}(c_{11} + c_{12}) + c_{44}$	(0, 0, 1)	$(p_{12} + p_{13})^2/4C$
(1, 1, 0)/ $\sqrt{2}$	$\frac{1}{2}(c_{11} + c_{12}) + c_{44}$	(1, -1, 0)/ $\sqrt{2}$	$(2p_{11} + p_{12} + p_{13} - 4p_{44})^2/16C$
(1, 1, 1)/ $\sqrt{3}$	$\frac{1}{3}(c_{11} + 2c_{12} + 4c_{44})$	(1, 1, -2)/ $\sqrt{6}$	$(p_{11} + p_{12} + p_{13} - 2p_{44})^2/9C$
(1, 1, 1)/ $\sqrt{3}$	$\frac{1}{3}(c_{11} + 2c_{12} + 4c_{44})$	(1, -1, 0)/ $\sqrt{2}$	$(p_{11} + p_{12} + p_{13} - 2p_{44})^2/9C$

Table 2.4.5.3. *Tetragonal  $T_1$  and hexagonal  $H_1$  Laue classes: longitudinal modes*

This table, written for the class  $T_1$ , is also valid for the class  $H_1$  with the additional relations  $c_{66} = \frac{1}{2}(c_{11} - c_{12})$ ;  $p_{66} = \frac{1}{2}(p_{11} - p_{12})$ .

$\hat{\mathbf{Q}} = \hat{\mathbf{u}}$	$C$	$\mathbf{e} = \mathbf{e}'$	$\beta$
(1, 0, 0)	$c_{11}$	(0, 1, 0)	$p_{12}^2/c_{11}$
(1, 0, 0)	$c_{11}$	(0, 0, 1)	$p_{31}^2/c_{11}$
(0, 0, 1)	$c_{33}$	(1, 0, 0)	$p_{13}^2/c_{33}$
(0, 0, 1)	$c_{33}$	(0, 1, 0)	$p_{13}^2/c_{33}$
(1, 1, 0)/ $\sqrt{2}$	$\frac{1}{2}(c_{11} + c_{12}) + c_{66}$	(0, 0, 1)	$p_{31}^2/C$
(1, 1, 0)/ $\sqrt{2}$	$\frac{1}{2}(c_{11} + c_{12}) + c_{66}$	(1, -1, 0)/ $\sqrt{2}$	$(p_{11} + p_{12} - 2p_{66})^2/4C$

Table 2.4.5.4. *Hexagonal Laue class  $H_2$ : longitudinal modes*

$\hat{\mathbf{Q}} = \hat{\mathbf{u}}$	$C$	$\mathbf{e} = \mathbf{e}'$	$\beta$
(1, 0, 0)	$c_{11}$	(0, 1, 0)	$p_{12}^2/c_{11}$
(1, 0, 0)	$c_{11}$	(0, 0, 1)	$p_{31}^2/c_{11}$
(0, 0, 1)	$c_{33}$	(1, 0, 0)	$p_{13}^2/c_{33}$
(0, 0, 1)	$c_{33}$	(0, 1, 0)	$p_{13}^2/c_{33}$
(1, 1, 0)/ $\sqrt{2}$	$c_{11}$	(0, 0, 1)	$p_{31}^2/c_{11}$
(1, 1, 0)/ $\sqrt{2}$	$c_{11}$	(1, -1, 0)/ $\sqrt{2}$	$p_{12}^2/c_{11}$

Table 2.4.5.5. *Tetragonal Laue class  $T_2$ : longitudinal modes*

$\hat{\mathbf{Q}} = \hat{\mathbf{u}}$	$C$	$\mathbf{e} = \mathbf{e}'$	$\beta$
(0, 0, 1)	$c_{33}$	(1, 0, 0)	$p_{13}^2/c_{33}$
(0, 0, 1)	$c_{33}$	(0, 1, 0)	$p_{13}^2/c_{33}$

Table 2.4.5.6. *Orthorhombic Laue class  $O$ : longitudinal modes*

$\hat{\mathbf{Q}} = \hat{\mathbf{u}}$	$C$	$\mathbf{e} = \mathbf{e}'$	$\beta$
(1, 0, 0)	$c_{11}$	(0, 1, 0)	$p_{21}^2/c_{11}$
(1, 0, 0)	$c_{11}$	(0, 0, 1)	$p_{31}^2/c_{11}$
(0, 1, 0)	$c_{22}$	(0, 0, 1)	$p_{32}^2/c_{22}$
(0, 1, 0)	$c_{22}$	(1, 0, 0)	$p_{12}^2/c_{22}$
(0, 0, 1)	$c_{33}$	(1, 0, 0)	$p_{13}^2/c_{33}$
(0, 0, 1)	$c_{33}$	(0, 1, 0)	$p_{23}^2/c_{33}$

