Comparing (2.4.2.11) and (2.4.2.2), one sees that the effective elastic tensor $\mathbf{c}^{(e)}$ now depends on the propagation direction $\hat{\mathbf{Q}}$. Otherwise, all considerations of the previous section, starting from (2.4.2.6), remain, with $\mathbf{c}$ simply replaced by $\mathbf{c}^{(e)}$.

2.4.3. Coupling of light with elastic waves

2.4.3.1. *Direct coupling to displacements*

The change in the relative optical dielectric tensor $\boldsymbol{\kappa}$ produced by an elastic wave is usually expressed in terms of the strain, using the Pockels piezo-optic tensor $\mathbf{p}$, as

$$(\Delta\kappa^{-1})_{ij} = p_{ijk\ell}S_{k\ell}. \qquad (2.4.3.1)$$

The elastic wave should, however, be characterized by both strain $\mathbf{S}$ *and* rotation $\mathbf{A}$ (Nelson & Lax, 1971; see also Section 1.3.1.3):

$$A_{[k\ell]} = \tfrac{1}{2}\left(\frac{\partial u_k}{\partial x_\ell} - \frac{\partial u_\ell}{\partial x_k}\right). \qquad (2.4.3.2)$$

The square brackets on the left-hand side are there to emphasize that the component is antisymmetric upon interchange of the indices, $A_{[k\ell]} = -A_{[\ell k]}$. For birefringent crystals, the rotations induce a change of the local $\boldsymbol{\kappa}$ in the laboratory frame. In this case, (2.4.3.1) must be replaced by

$$(\Delta\kappa^{-1})_{ij} = p'_{ijk\ell}\frac{\partial u_k}{\partial x_\ell}, \qquad (2.4.3.3)$$

where $\mathbf{p}'$ is the new piezo-optic tensor given by

$$p'_{ijk\ell} = p_{ijk\ell} + p_{ij[k\ell]}. \qquad (2.4.3.4)$$

One finds for the rotational part

$$p_{ij[k\ell]} = \tfrac{1}{2}[(\kappa^{-1})_{i\ell}\delta_{kj} + (\kappa^{-1})_{\ell j}\delta_{ik} - (\kappa^{-1})_{ik}\delta_{\ell j} - (\kappa^{-1})_{kj}\delta_{i\ell}]. \qquad (2.4.3.5)$$

If the principal axes of the dielectric tensor coincide with the crystallographic axes, this gives

$$p_{ij[k\ell]} = \tfrac{1}{2}(\delta_{i\ell}\delta_{kj} - \delta_{ik}\delta_{\ell j})(1/n_i^2 - 1/n_j^2). \qquad (2.4.3.6)$$

This is the expression used in this chapter, as monoclinic and triclinic groups are not listed in the tables below.

For the calculation of the Brillouin scattering, it is more convenient to use

$$(\Delta\kappa)_{mn} = -\kappa_{mi}\kappa_{nj}p'_{ijk\ell}\frac{\partial u_k}{\partial x_\ell}, \qquad (2.4.3.7)$$

which is valid for small $\Delta\kappa$.

2.4.3.2. *Coupling via the electro-optic effect*

Piezoelectric media also exhibit an electro-optic effect linear in the applied electric field or in the field-induced crystal polarization. This effect is described in terms of the third-rank electro-optic tensor $\mathbf{r}$ defined by

$$(\Delta\kappa^{-1})_{ij} = r_{ijm}E_m. \qquad (2.4.3.8)$$

Using the same approach as in (2.4.2.10), for long waves E_m can be expressed in terms of $S_{k\ell}$, and (2.4.3.8) leads to an effective Pockels tensor $\mathbf{p}^e$ accounting for both the piezo-optic and the electro-optic effects:

$$p^e_{ijk\ell} = p_{ijk\ell} - \frac{r_{ijm}e_{nk\ell}\hat{Q}_m\hat{Q}_n}{\varepsilon_{gh}\hat{Q}_g\hat{Q}_h}. \qquad (2.4.3.9)$$

The total change in the inverse dielectric tensor is then

$$(\Delta\kappa^{-1})_{ij} = (p^e_{ijk\ell} + p_{ij[k\ell]})\frac{\partial u_k}{\partial x_\ell} = p'_{ijk\ell}\frac{\partial u_k}{\partial x_\ell}. \qquad (2.4.3.10)$$

The same equation (2.4.3.7) applies.