Table 2.4.5.7. *Trigonal Laue class  $R_1$ : longitudinal modes*

$\hat{\mathbf{Q}} = \hat{\mathbf{u}}$	$C$	$\mathbf{e}$	$\mathbf{e}'$	$\beta$
(1, 0, 0)	$c_{11}$	(0, 1, 0)	(0, 1, 0)	$p_{12}^2/c_{11}$
(1, 0, 0)	$c_{11}$	(0, 0, 1)	(0, 0, 1)	$p_{31}^2/c_{11}$
(1, 0, 0)	$c_{11}$	(0, 1, 0)	(0, 0, 1)	$p_{11}^2/c_{11}$
(0, 0, 1)	$c_{33}$	(1, 0, 0)	(1, 0, 0)	$p_{13}^2/c_{33}$
(0, 0, 1)	$c_{33}$	(0, 1, 0)	(0, 1, 0)	$p_{13}^2/c_{33}$

Table 2.4.5.8. *Trigonal Laue class  $R_2$ : longitudinal modes*

$\hat{\mathbf{Q}} = \hat{\mathbf{u}}$	$C$	$\mathbf{e}$	$\mathbf{e}'$	$\beta$
(0, 0, 1)	$c_{33}$	(1, 0, 0)	(1, 0, 0)	$p_{13}^2/c_{33}$
(0, 0, 1)	$c_{33}$	(0, 1, 0)	(0, 1, 0)	$p_{13}^2/c_{33}$

resolutions are required, one uses a spherical interferometer as described below.

A major limitation of the Fabry–Perot interferometer is its poor contrast, namely the ratio between the maximum and the minimum of the Airy function, which is typically  $\sim 1000$ . This limits the use of this instrument to samples of very high optical quality, as otherwise the generally weak Brillouin signals are masked by the elastically scattered light. To avert this effect, several passes are made through the same instrument, thus elevating the Airy function to the corresponding power (Hariharan & Sen, 1961; Sandercock, 1971). Multiple-pass instruments with three, four or five passes are common. Another limitation of the standard Fabry–Perot interferometer is that the interference pattern is repeated at each order. Hence, if the spectrum has a broad spectral spread, the overlap of adjacent orders can greatly complicate the interpretation of measurements. In this case, tandem instruments can be of considerable help. They consist of two Fabry–Perot interferometers with combs of different periods placed in series (Chantrel, 1959; Mach *et al.*, 1963). These are operated around a position where the peak transmission of the first interferometer coincides with that of the second one. The two Fabry–Perot interferometers are scanned simultaneously. With this setup, the successive orders are reduced to small ghosts and overlap is not a problem. A convenient commercial instrument has been designed by Sandercock (1982).

To achieve higher resolutions, one uses the spherical Fabry–Perot interferometer (Connes, 1958; Hercher, 1968). This consists

of two spherical mirrors placed in a near-confocal configuration. Their spacing  $\ell$  is scanned over a distance of the order of  $\lambda$ . The peculiarity of this instrument is that its luminosity increases with its resolution. One obvious drawback is that a change of resolving power, *i.e.* of  $\ell$ , requires other mirrors. Of course, the single spherical Fabry–Perot interferometer suffers the same limitations regarding contrast and order overlap that were discussed above for the planar case. Multipassing the spherical Fabry–Perot interferometer is possible but not very convenient. It is preferable to use tandem instruments that combine a multipass planar instrument of low resolution followed by a spherical instrument of high resolution (Pine, 1972; Vacher, 1972). To analyse the linewidth of narrow phonon lines, the planar standard is adjusted dynamically to transmit the Brillouin line and the spherical interferometer is scanned across the line. With such a device, resolving powers of  $\sim 10^8$  have been achieved. For the dynamical adjustment of this instrument one can use a reference signal near the frequency of the phonon line, which is derived by electro-optic modulation of the exciting laser (Sussner & Vacher, 1979). In this case, not only the width of the phonon, but also its absolute frequency shift, can be determined with an accuracy of  $\sim 1$  MHz. It is obvious that to achieve this kind of resolution, the laser source itself must be appropriately stabilized.

In closing, it should be stressed that the practice of interferometry is still an art that requires suitable skills and training in spite of the availability of commercial instruments. The experimenter must take care of a large number of aspects relating to the

## 2.4. BRILLOUIN SCATTERING

Table 2.4.5.9. *Cubic Laue classes  $C_1$  and  $C_2$ : transverse modes, backscattering*

This table, written for the class  $C_2$ , is also valid for the class  $C_1$  with the additional relation  $p_{12} = p_{13}$ .  
It can also be used for the spherical system where  $c_{44} = \frac{1}{2}(c_{11} - c_{12})$ ,  $p_{44} = \frac{1}{2}(p_{11} - p_{12})$ .

$\hat{\mathbf{Q}}$	$\hat{\mathbf{u}}$	$C$	$\mathbf{e}$	$\mathbf{e}'$	$\beta$
$(1, 1, 0)/\sqrt{2}$	$(1, -1, 0)/\sqrt{2}$	$\frac{1}{2}(c_{11} - c_{12})$	$(0, 0, 1)$	$(0, 0, 1)$	$(p_{12} - p_{13})^2/2(c_{11} - c_{12})$
$(1, 1, 1)/\sqrt{3}$	$D$	$\frac{1}{3}(c_{11} - c_{12} + c_{44})$	$(1, 1, -2)/\sqrt{6}$	$(1, -1, 0)/\sqrt{2}$	$[3(p_{12} - p_{13})^2 + (p_{12} + p_{13} + 4p_{44} - 2p_{11})^2]/72C$

Table 2.4.5.10. *Tetragonal  $T_1$  and hexagonal  $H_1$  Laue classes: transverse modes, backscattering*

This table, written for the class  $T_1$ , is also valid for the class  $H_1$  with the additional relations  $c_{66} = \frac{1}{2}(c_{11} - c_{12})$ ;  $p_{66} = \frac{1}{2}(p_{11} - p_{12})$ .

$\hat{\mathbf{Q}}$	$\hat{\mathbf{u}}$	$C$	$\mathbf{e}$	$\mathbf{e}'$	$\beta$
$(0, 1, 1)/\sqrt{2}$	$(1, 0, 0)$	$\frac{1}{2}(c_{44} + c_{66})$	$(1, 0, 0)$	$(0, 1, -1)/\sqrt{2}$	$[(n_1^2 + n_3^2)^2/16n_1^4n_3^4C](n_1^2p_{66} - n_3^2p'_{44})^2$

Table 2.4.5.11. *Hexagonal Laue class  $H_2$ : transverse modes, backscattering*

$$c_{66} = \frac{1}{2}(c_{11} - c_{12}); p_{66} = \frac{1}{2}(p_{11} - p_{12}).$$

$\hat{\mathbf{Q}}$	$\hat{\mathbf{u}}$	$C$	$\mathbf{e}$	$\mathbf{e}'$	$\beta$
$(1, 0, 0)$	$(0, 1, 0)$	$c_{66}$	$(0, 1, 0)$	$(0, 1, 0)$	$p_{16}^2/c_{66}$
$(1, 0, 0)$	$(0, 0, 1)$	$c_{44}$	$(0, 1, 0)$	$(0, 0, 1)$	$p_{45}^2/c_{44}$
$(0, 1, 1)/\sqrt{2}$	$(1, 0, 0)$	$\frac{1}{2}(c_{44} + c_{66})$	$(1, 0, 0)$	$(1, 0, 0)$	$p_{16}^2/(c_{44} + c_{66})$
$(0, 1, 1)/\sqrt{2}$	$(1, 0, 0)$	$\frac{1}{2}(c_{44} + c_{66})$	$(1, 0, 0)$	$(0, 1, -1)/\sqrt{2}$	$[(n_1^2 + n_3^2)^2/16n_1^4n_3^4C](n_1^2p_{66} - n_3^2p'_{44})^2$

Table 2.4.5.12. *Tetragonal Laue class  $T_2$ : transverse modes, backscattering*

$\hat{\mathbf{Q}}$	$\hat{\mathbf{u}}$	$C$	$\mathbf{e}$	$\mathbf{e}'$	$\beta$
$(1, 0, 0)$	$(0, 0, 1)$	$c_{44}$	$(0, 1, 0)$	$(0, 0, 1)$	$p_{45}^2/c_{44}$
$(1, 1, 0)/\sqrt{2}$	$(0, 0, 1)$	$c_{44}$	$(0, 0, 1)$	$(1, -1, 0)/\sqrt{2}$	$p_{45}^2/c_{44}$

Table 2.4.5.13. *Orthorhombic Laue class  $O$ : transverse modes, backscattering*