2.4.4. Brillouin scattering in crystals

2.4.4.1. *Kinematics*

Brillouin scattering occurs when an incident photon at frequency ν_i interacts with the crystal to either produce or absorb an acoustic phonon at $\delta\nu$, while a scattered photon at ν_s is simultaneously emitted. Conservation of energy gives

$$\delta\nu = \nu_s - \nu_i, \qquad (2.4.4.1)$$

where positive $\delta\nu$ corresponds to the anti-Stokes process. Conservation of momentum can be written

$$\mathbf{Q} = \mathbf{k}_s - \mathbf{k}_i, \qquad (2.4.4.2)$$

where $\mathbf{Q}$ is the wavevector of the emitted phonon, and $\mathbf{k}_s$, $\mathbf{k}_i$ are those of the scattered and incident photons, respectively. One can define unit vectors $\mathbf{q}$ in the direction of the wavevectors $\mathbf{k}$ by

$$\mathbf{k}_i = 2\pi\mathbf{q}n/\lambda_0, \qquad (2.4.4.3a)$$

$$\mathbf{k}_s = 2\pi\mathbf{q}'n'/\lambda_0, \qquad (2.4.4.3b)$$

where n and n' are the appropriate refractive indices, and λ_0 is the vacuum wavelength of the radiation. Equation (2.4.4.3*b*) assumes that $\delta\nu \ll \nu_i$ so that λ_0 is not appreciably changed in the scattering. The incident and scattered waves have unit polarization vectors $\mathbf{e}$ and $\mathbf{e}'$, respectively, and corresponding indices n and n'. The polarization vectors are the principal directions of vibration derived from the sections of the ellipsoid of indices by planes perpendicular to $\mathbf{q}$ and $\mathbf{q}'$, respectively. We assume that the electric vector of the light field $\mathbf{E}_{\rm opt}$ is parallel to the displacement $\mathbf{D}_{\rm opt}$. This is exactly true for many cases listed in the tables below. In the other cases (such as skew directions in the orthorhombic group) this assumes that the birefringence is sufficiently small for the effect of the angle between $\mathbf{E}_{\rm opt}$ and $\mathbf{D}_{\rm opt}$ to be negligible. A full treatment, including this effect, has been given by Nelson *et al.* (1972).

After substituting (2.4.4.3) in (2.4.4.2), the unit vector in the direction of the phonon wavevector is given by

$$\hat{\mathbf{Q}} = \frac{n'\mathbf{q}' - n\mathbf{q}}{|n'\mathbf{q}' - n\mathbf{q}|}. \qquad (2.4.4.4)$$

The Brillouin shift $\delta\nu$ is related to the phonon velocity V by

$$\delta\nu = VQ/2\pi. \qquad (2.4.4.5)$$

Since $\nu\lambda_0 = c$, from (2.4.4.5) and (2.4.4.3), (2.4.4.4) one finds

$$\delta\nu \cong (V/\lambda_0)[n^2 + (n')^2 - 2nn'\cos\theta]^{1/2}, \qquad (2.4.4.6)$$

where θ is the angle between $\mathbf{q}$ and $\mathbf{q}'$.

2.4.4.2. *Scattering cross section*

The power $dP_{\rm in}$, scattered from the illuminated volume V in a solid angle $d\Omega_{\rm in}$, where $P_{\rm in}$ and $\Omega_{\rm in}$ are measured inside the sample, is given by

$$\frac{dP_{\rm in}}{d\Omega_{\rm in}} = V\frac{k_BT\pi^2n'}{2n\lambda_0^4C}MI_{\rm in}, \qquad (2.4.4.7)$$

where $I_{\rm in}$ is the incident light intensity inside the material, $C = \rho V^2$ is the appropriate elastic constant for the observed phonon, and the factor k_BT results from taking the fluctuation–