$\hat{\mathbf{Q}}$	$\hat{\mathbf{u}}$	$C$	$\mathbf{e}$	$\mathbf{e}'$	$\beta$
$(1, 1, 0)/\sqrt{2}$	$(0, 0, 1)$	$\frac{1}{2}(c_{44} + c_{55})$	$(0, 0, 1)$	$(1, -1, 0)/\sqrt{2}$	$[(n_1^2 + n_2^2)^2/16n_1^4n_2^4C](n_1^2p'_{55} - n_2^2p'_{44})^2$
$(0, 1, 1)/\sqrt{2}$	$(1, 0, 0)$	$\frac{1}{2}(c_{55} + c_{66})$	$(1, 0, 0)$	$(0, 1, -1)/\sqrt{2}$	$[(n_2^2 + n_3^2)^2/16n_2^4n_3^4C](n_2^2p'_{66} - n_3^2p'_{55})^2$
$(1, 0, 1)/\sqrt{2}$	$(0, 1, 0)$	$\frac{1}{2}(c_{44} + c_{66})$	$(0, 1, 0)$	$(-1, 0, 1)/\sqrt{2}$	$[(n_1^2 + n_3^2)^2/16n_1^4n_3^4C](n_3^2p'_{44} - n_1^2p'_{66})^2$

Table 2.4.5.14. *Trigonal Laue class  $R_1$ : transverse modes, backscattering*

$$c_{66} = \frac{1}{2}(c_{11} - c_{12}); p_{66} = \frac{1}{2}(p_{11} - p_{12}).$$

$\hat{\mathbf{Q}}$	$\hat{\mathbf{u}}$	$C$	$\mathbf{e}$	$\mathbf{e}'$	$\beta$
$(0, 1, 0)$	$(1, 0, 0)$	$c_{66}$	$(0, 0, 1)$	$(1, 0, 0)$	$p_{41}^2/c_{66}$
$(0, 0, 1)$	$D$	$c_{44}$	$(1, 0, 0)$	$(1, 0, 0)$	$p_{14}^2/c_{44}$
$(0, 0, 1)$	$D$	$c_{44}$	$(0, 1, 0)$	$(1, 0, 0)$	$p_{14}^2/c_{44}$
$(0, 1, 1)/\sqrt{2}$	$(1, 0, 0)$	$\frac{1}{2}(c_{44} + c_{66}) + c_{14}$	$(1, 0, 0)$	$(0, 1, -1)/\sqrt{2}$	$[(n_1^2 + n_3^2)^2/16n_1^4n_3^4C][n_1^2(p_{66} + p_{14}) - n_3^2(p'_{44} + p_{41})]^2$
$(0, 1, -1)/\sqrt{2}$	$(1, 0, 0)$	$\frac{1}{2}(c_{44} + c_{66}) - c_{14}$	$(1, 0, 0)$	$(0, 1, 1)/\sqrt{2}$	$[(n_1^2 + n_3^2)^2/16n_1^4n_3^4C][n_1^2(p_{66} - p_{14}) + n_3^2(p_{41} - p'_{44})]^2$

Table 2.4.5.15. *Trigonal Laue class  $R_2$ : transverse modes, backscattering*

$\hat{\mathbf{Q}}$	$\hat{\mathbf{u}}$	$C$	$\mathbf{e}$	$\mathbf{e}'$	$\beta$
$(0, 0, 1)$	$D$	$c_{44}$	$(1, 0, 0)$	$(1, 0, 0)$	$(p_{14}^2 + p_{15}^2)/c_{44}$
$(0, 0, 1)$	$D$	$c_{44}$	$(0, 1, 0)$	$(1, 0, 0)$	$(p_{14}^2 + p_{15}^2)/c_{44}$

## 2. SYMMETRY ASPECTS OF EXCITATIONS

Table 2.4.5.16. *Cubic Laue classes  $C_1$  and  $C_2$ : transverse modes, right-angle scattering*

This table, written for the class  $C_2$ , is also valid for the class  $C_1$  with the additional relation  $p_{12} = p_{13}$ .  
It can also be used for the spherical system where  $c_{44} = \frac{1}{2}(c_{11} - c_{12})$ ,  $p_{44} = \frac{1}{2}(p_{11} - p_{12})$ .

$\hat{\mathbf{Q}}$	$\hat{\mathbf{u}}$	$C$	Scattering plane	$\mathbf{e}$	$\mathbf{e}'$	$\beta$
(1, 0, 0)	$D$	$c_{44}$	(001)	(0, 0, 1)	$(1, -1, 0)/\sqrt{2}$	$p_{44}^2/2c_{44}$
(1, 0, 0)	$D$	$c_{44}$	(010)	(0, 1, 0)	$(1, 0, 1)/\sqrt{2}$	$p_{44}^2/2c_{44}$
$(1, 1, 0)/\sqrt{2}$	(0, 0, 1)	$c_{44}$	(001)	(0, 0, 1)	(1, 0, 0)	$p_{44}^2/2c_{44}$
$(1, 1, 0)/\sqrt{2}$	$(-1, 1, 0)/\sqrt{2}$	$\frac{1}{2}(c_{11} - c_{12})$	(001)	(0, 0, 1)	(0, 0, 1)	$(p_{12} - p_{13})^2/4C$
$(1, 1, 0)/\sqrt{2}$	$(-1, 1, 0)/\sqrt{2}$	$\frac{1}{2}(c_{11} - c_{12})$	(1-10)	$(1, -1, 0)/\sqrt{2}$	$(1, 1, -\sqrt{2})/2$	$(2p_{11} - p_{12} - p_{13})^2/32C$

Table 2.4.5.17. *Tetragonal  $T_1$  and hexagonal  $H_1$  Laue classes: transverse modes, right-angle scattering*

This table, written for the class  $T_1$ , is also valid for the class  $H_1$  with the additional relations  $c_{66} = \frac{1}{2}(c_{11} - c_{12})$ ;  $p_{66} = \frac{1}{2}(p_{11} - p_{12})$ .

$\hat{\mathbf{Q}}$	$\hat{\mathbf{u}}$	$C$	Scattering plane	$\mathbf{e}$	$\mathbf{e}'$	$\beta$
(1, 0, 0)	(0, 0, 1)	$c_{44}$	(001)	(0, 0, 1)	$(q_1^{(1)}, q_2^{(1)}, 0)$	$(q_1^{(1)} p_{44}^{\prime})^2/c_{44}$
(1, 0, 0)	(0, 1, 0)	$c_{66}$	(010)	(0, 1, 0)	$(q_1^{(2)}, 0, q_3^{(2)})$	$\{[(n_3 q_1^{(2)})^2 + (n_1 q_3^{(2)})^2]^2/n_3^4 c_{66}\} (q_1^{(2)} p_{66}^{\prime})^2$
(0, 0, 1)	$D$	$c_{44}$	(010)	(0, 1, 0)	$(q_1^{(5)}, 0, q_3^{(5)})$	$\{[(n_3 q_1^{(5)})^2 + (n_1 q_3^{(5)})^2]^2/n_3^4 c_{44}\} (q_3^{(5)} p_{44}^{\prime})^2$
$(1, 1, 0)/\sqrt{2}$	(0, 0, 1)	$c_{44}$	(001)	(0, 0, 1)	$(q_1^{(7)}, q_2^{(7)}, 0)$	$[(q_1^{(7)} + q_2^{(7)}) p_{44}^{\prime}]^2/2c_{44}$
$(1, 1, 0)/\sqrt{2}$	$(1, -1, 0)/\sqrt{2}$	$\frac{1}{2}(c_{11} - c_{12})$	(1-10)	$(1, -1, 0)/\sqrt{2}$	$(q_1^{(10)}, q_1^{(10)}, q_3^{(10)})$	$\{[2(n_3 q_1^{(10)})^2 + (n_1 q_3^{(10)})^2]^2/n_3^4 (c_{11} - c_{12})\} [q_1^{(10)} (p_{11} - p_{12})]^2$
$(0, 1, 1)/\sqrt{2}$	(1, 0, 0)	$\frac{1}{2}(c_{44} + c_{66})$	(100)	(1, 0, 0)	(0, 1, 0)	$p_{66}^2/(c_{44} + c_{66})$

Table 2.4.5.18. *Hexagonal  $H_2$  Laue class: transverse modes, right-angle scattering*

$c_{66} = \frac{1}{2}(c_{11} - c_{12})$ ;  $p_{66} = \frac{1}{2}(p_{11} - p_{12})$ .

$\hat{\mathbf{Q}}$	$\hat{\mathbf{u}}$	$C$	Scattering plane	$\mathbf{e}$	$\mathbf{e}'$	$\beta$
(1, 0, 0)	(0, 0, 1)	$c_{44}$	(001)	(0, 0, 1)	$(q_1^{(1)}, q_2^{(1)}, 0)$	$(q_1^{(1)} p_{44}^{\prime} + q_2^{(1)} p_{45}^{\prime})^2/c_{44}$
(1, 0, 0)	(0, 1, 0)	$c_{66}$	(001)	$(1, 1, 0)/\sqrt{2}$	$(1, -1, 0)/\sqrt{2}$	$p_{16}^2/c_{66}$
(1, 0, 0)	(0, 1, 0)	$c_{66}$	(010)	(0, 1, 0)	$(q_1^{(2)}, 0, q_3^{(2)})$	$\{[(n_3 q_1^{(2)})^2 + (n_1 q_3^{(2)})^2]^2/n_3^4 c_{66}\} (q_1^{(2)} p_{66}^{\prime})^2$
(0, 0, 1)	$D$	$c_{44}$	(010)	(0, 1, 0)	$(q_1^{(5)}, 0, q_3^{(5)})$	$\{[(n_3 q_1^{(5)})^2 + (n_1 q_3^{(5)})^2]^2/n_3^4 c_{44}\} (q_3^{(5)} p_{44}^{\prime})^2 + p_{45}^2$
$(0, 1, 1)/\sqrt{2}$	(1, 0, 0)	$\frac{1}{2}(c_{44} + c_{66})$	(100)	(1, 0, 0)	(1, 0, 0)	$p_{16}^2/(c_{44} + c_{66})$
$(0, 1, 1)/\sqrt{2}$	(1, 0, 0)	$\frac{1}{2}(c_{44} + c_{66})$	(100)	(1, 0, 0)	(0, 1, 0)	$p_{66}^2/(c_{44} + c_{66})$

Table 2.4.5.19. *Tetragonal  $T_2$  Laue class: transverse modes, right-angle scattering*

$\hat{\mathbf{Q}}$	$\hat{\mathbf{u}}$	$C$	Scattering plane	$\mathbf{e}$	$\mathbf{e}'$	$\beta$
(1, 0, 0)	(0, 0, 1)	$c_{44}$	(001)	(0, 0, 1)	$(q_1^{(1)}, q_2^{(1)}, 0)$	$(q_1^{(1)} p_{44}^{\prime} + q_2^{(1)} p_{45}^{\prime})^2/c_{44}$
(1, 0, 0)	(0, 0, 1)	$c_{44}$	(010)	(0, 1, 0)	$(q_1^{(2)}, 0, q_3^{(2)})$	$\{[(n_3 q_1^{(2)})^2 + (n_1 q_3^{(2)})^2]^2/n_3^4 c_{44}\} (q_3^{(2)} p_{45}^{\prime})^2$
(0, 0, 1)	$D$	$c_{44}$	(010)	(0, 1, 0)	$(q_1^{(5)}, 0, q_3^{(5)})$	$\{[(n_3 q_1^{(5)})^2 + (n_1 q_3^{(5)})^2]^2/n_3^4 c_{44}\} (q_3^{(5)} p_{44}^{\prime})^2 + p_{45}^2$
$(1, 1, 0)/\sqrt{2}$	(0, 0, 1)	$c_{44}$	(001)	(0, 0, 1)	$(q_1^{(7)}, q_2^{(7)}, 0)$	$[(q_1^{(7)} + q_2^{(7)}) p_{44}^{\prime} + (q_2^{(7)} - q_1^{(7)}) p_{45}^{\prime}]^2/2c_{44}$

Table 2.4.5.20. *Orthorhombic Laue class  $O$ : transverse modes, right-angle scattering*

$\hat{\mathbf{Q}}$	$\hat{\mathbf{u}}$	$C$	Scattering plane	$\mathbf{e}$	$\mathbf{e}'$	$\beta$
(1, 0, 0)	(0, 0, 1)	$c_{55}$	(001)	(0, 0, 1)	$(q_1^{(1)}, q_2^{(1)}, 0)$	$\{[(n_2 q_1^{(1)})^2 + (n_1 q_2^{(1)})^2]^2/n_2^4 c_{55}\} (q_1^{(1)} p_{55}^{\prime})^2$
(1, 0, 0)	(0, 1, 0)	$c_{66}$	(010)	(0, 1, 0)	$(q_1^{(2)}, 0, q_3^{(2)})$	$\{[(n_3 q_1^{(2)})^2 + (n_1 q_3^{(2)})^2]^2/n_3^4 c_{66}\} (q_1^{(2)} p_{66}^{\prime})^2$
(0, 1, 0)	(1, 0, 0)	$c_{66}$	(100)	(1, 0, 0)	$(0, q_2^{(3)}, q_3^{(3)})$	$\{[(n_3 q_2^{(3)})^2 + (n_2 q_3^{(3)})^2]^2/n_3^4 c_{66}\} (q_3^{(3)} p_{66}^{\prime})^2$
(0, 1, 0)	(0, 0, 1)	$c_{44}$	(001)	(0, 0, 1)	$(q_1^{(4)}, q_2^{(4)}, 0)$	$\{[(n_2 q_1^{(4)})^2 + (n_1 q_2^{(4)})^2]^2/n_2^4 c_{44}\} (q_2^{(4)} p_{44}^{\prime})^2$
(0, 0, 1)	(0, 1, 0)	$c_{44}$	(010)	(0, 1, 0)	$(q_1^{(5)}, 0, q_3^{(5)})$	$\{[(n_3 q_1^{(5)})^2 + (n_1 q_3^{(5)})^2]^2/n_3^4 c_{44}\} (q_3^{(5)} p_{44}^{\prime})^2$
(0, 0, 1)	(1, 0, 0)	$c_{55}$	(100)	(1, 0, 0)	$(0, q_2^{(6)}, q_3^{(6)})$	$\{[(n_3 q_2^{(6)})^2 + (n_2 q_3^{(6)})^2]^2/n_3^4 c_{55}\} (q_3^{(6)} p_{55}^{\prime})^2$
$(1, 1, 0)/\sqrt{2}$	(0, 0, 1)	$\frac{1}{2}(c_{44} + c_{55})$	(001)	(0, 0, 1)	$(q_1^{(7)}, q_2^{(7)}, 0)$	$\{[(n_2 q_1^{(7)})^2 + (n_1 q_2^{(7)})^2]^2/n_2^4 n_3^4 (c_{44} + c_{55})\} \times (n_1^2 q_1^{(7)} p_{55}^{\prime} + n_2^2 q_2^{(7)} p_{44}^{\prime})^2$
$(0, 1, 1)/\sqrt{2}$	(1, 0, 0)	$\frac{1}{2}(c_{55} + c_{66})$	(100)	(1, 0, 0)	$(0, q_2^{(8)}, q_3^{(8)})$	$\{[(n_3 q_2^{(8)})^2 + (n_2 q_3^{(8)})^2]^2/n_3^4 n_2^4 (c_{55} + c_{66})\} \times (n_2^2 q_2^{(8)} p_{66}^{\prime} + n_3^2 q_3^{(8)} p_{55}^{\prime})^2$
$(1, 0, 1)/\sqrt{2}$	(0, 1, 0)	$\frac{1}{2}(c_{44} + c_{66})$	(010)	(0, 1, 0)	$(q_1^{(9)}, 0, q_3^{(9)})$	$\{[(n_1 q_3^{(9)})^2 + (n_3 q_1^{(9)})^2]^2/n_1^4 n_3^4 (c_{44} + c_{66})\} \times (n_3^2 q_3^{(9)} p_{44}^{\prime} + n_1^2 q_1^{(9)} p_{66}^{\prime})^2$

## 2.4. BRILLOUIN SCATTERING

Table 2.4.5.21. *Trigonal Laue class R<sub>1</sub>: transverse modes, right-angle scattering*

$$c_{66} = \frac{1}{2}(c_{11} - c_{12}); p_{66} = \frac{1}{2}(p_{11} - p_{12}).$$

$\hat{\mathbf{Q}}$	$\hat{\mathbf{u}}$	$C$	Scattering plane	$\mathbf{e}$	$\mathbf{e}'$	$\beta$
(0, 1, 0)	(1, 0, 0)	$c_{66}$	(100)	(1, 0, 0)	$(0, q_2^{(3)}, q_3^{(3)})$	$\{(n_3 q_2^{(3)})^2 + (n_1 q_3^{(3)})^2\} / n_1^4 n_3^4 c_{66} (n_2^2 q_2^{(3)} p_{66} + n_3^2 q_3^{(3)} p_{41})^2$
(0, 1, 0)	(1, 0, 0)	$c_{66}$	(001)	(0, 0, 1)	$(q_1^{(4)}, q_2^{(4)}, 0)$	$(q_1^{(4)} p_{41})^2 / c_{66}$
(0, 0, 1)	$D$	$c_{44}$	(010)	(0, 1, 0)	(0, 1, 0)	$p_{14}^2 / c_{44}$
(0, 0, 1)	$D$	$c_{44}$	(010)	(0, 1, 0)	$(q_1^{(5)}, 0, q_3^{(5)})$	$\{(n_3 q_1^{(5)})^2 + (n_1 q_3^{(5)})^2\} / n_1^4 n_3^4 c_{44} [n_2^4 (q_1^{(5)} p_{14})^2 + n_3^4 (q_3^{(5)} p_{44}')^2]$
$(0, 1, 1) / \sqrt{2}$	(1, 0, 0)	$\frac{1}{2}(c_{44} + c_{66}) + c_{14}$	(100)	(1, 0, 0)	(0, 1, 0)	$(p_{66} + p_{14})^2 / 2C$
$(0, -1, 1) / \sqrt{2}$	(1, 0, 0)	$\frac{1}{2}(c_{44} + c_{66}) - c_{14}$	(100)	(1, 0, 0)	(0, 1, 0)	$(p_{66} - p_{14})^2 / 2C$

Table 2.4.5.22. *Trigonal Laue class R<sub>2</sub>: transverse modes, right-angle scattering*

$$c_{66} = \frac{1}{2}(c_{11} - c_{12}); p_{66} = \frac{1}{2}(p_{11} - p_{12}).$$

$\hat{\mathbf{Q}}$	$\hat{\mathbf{u}}$	$C$	Scattering plane	$\mathbf{e}$	$\mathbf{e}'$	$\beta$
(0, 0, 1)	$D$	$c_{44}$	(010)	(0, 1, 0)	(0, 1, 0)	$p_{14}^2 / c_{44}$
(0, 0, 1)	$D$	$c_{44}$	(010)	(0, 1, 0)	$(q_1^{(5)}, 0, q_3^{(5)})$	$\{(n_3 q_1^{(5)})^2 + (n_1 q_3^{(5)})^2\} / n_1^4 n_3^4 c_{44} [n_2^4 (q_1^{(5)} p_{14})^2 + n_3^4 (q_3^{(5)} p_{44}')^2]$

Table 2.4.5.23. *Particular directions of incident light used in Tables 2.4.5.17 to 2.4.5.22*

$$\varepsilon_1 = (n_2 + n_3 - 2n_1) / 4n_1, \varepsilon_2 = (n_1 + n_3 - 2n_2) / 4n_2, \varepsilon_3 = (n_1 + n_2 - 2n_3) / 4n_3.$$

Notation	$q_1$	$q_2$	$q_3$
$\mathbf{q}^{(1)}$	$-2^{(-1/2)}(1 - \varepsilon_3)$	$2^{(-1/2)}(1 + \varepsilon_3)$	0
$\mathbf{q}^{(2)}$	$-2^{(-1/2)}(1 - \varepsilon_2)$	0	$2^{(-1/2)}(1 + \varepsilon_2)$
$\mathbf{q}^{(3)}$	0	$-2^{(-1/2)}(1 - \varepsilon_1)$	$2^{(-1/2)}(1 + \varepsilon_1)$
$\mathbf{q}^{(4)}$	$2^{(-1/2)}(1 + \varepsilon_3)$	$-2^{(-1/2)}(1 - \varepsilon_3)$	0
$\mathbf{q}^{(5)}$	$2^{(-1/2)}(1 + \varepsilon_2)$	0	$-2^{(-1/2)}(1 + \varepsilon_2)$
$\mathbf{q}^{(6)}$	0	$2^{(-1/2)}(1 + \varepsilon_1)$	$-2^{(-1/2)}(1 - \varepsilon_1)$
$\mathbf{q}^{(7)}$	$-2^{(-1/2)}(n_1 + n_3)(n_1^2 + n_3^2)^{(-1/2)}$	$2^{(-1/2)}(n_1 - n_3)(n_1^2 + n_3^2)^{(-1/2)}$	0
$\mathbf{q}^{(8)}$	0	$-2^{(-1/2)}(n_1 + n_2)(n_1^2 + n_2^2)^{(-1/2)}$	$2^{(-1/2)}(n_2 - n_1)(n_1^2 + n_2^2)^{(-1/2)}$
$\mathbf{q}^{(9)}$	$2^{(-1/2)}(n_3 - n_2)(n_2^2 + n_3^2)^{(-1/2)}$	0	$-2^{(-1/2)}(n_2 + n_3)(n_2^2 + n_3^2)^{(-1/2)}$
$\mathbf{q}^{(10)}$	$-\frac{1}{2}(1 - \varepsilon_2)$	$-\frac{1}{2}(1 - \varepsilon_2)$	$2^{(-1/2)}(1 + \varepsilon_2)$

optical setup, the collection and acceptance angles of the instruments, spurious reflections and spurious interferences, etc. A full list is too long to be given here. However, when properly executed, interferometry is a fine tool, the performance of which is unequalled in its frequency range.

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